FUZZY INSURANCE

By Jean Lemaire

Wharton School
University of Pennsylvania, U.S.A.

Abstract

Fuzzy set theory is a recently developed field of mathematics, that introduces sets of objects whose boundaries are not sharply defined. Whereas in ordinary Boolean algebra an element is either contained or not contained in a given set, in fuzzy set theory the transition between membership and non-membership is gradual. The theory aims at modelizing situations described in vague or imprecise terms, or situations that are too complex or ill-defined to be analysed by conventional methods. This paper aims at presenting the basic concepts of the theory in an insurance framework. First the basic definitions of fuzzy logic are presented, and applied to provide a flexible definition of a "preferred policyholder" in life insurance. Next, fuzzy decision-making procedures are illustrated by a reinsurance application, and the theory of fuzzy numbers is extended to define fuzzy insurance premiums.

Keywords

Fuzzy set theory; preferred policyholders in life insurance, optimal XL-retentions; net single premiums for pure endowment insurance

1 INTRODUCTION

In 1965, Zadeh published a paper entitled "Fuzzy Sets" in a little known journal, Information and Control, introducing for the first time sets of objects whose boundaries are not sharply defined. This paper gave rise to an enormous interest among researchers, and initiated the fulgurant growth of a new discipline of mathematics, fuzzy set theory. The number of papers related to the field exploded from 240 in 1975 (Zadeh et al.), to 760 in 1977 (Gupta et al.), 2500 in 1980 (Chen et al.), and 5000 in 1987 (Zimmerman). Today, there are many more researchers in fuzzy set theory than in actuarial science, and they form a much more international group, with important contributions from China, Japan, and the Soviet Union. Two monthy scientific journals publish new theoretical developments and applications, that are to be found in linguistics, risk analysis, artificial intelligence (approximate reasoning, expert systems), pattern analysis and classification (pattern recognition, clustering, image processing, computer vision), information processing, and decision-making. In this paper we will explore some possible applications of fuzzy set theory to insurance.

In ordinary Boolean algebra, an element is either contained or not contained in a given set. the transition from membership to non-membership is abrupt. Fuzzy sets, on the other hand, describe sets of elements or variables whose limits are ill-defined or imprecise The transition between membership and non-membership is gradual: an element can "more or less" belong to a set Consider for instance the set of "young drivers". In Boolean algebra, it is assumed that any individual either belongs or does not belong to the set of young drivers. This implies that the individual will move from the category of "young drivers" to the complementary set of "not young drivers" overnight Fuzzy set theory allows for grades of membership. Depending on the specific application, one might for instance decide that drivers under 20 are definitely young, that drivers over 30 are definitely not young, and that a 23-year-old driver is "more or less" young, or is young with a grade membership of 0.7, on a scale from 0 to 1

Fuzzy set theory thus aims at modelizing imprecise, vague, fuzzy information, which abound in real world situations. Indeed, many practical problems are extremely complex and ill-designed, hence difficult to modelize with precision To quote ZADEH, "as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance become almost exclusive characteristics" Computers cannot adequately handle such problems, because machine intelligence still employs sequential (Boolean) logic. The superiority of the human brain results from its capacity of handling fuzzy statements and decisions, by adding to logic parallel and simultaneous information sources and thinking processes, and by filtering and selecting only those that are useful and relevant to its purposes. The human brain has many more thinking processes available and has developed a far greater filtering capacity than the machine A group of individuals is able to resolve the command "tall people in the back, short people in the front", a machine is not Fuzzy set theory explicitly introduces vagueness in the reasoning, hoping to provide decision-making procedures that are closer to the way the human brain performs.

