E-2

We have:

$$\overline{X}_{(i,j),(u,v)}(s,t) = \sum_{(K,:)\ in\ II\ (i,j)} Y_{(k,w),(u,v)}(s,t-1) + Z_{(i,j),\ldots}\ldots \tag{37}$$

and

$$Y_{(i,j),(u,v)}(s,t) = Y_{(i,j)}(t)\ \frac{X_{(i,j),(u,v)}(s,t)}{X_{(i,j)}(t)} \tag{38}$$

As in the previous section $Y_{(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. The remaining quantity to compute is:

$$X_{(i,j)}(t) = \sum_{\substack{(u,v)\ in \\ II(i,j)\ set}} X_{(i,j),(u,v)}(s,t) + Z_{(i,j)}(t) \tag{39}$$

These equations have been programmed for the grid of repeaters at the lattice points of the Euclidean plane, as earlier. The program is for the case where a repeater has a single fixed path to the origin or ground station. The program was run for ten time points using a single sample at each point. The numerical results are, therefore, subject to some variability. Some of the results of a single run are given in Tables 28 - 34. As is evident from the tables, saturation of the channel begins early for $\lambda = 5$. In Table 31 for example the proba-bility that a message originating at d = 5 at time 1 gets to the ori-gin with a delay of less than four time units is only about .44. For $\lambda = 5$ the situation deteriorates rapidly with time. To obtain a large set of representative data would require running the program for many time points, probably at least 15 or 20 for different values of λ and slot size. This can be done using the available program.

10.3.   Delays and Average Delays as a Function of Distance

We can extend the calculations and analyses described in the pre-vious two sections to include calculations of delay distributions and average delay. In addition to studying delays, we can develop equa-tions to study bottlenecks in a given network. These formulae have been programmed and numerical results can be obtained.

Let $D_{(i,j)}(t)$ be the random variable delay of a message which originates at (i,j) at time t. We assume that the probability that a message is delayed by k-units of time is given by the proportion of

Network Analysis Corporation

Retransmissions at Point of Wipeout

λ=5, m=100, Model

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	6	6	0	0	6	0
4	6	11	6	1	3	0
5	43	26	18	1	1	0
6		19	19	7	2	0
7		16	29	13	0	0
8		91	47	3	0	0
9		67	67	1	0	0
10		151	64	9	2	0

TABLE 28

λ=5, m=100, Mode 2

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	7	2	1	0	0	0
4	4	10	1	0	2	0
5	34	20	2	2	1	0
6		37	2	1	1	0
7		19	5	0	1	0
8		28	6	2	1	1
9		33	18	1	1	0
10		41	4	0	1	1

TABLE   29

Delay Probability Tables for a Message Being

Accepted d units from the Ground Station

$\lambda=5$, m=100, Mode 1

A Message Originating at d=5 at time 0.

delay \ dist	0	1	2	3	4	5
0	.159	.271	.524	.933	1.000	1.000
1	.235	.247	.325	.059		
2	.134	.157	.084	.004		
3	.097	.076	.028	.002		
4	.089	.045	.016	.001		
5		.041	.009			
6			.007			

TABLE   30

A Message Origination at d=4 at time 0.

delay \ dist	0	1	2	3	4
0	.249	.454	.727	1.000	1.000
1	.251	.221	.153		
2	.196	.123	.083		
3	.083	.070	.021		
4	.054	.033	.007		
5	.046	.018	.004		
6		.016	.002		
7			.002		

TABLE 31

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A Message Originating at d=3 at time 0

dist delay	0	1	2	3
0	.916	.950	1.000	1.000
1	.036	.031		
2	.023	.010		
3	.013	.004		
4	.004	.002		
5	.003	.001		
6	.002	.001		
7		.001		

TABLE 32

A Message Originating at d=2 at time 0

dist delay	0	1	2
0	.823	1.000	1.000
1	.170		
2	.004		
3	.002		
4	.001		

TABLE 33

A Message Originating at d=1, at time 0

dist delay	0	1
0	.778	1.000
1	.183	
2	.038	
3	.001	

TABLE 34

11.78

THE NUMBER OF MESSAGES ACCEPTED AT THE ORIGIN

$\lambda = 5$, m = 100, mode 2

Originating at d = 5 at t = 0:   3 messages

Time of Acceptance at Origin	Number	Delay
t = 5	.814	0
t = 6	1.130	1
t = 7	.502	2
t = 8	.253	3
t = 9	.161	4
	2.860	

