EXHIBIT 1

Bit-Interleaved Coded Modulation with Iterative Decoding

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***Abstract*—A simple iterative decoding technique using hard-decision feedback is presented for bit-interleaved coded modulation (BICM). With an 8-state, rate-2/3 convolutional code, and 8-PSK modulation, the improvement over the conventional BICM scheme exceeds 1 dB for a fully-interleaved Rayleigh flat-fading channel and exceeds 1.5 dB for a channel with additive white Gaussian noise. This robust performance makes BICM with iterative decoding suitable for both types of channels.**

***Index Terms*— Interleaving, iterative decoding, trellis-coded modulation.**

I. INTRODUCTION

IN UNGERBOECK'S trellis-coded modulation (TCM) scheme [1], the convolutional code and modulation are jointly optimized to maximize the minimum Euclidean distance between coded signal sequences. For fully-interleaved Rayleigh-fading channels, the performance of a coded system strongly depends on the code diversity [2]. Since the original Ungerboeck codes usually yield a low diversity, adaptations have been suggested to improve the TCM performance over fading channels [2], [3]. However, these are often achieved at the expense of a reduced minimum Euclidean distance, and therefore result in a degraded performance over additive white Gaussian noise (AWGN) channels. A higher outage probability may also occur when the fading is very slow and the channel time diversity is limited by the interleaver depth.

Among the many adaptations suggested, the bit-interleaved coded modulation (BICM) scheme initially proposed by Zehavi [3] seemingly gives the largest improvement for Rayleigh-fading channels. In BICM, the diversity order is increased significantly by using bit interleavers in place of conventional symbol interleavers. However, the minimum Euclidean distance is also reduced due to the random modulation caused by the bit interleavers. Besides, the decoder widely used for BICM does not fully exploit the advantages provided by bit interleaving and can therefore be improved.

In this letter, we show that these drawbacks can be overcome by a simple iterative decoding technique (BICM-ID) using hard-decision feedback. Our simulation results show that gains exceeding 1 dB over conventional BICM are achieved for fully-interleaved Rayleigh flat-fading channels. The improvement for AWGN channels is even more impressive—BICM-ID with two iterations of decoding attains the performance of Ungerboeck's TCM. We further show that for BICM-ID, a "mixed" signal labeling method outperforms Gray labeling, the BICM standard [3].

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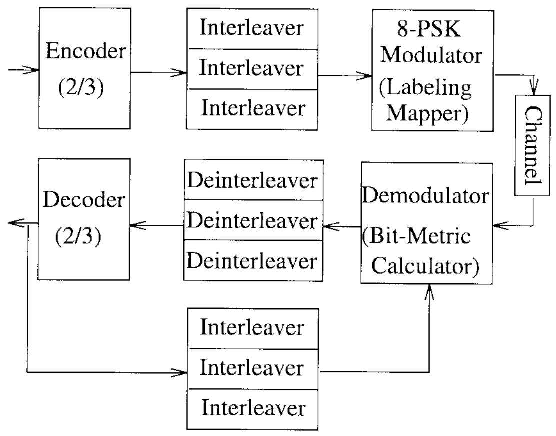

Fig. 1. Block diagram of the BICM-ID scheme.

II. BIT-INTERLEAVED CODED MODULATION WITH ITERATIVE DECODING

A. System Description

Our system block diagram is shown in Fig. 1. Note the addition of a feedback loop compared with the conventional decoder. Although we show only a system with a rate-2/3 code and 8-PSK modulation, the extension to other coding rates and modulation types is straightforward.

We represent the output of the rate-2/3 convolutional encoder by

$$\mathbf{C} = [c_0^1, c_0^2, c_0^3, \cdots, c_t^1, c_t^2, c_t^3, \cdots] = [C_0, \cdots, C_t, \cdots] \quad (1)$$

where c_t^i is the ith output bit at time position t and $C_t = [c_t^1, c_t^2, c_t^3]$. Three independent bit interleavers permute bits to break the correlation of the fading channel as well as the correlation between the bits in the same symbol. At the deinterleavers, the permutation is inverted. The output of the interleavers is represented by

$$\mathbf{V} = [v_0^1, v_0^2, v_0^3, \cdots, v_t^1, v_t^2, v_t^3, \cdots] = [V_0, \cdots, V_t, \cdots] \quad (2)$$

where v_t^i is the ith output bit at time position t and $V_t = [v_t^1, v_t^2, v_t^3]$. The interleaving is followed by a signal labeling map μ, an isomorphism between a 3-tuple $V_t = [v_t^1, v_t^2, v_t^3]$ and a signal constellation point x_t

$$x_t = \mu(V_t), \; x_t \in \chi \tag{3}$$

where the 8-PSK signal set $\chi = \{\sqrt{E_s}e^{j2n\pi/8}, n = 0, \cdots, 7\}$ and E_s is the energy per channel symbol. With a rate-2/3 convolutional code, the energy per information bit is $E_b = E_s/2$.