A clear distinction has to be made between fuzzy sets and probability theory. Uncertainty should not be confused with imprecision Probabilities are primarily intended to represent a degree of knowledge about real entities, while the degrees of membership defining the strength of participation of an entity in a class are the representation of the degree by which a proposition is partially true Probability concepts are derived from considerations about the uncertainty of propositions about the real world Fuzzy concepts are closely related to the multivalued logic treatments of issues of imprecision in the definition of entities Hence, fuzzy set theory provides a better framework than probability theory for modelling problems that have some inherent imprecision The traditional approach to risk analysis, for instance, is based on the premise that probability theory provides the necessary and sufficient tools for dealing with the uncertainty and imprecision which underline the concept of risk in decision analysis The theory of fuzzy sets calls into question the validity of this premise. It does not equate imprecision with randomness It suggests that much of the uncertainty which is intrinsic in risk analysis is rooted in the fuzziness of the information which is resident in the data base and in the imprecision of the underlying probabilities. Classical probability theory has its effectiveness limited when dealing with problems in which some of the principal sources of uncertainty are non-statistical in nature

In the sequel we will present the basic principles of fuzzy logic, fuzzy decision-making, and fuzzy arithmetics, while developing three insurance examples We will show that fuzzy set theory could provide decision procedures that are much more flexible than those originating from conventional set theory Indeed, insurance executives and actuaries, much better trained to deal with uncertainty than with vagueness, have often transformed imprecise statements into "all-or-nothing" rules. For instance, Belgian insurers have used the fuzzy statistical evidence "Young drivers provoke more automobile accidents" to set up the a posteriori rating rule "Drivers under 23 years of age will pay a $ 150 deductible if they provoke an accident". Hence "young" was equated with "under 23", a definite distorsion of the initial statement As another example, Belgian regulatory authorities define, for statistical purposes, a "severely wounded person" as "any person, wounded in an automobile accident, whose condition requires a hospital stay longer than 24 hours", a very arguable "de-fuzzification" of a fuzzy health condition

In Section 2 we will present the basic definitions of fuzzy logic and apply them to provide a more flexible definition of a "preferred policyholder" than the one currently used by some American life insurers Section 3 introduces the main concepts of fuzzy decision-making, and uses them to select an optimal Excess of Loss retention. Fuzzy arithmetics are presented in Section 4, and applied to compute the fuzzy premium of a pure endowment policy

First, let us introduce our three examples.

Problem 1 Definition of a preferred policyholder in life insurance

Heavy competition between American life insurers has resulted in a greater subdivison of policyholders than in Europe U.S. insurers first began, in the mid 1960s, to award substantial discounts to nonsmokers purchasing a term or a whole life insurance. Then the "preferred policyholder" category was further refined, and more discounts were granted to applicants who met very stringent health requirements, such as a cholesterol level not exceeding 200, a blood pressure not exceeding 130/80, . For instance, one company offers a non-smoker bonus of 65% more insurance coverage with no increase in premium if the applicant has not smoked for 12 months prior to application A bonus of 100% is offered if the applicant:

— has not smoked for the past 12 months, and
— has a resting pulse of 72 or below, and
— has a blood pressure that does not exceed 134/80, and
— has a total cholesterol reading not exceeding 200, and

— does not engage in hazardous sports, and
— rigorously follows a 3-times-a-week exercise program of at least 20 minutes, and
— is within specified height and weight limits, and
— has no more than one death in immediate family prior to 60 years of age due to kidney or heart disease, stroke or diabetes

Again this is a distorsion, or a least a very strict interpretation, of the medical statement "People who exercise, who do not smoke, who have a low level of cholesterol, low blood pressure, who are neither overweight nor severely underweight, ... have a higher life expectancy". Insurers demand all conditions to be strictly met, the slightest infringement leads to automatic rejection of the preferred category For instance, a cholesterol level of 201 implies that the preferred rates won't apply, even if the applicant meets all other requirements. A cholesterol level of 200 is accepted, a level of 201 is not! We will show that fuzzy set theory can be used to provide a more flexible definition of a preferred policyholder, that allows for some form of compensation between the selected criteria.

Problem 2. Selection of an optimal excess of loss retention

Imprecise statements seem to be pervasive in reinsurance practive, where vague recommendations and rules abound. "As a rule of thumb, an excess of loss (XL) retention should approximatively equal 1% of the premium income", "Our long-term relationship with our present reinsurer should in principle be maintained", "We could accept those conditions providing substantial retrocessions are offered", "A ball-park figure for the cost of this reinsurance program is $10 million", are fuzzy sentences frequently heard in practice. To illustrate fuzzy decision-making procedures, we shall consider the problem of the selection of the optimal retention of a pure XL treaty, given the four following fuzzy goals and constraints.

Goal 1: The ruin probability should be substantially decreased, ideally down to be neighbourhood of 10^{-5}.