TABLE 35

Originating at d = 5 at t = 1:   6 messages

Time of Acceptance at Origin	Number	Delay
t = 6	2.010	0
t = 7	1.238	1
t = 8	.988	2
t = 9	.805	3
	5.061	

TABLE 36

Originating at d = 5 at t = 2:   2 messages

Time of Acceptance at Origin	Number	Delay
t = 7	.115	0
t = 8	.249	1
t = 9	.256	2
	.620	

TABLE 37

11.79

DELAY PROBABILITY TABLES FOR A MESSAGE

BEING ACCEPTED d UNITS FROM THE GROUND STATION

$\lambda = 5$, m = 100, mode 2

Dist Delay	0	1	2	3	4	5
A Message Originating at d = 5 at time 0						
0	.271	.496	.854	1.000	1.000	1.000
1	.377	.360	.140	-	-	-
2	.167	.071	.003	-	-	-
3	.084	.041	.003	-	-	-
4	.054	.018	-	-	-	-
5		.007	-	-	-	-

TABLE 38

Dist Delay	0	1	2	3	4	
A Message Originating at d = 4 at time 0						
0	.407	.452	.792	1.000	1.000	
1	.189	.300	.178	-	-	
2	.216	.178	.029	-	-	
3	.092	.035	.001	-	-	
4	.045	.020	.001	-	-	
5	.028	.008	-	-	-	
6	-	.003	-	-	-	

TABLE 39

11.80

A Message Originating at d = 3 at time 0

Dist Delay	0	1	2	3
0	.511	.962	1.000	1.000
1	.425	.022	-	-
2	.031	.009	-	-
3	.020	.005	-	-
4	.007	.001	-	-
5	.003	.001	-	-
6	.002	-	-	-

TABLE 40

A Message Originating at d = 2 at time 0

Dist Delay	0	1	2	3
0	.883	.923	1.000	
1	.061	.074	-	
2	.050	.002	-	
3	.003	-	-	
4	.	-	-	
5	.001	-	-	

TABLE 41

A Message Originating at d = 1 at time 0

Dist Delay	0	1		
0	.800	1.000		
1	.191	-		
2	.005	-		
3	.004	-		

TABLE 42

A Message Originating at d = 0 at time 0

Dist Delay	0	1		
0	1.000			
1	.000			

TABLE 43

A Message Originating at d = 5 at time 1

Dist Delay	0	1	2	3	4	5	
0	.335	.520	.718	.752	.846	1.000	
1	.206	.159	.116	.207	.132	-	
2	.165	.161	.125	.039	.020	-	
3	.134	.081	.024	.002	.002	-	
4	-	.036	.012	.003	-	-	
5	-	-	.003				

TABLE 44

11.82

PROBABILITY OF ZERO DELAY VS. DISTANCE OF ORIGINATION

MODE 1

t \ d	0	1	2	3	4	5
0	1.000	.778	.823	.916	.349	.159
1	.346	.329	.458	.125	.091	.123
2	.242	.229	,045	.085	.053	.047
3	.207	.043	.013	.058	.021	.024
4	.177	.033	.025	.021	.013	.037
5	.092	.019	.011	.006	.015	
6	.021	.005	.002	.008		
7	.029	.002	.001			
8	.031	.001				
9	.047					

TABLE 45

MODE 2

t \ d	0	1	2	3	4	5
0	1.000	.800	.883	.511	.407	.271
1	.373	.136	.118	.356	.170	.335
2	.333	.118	.102	.030	.180	.115
3	.100	.044	.033	.154	.066	.218
4	.112	.026	.061	.045	.109	.161
5	.062	.032	.005	.100	.073	
6	.021	.011	.022	.041		
7	.045	.022	.019			
8	.005	.018				
9	.021					

TABLE 46

messages which originate at time t which are delayed by k-units. If enough random samples are taken, this estimate becomes quite good. In notation, define $Y_{(o,o)\ (i,j)\ (s,t)}$ as the number of messages which are accepted at the ground station at time s, which originated at (i,j) at time t. Then,

$$P\{D_{(i,j)}(t)=k\} = \frac{Y_{(o,o),(i,j)\ (t+k+i,t)}}{O_{(i,j)}(t)}; \quad k=0,\ 1,\ 2,\ldots, \tag{40}$$

When doing computer trials to determine 40 we can average over all 4i repeaters at distance i and compute the delay distribution as a function only of distance to the ground station, i.e.

$$P\{D_i(t)=k\}=\frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(t)=k\}, \text{ where} \tag{41}$$

$D_i(t)$ is the random variable delay of a message originating at distance i at time t.