For a Rayleigh-fading channel with coherent detection, the received discrete-time signal is

$$y_t = \rho_t x_t + n_t \tag{4}$$

where ρ_t is the Rayleigh-distributed fading amplitude and n_t is complex AWGN with variance $\sigma_I^2 = \sigma_Q^2 = N_0/2$. For an AWGN channel, $\rho_t = 1$. Throughout the paper, we assume that ρ_t is known at the decoder from pilot symbols.

B. Conventional Decoding

For each received signal y_t, a log likelihood function is calculated for the two possible binary values of each coded bit

$$\begin{aligned}\lambda(v_t^i = b) &= \log \sum_{x \in \chi(i,b)} P(x|y_t, \rho_t) \\ &= \log \sum_{x \in \chi(i,b)} P(y_t|x, \rho_t)P(x), \\ &\qquad i = 1, 2, 3; \;\; b = 0, 1\end{aligned} \tag{5}$$

where the signal subset $\chi(i,b) = \{\mu([v^1, v^2, v^3])|v^i = b\}$ and the terms common to all i and b are disregarded. For 8-PSK, the size of each signal subset is 4. In conventional decoding, the *a priori* probability $P(x)$ is assumed equal for any $x \in \chi(i,b)$. Then, the bit metric becomes

$$\begin{aligned}\lambda(v_t^i = b) &= \log \sum_{x \in \chi(i,b)} P(y_t|x, \rho_t) \\ &\approx \max_{x \in \chi(i,b)} \log P(y_t|x, \rho_t) \\ &= -\min_{x \in \chi(i,b)} \| y_t - \rho_t x \|^2\end{aligned} \tag{6}$$

where a constant scalar is disregarded and the approximation is good at high signal-to-noise ratio [3]. At the Viterbi decoder, the branch metric corresponding to each of the eight possible 3-tuples $C_t = [c_t^1, c_t^2, c_t^3]$ is the sum of the corresponding bit metrics after deinterleaving.

C. Iterative Decoding with Hard-decision Feedback

Convolutional encoding introduces redundancy and memory into the signal sequence x_t. Yet, the equally likely assumption for $P(x)$ in (6) fails to use this information, primarily because it is difficult to specify in advance of *any* decoding. The *a priori* information is reflected in the decoding results and therefore can be included through iterative decoding. One approach is to use the Viterbi algorithm with soft outputs, although this is computationally complex. Instead, we consider only binary-decision feedback for the calculation of the bit calculate $\lambda(v_t^1 = 0)$, we assume, for any $x = \mu(v^1 = 0, v^2, v^3) \in \chi(1, 0)$,

$$P(x) = \begin{cases} 1, & \text{if } v^1 = 0, v^2 = \hat{v}_t^2, v^3 = \hat{v}_t^3 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where $\hat{v}_t^2$ and $\hat{v}_t^3$ are the first-round decoding decisions. Then the bit metric with the decision feedback becomes

$$\lambda(v_t^1 = 0) = - \| y_t - \rho_t \mu([0, \hat{v}_t^2, \hat{v}_t^3]) \|^2 . \tag{8}$$

The bit metrics for other bit positions and bit values follow similarly.

Note that all the distance values (symbol metrics) used in (8) have been calculated and stored during the first-round decoding. In the second round, only a reselection of the distance values is involved. A multiple run of the Viterbi algorithm is needed. But the only extra hardware requirement is a set of interleavers.

Given the feedback $\hat{v}_t^2$ and $\hat{v}_t^3$, the Euclidean distance between the signals $\mu(0, \hat{v}_t^2, \hat{v}_t^3)$ and $\mu(1, \hat{v}_t^2, \hat{v}_t^3)$ can be significantly larger than the minimum Euclidean distance between the signals in the subset $\chi(1, 0)$ and those in $\chi(1, 1)$. With an appropriate signal labeling, the minimum Euclidean distance between coded sequences can be made large for BICM-ID. This is the key that BICM-ID outperforms conventional BICM and that BICM-ID is suitable for both Rayleigh fading and AWGN channels.

To avoid severe error propagation, the bits feedback should be independent of the bit for which the bit metric is calculated. This is made possible by the independent bit interleavers—the three bits making up a channel symbol are typically far apart in the coded sequence. This is clearly a feature not available in a symbol-interleaved system.