Goal 2: The coefficient of variation of the retained portfolio should be reduced; if possible it should not exceed 3

Constraint 1. The reinsurance premium should not exceed 2 5% of the line's premium income by much.

Constraint 2. As a rule of thumb, the retention should approximatively be equal to 1% of the line's premium income

Problem 3 Computation of the fuzzy premium of a pure endowment policy

Forecasting interest rates is undoubtedly one of the most complex modelling problems. Money market interest rates seem to fluctuate according to monthly U.S. unemployment and trade deficit figures, vague statements made by Mr Kohl or Mr Greenspan, the markets' perception of Mr Bush's willingness to tackle the deficit problem, the mood of the participants to an OPEC meeting, etc. To compute insurance premiums over a 40-year span with a fixed interest rate of 4.75% then seems to be an exercise in futility. We will show that the introduction of fuzzy interest rates (and fuzzy survival probabilities) at least allows us to obtain a partial measure of our ignorance.

As illustrated by our examples, fuzzy set theory attempts to modelize imprecise expressions like "more or less young", "neither overweight nor underweight", "in the neighbourhood of", "in principle". In retreating from precision in the face of overpowering complexity, the theory explores the use of what might be called linguistic variables, that is, variables whose values are not numbers but words or sentences. In summary, fuzzy set theory endorses Bertrand Russell's opinion that

> "All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence"

and rejects Yves Le Dantec's aphorism

> "That only is science which deals with the measurable".

2 FUZZY LOGIC AND FUZZY PREFERRED POLICYHOLDERS

2.1. Basic definitions

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. More precisely, let $X = \{x\}$ denote a collection of objects denoted generically by x A fuzzy set A in X is a set of ordered pairs

$$A = \{x, U_A(x)\}, \qquad x \in X$$

where $U_A(x)$ is termed the grade of membership of x in A, and $U_A : X \to M$ is a function from X to a space M, called the membership space Hence a fuzzy set A on a referential set X can be viewed as a mapping U_A from X to M. (Examples of membership functions are presented in all figures).

For our purposes it is sufficient to assume that M is the interval [0, 1], with 0 and 1 representing, respectively, the lowest and highest grade of membership The degree of membership of x in A corresponds to a "truth value" of the statement "x is a member of A". When M only contains the two points 0 and 1, A is nonfuzzy.

Problem 1

Let X be a set of prospective policyholders, $x = x(t_1, t_2, t_3, t_4)$. For simplicity, assume that the requirements for the status of "preferred policyholder" will be based on the values taken by 4 variables

t_1, the total level of cholesterol in the blood, in mg/dl,
t_2, the systolic blood pressure, in mm of Hg

DOCKET ALARM

Explore Litigation Insights

Docket Alarm provides insights to develop a more informed litigation strategy and the peace of mind of knowing you're on top of things.

Real-Time Litigation Alerts

Keep your litigation team up-to-date with **real-time alerts** and advanced team management tools built for the enterprise, all while greatly reducing PACER spend.

Our comprehensive service means we can handle Federal, State, and Administrative courts across the country.

Advanced Docket Research

With over 230 million records, Docket Alarm's cloud-native docket research platform finds what other services can't. Coverage includes Federal, State, plus PTAB, TTAB, ITC and NLRB decisions, all in one place.

Identify arguments that have been successful in the past with full text, pinpoint searching. Link to case law cited within any court document via Fastcase.

Analytics At Your Fingertips

Learn what happened the last time a particular judge, opposing counsel or company faced cases similar to yours.

Advanced out-of-the-box PTAB and TTAB analytics are always at your fingertips.

API

Docket Alarm offers a powerful API (application programming interface) to developers that want to integrate case filings into their apps.

LAW FIRMS

Build custom dashboards for your attorneys and clients with live data direct from the court.

Automate many repetitive legal tasks like conflict checks, document management, and marketing.

FINANCIAL INSTITUTIONS

Litigation and bankruptcy checks for companies and debtors.

E-DISCOVERY AND LEGAL VENDORS

Sync your system to PACER to automate legal marketing.

WHAT WILL YOU BUILD? | **sales@docketalarm.com** | **1-866-77-FASTCASE**

fastcase®
Smarter legal research.