To obtain a time invariant measure, we can average 41 over time and obtain the probability distribution of the random variable D_i and its expectation, given by:

$$E(D_i) = \sum_{k=o}^{\infty} k\ P\{D_i=k\} \tag{42}$$

The same kind of analysis can be used to study "bottlenecks." Let $D_{(i,j)}(w,t)$ be the random variable "delay of a message" originating at (i,j) at time t in getting w units from the ground station for w=0, 1, 2,..., i. As earlier we have:

$$P\{D_{(i,j)}(w,t)=k\} = \frac{Y_{(w,u),(i,j)}(t+k+i-w,\ t)}{O_{(i,j)}(t)}; \quad k= 0,\ 1,\ 2,\ldots, \tag{43}$$

where (w,u) is the unique repeater on the path from (i,j) to (o,o) at distance w. Similarly as in 41 and 42 :

$$P\{D_i(w,t)=k\} = \frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(w,t)=k\} \text{ and} \tag{44}$$

$$E\left[D_i(w,t)\right]=\sum_{k=1}^{\infty} kP\{D_i(w,t)=k\}. \tag{45}$$

11.85

11.   MESSAGES OUTWARD FROM THE ORIGIN

11.1. A Single Message Originates at the Ground Station.

The next major part of our study is to model the situation when message flow is outward from the origin. We begin our study with the dynamics of the simple model where the repeaters are at the lattice points of the plane. A single message originates at the ground station at time t=0. Every message received by a repeater is accepted and perfectly retransmitted to each of its four nearest neighbors. We determine:

a)   The number of repeaters which receive the message for the first time at time t: t = 0, 1, 2,....

b)   The number of repeaters which have seen the single message by time t.

c)   The number of copies of the single message received by any repeater at time t.

The assumptions are:

1)   A single message arrives at a given node at time t = 0, no other messages are introduced into the network. We assume the message originated at the origin,(Cartesian coordinates (0,0)).

2)   Message transmission is perfect, i.e., after one time unit each of the four neighbors to any repeater receive all messages transmitted by the repeater at the previous time point.

Some diagrams and numbers are helpful to fix ideas.

t = 0,
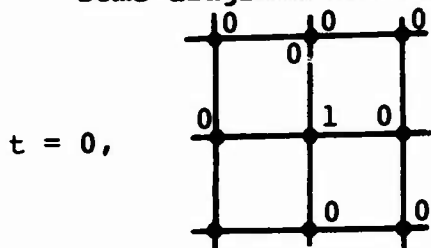
(one message at origin, no messages elsewhere)

t = 1,
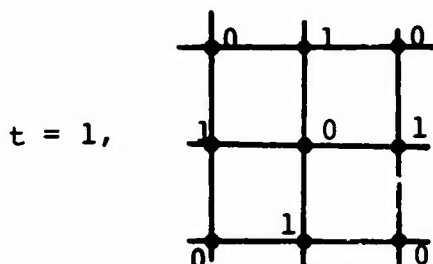
(a single message at each of the four neighbors, no messages elsewhere)
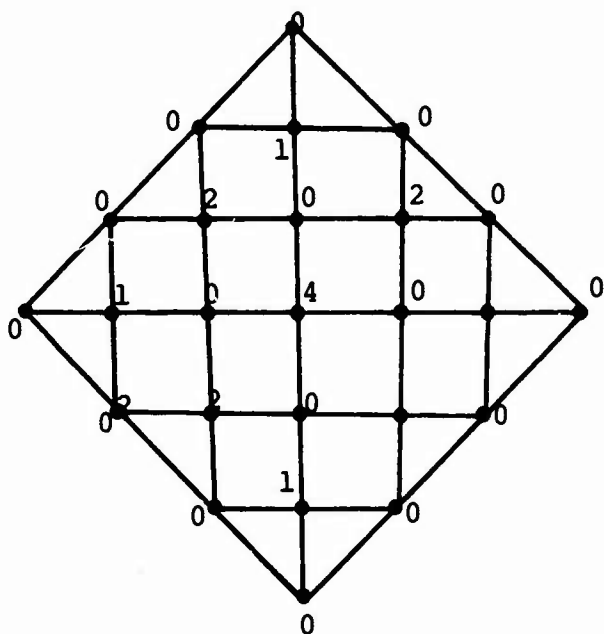
(4 messages at the origin, 0 Message at all repeaters 1 step from origin, 1 message at each of four repeaters 2 steps in either the horizontal or vertical directions, 2 messages at repeaters, 1 unit in horizontal direction and 1 unit in vertical direction.