D. Signal Labeling

The performance of BICM-ID strongly depends on the signal labeling methods. We find that a mixed labeling method outperforms both Gray labeling and set-partitioning labeling. With mixed labeling, the eight sequential labels for 8-PSK signals are $\{000, 001, 010, 011, 110, 111, 100, 101\}$. The details of the labeling design is discussed in [5].

III. Simulation Results

We show the BER performance of 8-PSK BICM-ID for both Rayleigh fading and AWGN channels. The simulation results of Ungerboeck's TCM scheme and Zehavi's BICM scheme are also included for comparison. Mixed labeling is used for BICM-ID, while Gray labeling is used for BICM. The best 8-state rate-2/3 convolutional code [4, p. 331] is used for both schemes. For BICM-ID, a short interleaver may limit the improvement through multiple iterations. Therefore, random interleavers with length 2000 are used. Each bit-error rate (BER) data point is generated by a Monte Carlo simulation with more than 10^7 trials.

The performance for Rayleigh fading channels is shown in Fig. 2. Compared with Zehavi's BICM scheme, there is a 1-dB performance degradation after the first round of decod-

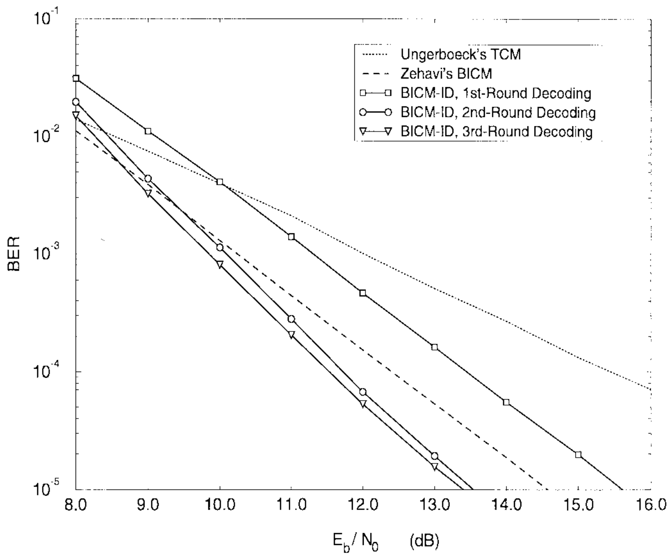

Fig. 2. The performance of BICM-ID with the 8-state rate-2/3 convolutional code and 8-PSK modulation for Rayleigh fading channels.

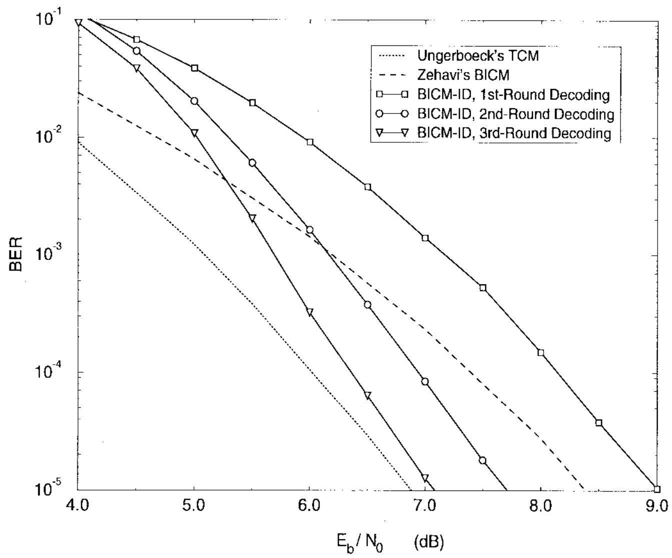

Fig. 3. The performance of BICM-ID with the 8-state, rate-2/3 convolutional code and 8-PSK modulation for AWGN channels.

labeling. However, with a second round of decoding, BICM-ID quickly catches up and outperforms BICM by 1-dB at BER $= 10^{-5}$. A third round of decoding adds a slight improvement.

The performance for AWGN channels is shown in Fig. 3. The E_b/N_0 gap between BICM-ID and Ungerboeck's TCM scheme is only 0.2 dB at BER $= 10^{-5}$. The gain of BICM-ID over BICM is more than 1.5 dB.

IV. Conclusion

We propose BICM-ID and demonstrate that it significantly outperforms conventional BICM. The robust performance of BICM-ID makes it suitable for both AWGN and Rayleigh fading channels. The effects of interleaver length are still under investigation. Significant results for other coding and modulation schemes have also been obtained and will be reported in [5].

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