ADJUSTING INDICATED INSURANCE RATES: FUZZY RULES THAT CONSIDER BOTH EXPERIENCE AND AUXILIARY DATA

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Abstract

This paper describes how an actuary can use fuzzy logic to adjust insurance rates by considering both claim experience data and supplementary information. This supplementary data may be financial or marketing data or statements that reflect the philosophy of the actuary's company or client. The paper shows how to build and fine-tune a rate-making model by using workers compensation insurance data from an insurance company.

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1. INTRODUCTION

Through the education programs of the Society of Actuaries and the Casualty Actuarial Society, actuaries are equipped with statistical tools to analyze experience data and to determine necessary rate changes for their insurance products. Students are often surprised to learn that those rate changes are frequently not accepted "as is" by company management. Actuaries work with sales, marketing, and underwriting personnel to develop rates that will be competitive and adequate.

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Actuaries frequently consider statistical data specific to rates, such as the results of experience studies. In setting premiums, actuaries also consider constraints that supplement experience data. These constraints may reflect company philosophy, such as "We wish to increase our market share *moderately* from year to year." They may also include financial data, such as "Raise the rates if we experience *high* loss ratios or *low* profit margins."

The theory of fuzzy sets provides a natural setting in which to handle such statements. Through fuzzy sets, one can account for vague notions whose boundaries are not clearly defined, such as "*large* amount of business." Fuzzy logic provides a uniform way to handle such factors that influence the indicated rate change (Zadeh [20]). A fuzzy logic system is a type of expert system. An advantage of using a fuzzy logic system is that it provides a systematic way to develop mathematical rules from linguistic ones. This paper describes step-by-step how an actuary can adjust rates by beginning with linguistic rules that consider both experience data and supplementary information.

Fuzzy sets have only recently been applied to problems in actuarial science. DeWit [5] and Lemaire [13] show how to apply fuzzy sets in individual underwriting, and Young [16] indicates how to use fuzzy sets in group health underwriting. Ostaszewski [15] suggests several areas in actuarial science in which fuzzy sets may prove useful. Cummins and Derrig [2] apply a form of fuzzy logic to calculate fuzzy trends in property-liability insurance. Derrig and Ostaszewski [4] employ fuzzy clustering in risk classification and provide an example in automobile insurance. Cummins and Derrig [3] use fuzzy arithmetic in pricing property-liability insurance. In an earlier paper [18], I show how to develop a fuzzy logic model with which actuaries can adjust insurance rates by considering only constraints or information that are ancillary to experience data.

Section 2 introduces fuzzy sets and defines operators corresponding to the linguistic connectors *and* and *or* and the modifier

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not. It also describes a simple fuzzy inference system. References for fuzzy sets include Dubois and Prade [7], Kosko [12], and Zadeh [19]. Some references for fuzzy logic and fuzzy inference are Bellman and Zadeh [1], Driankov et al. [6], Kandel and Langholz [9], Klir and Folger [11], Mamdani [14], Zadeh [20], and Zimmermann [21].

Section 3 describes how to construct and fine-tune a pricing model using fuzzy inference. Section 4 shows how to build a pricing model using workers compensation insurance data from an insurance company. Finally, Section 5 summarizes the paper's key points.

2. FUZZY INFERENCE

Fuzzy sets describe concepts that are vague (Zadeh [19]). The fuzziness of a set arises from the lack of well-defined boundaries. This lack is due to the imprecise nature of language; that is, objects can possess an attribute to various degrees. A fuzzy set corresponding to a given characteristic assigns a value to an object, the degree to which the object possesses the attribute.

Examples of fuzzy sets encountered in insurance pricing are *stable* rates, *large* profits, and *small* amounts of business renewed or written. Indeed, rates can be *stable* to different degrees depending on the relative or absolute changes in the premium rate. Also, profits can be *large* to different degrees depending on the relative or absolute amount of profits.

Fuzzy sets generalize nonfuzzy, or crisp, sets. A crisp set, C, is given by a characteristic function:

$$\chi_C : X \to \{0,1\},$$

in which $\chi_C(x) = 1$ if x is in C; otherwise, $\chi_C(x) = 0$. Fuzzy sets recognize that objects can belong to a given set to different degrees. They essentially expand the notion of set to allow partial membership in a set.

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DEFINITION 2.1 *A fuzzy set, A, in a universe of discourse, X, is a function m_A on X that takes values in the unit interval* [0,1]*:*

$$m_A : X \to [0,1].$$

The function m_A is called the membership function of A, and for any x in X, $m_A(x)$ in [0,1] *represents the grade of membership of x in A.*

EXAMPLE 2.1 One may define *stable* rates by the following hypothetical fuzzy set:

$$m_{stable}(r) = \begin{cases} 0, & \text{if} \quad r < -0.10, \\ \dfrac{r+0.10}{0.05}, & \text{if} \quad -0.10 \le r < -0.05, \\ 1, & \text{if} \quad -0.05 \le r < 0.05, \\ \dfrac{0.10-r}{0.05}, & \text{if} \quad 0.05 \le r < 0.10, \\ 0, & \text{if} \quad r \ge 0.10, \end{cases}$$

in which r is the relative rate change. For example, the degree to which a rate increase of 8% is *stable* is 0.40. It does *not* mean, however, that one will view an 8% rate increase as stable 40% of the time and unstable the rest of the time. See Figure 1 for the graph of this fuzzy set. The points ± 0.05 and ± 0.10 depend on the line of business. Also, one may want to use a fuzzy set that is not necessarily piecewise linear.

We now define three basic operations on fuzzy sets.

DEFINITION 2.2 *The union, $A \cup B$, of two fuzzy sets, A and B, is given by*

$$m_{A \cup B}(x) \equiv \max[m_A(x), m_B(x)], \qquad x \in X,$$

and the intersection, $A \cap B$, is given by

$$m_{A \cap B}(x) \equiv \min[m_A(x), m_B(x)], \qquad x \in X.$$

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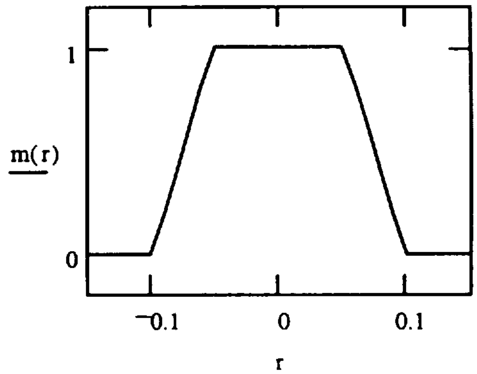
FIGURE 1

GRAPH OF FUZZY SET OF STABLE RATES, EXAMPLE 2.1



The complement, $-A$, of fuzzy set A is given by

$$m_{-A}(x) \equiv 1 - m_A(x), \qquad x \in X.$$

The union operation acts as an *or* operator, the intersection operation acts as *and*, and the complement operation acts as *not*. Thus, for example, $m_{A \cap B}(x)$ represents the degree to which x is a member of both A and B. The given definitions are not the only acceptable ones for these operations. Klir and Folger [11] specify axioms that union, intersection, and complement satisfy. Also, Dubois and Prade [7] and Young [16] discuss alternative operators. One in particular is the intersection operator called the algebraic product. The algebraic product of two fuzzy sets A and B is given by

$$m_{AB}(x) = m_A(x) \cdot m_B(x).$$

The algebraic product allows the fuzzy sets to interact in the intersection. That is, both fuzzy sets contribute to the value of the intersection, as opposed to the min operator in which the minimum of the two values determines the value of the intersection.

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