By examining the diagrams we are led to introduce a coordinate system based on distance as measured in steps to reach a repeater and horizontal distance of the repeater from the origin. The quadrant symmetry of the model also indicates use of these coordinates.

11.87

The coordinates of a repeater are denoted by (d,j) where d is the distance of the repeater from the origin measured in minimum time units a message needs to arrive at the repeater from the origin; the second coordinate j is the horizontal distance of the repeater from the origin again measured in time units but only in the horizontal direction.

For example, we give some coordinates:
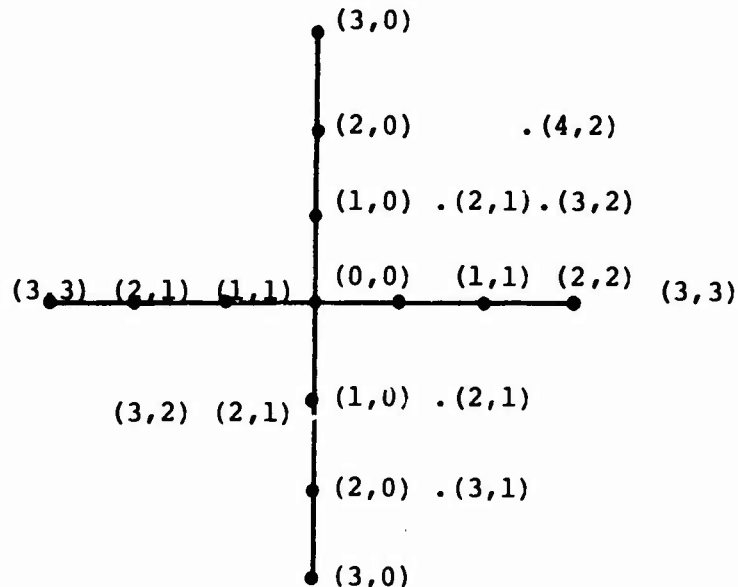
Some further notation which is necessary;

$B(t)$ = the number of repeaters which receive the message for the first time at time t.

Clearly $B(t)$ is the number of repeaters whose first coordinate is t. i.e. that are at distance t from the origin.

$A(t)$ = the number of repeaters which have seen the message by time t. Clearly $A(t) = \sum_{j=0}^{t} B(j)$.

$N_j^d(t)$ = number of copies of the message received by a given repeater at coordinates (d,j) at time t. Clearly

a) $N_j^d(t) = 0$ for $d > t$

b) $N_j^d(d+2k+1) = 0$ for $k = 1, 2, \ldots$.

Thus it is necessary to compute $N_j^d(d+2k)$ $k = 0, 1, 2, \ldots$

11.88

A)   Calculation of B(t):

The quantity B(t), (the number of repeaters at distance t from the origin is the number of repeaters which receive the message for the first time t, is easy to compute. This quantity is given by the number of integer solutions to $|i| + |j| = t$. Since a repeater is at distance d from the origin if and only if its' Cartesian coordinates(i,j) satisfy $|i| + |j| = t$, we can solve this equation and count solutions. Note that

i=0,	j=t or -t	2 solutions
i=1,	j=t-1 or -t+1	2 solutions
i=-1,	j=t-1 or -t+1	2 solutions
i=2,	j=t-2 or -t+2	2 solutions
i=-2,	j=t-2 or -t+2	2 solutions
.	. .	
.	. .	
.	. .	
i=t-1,	j=1 or j=-1	. . .
i=-t+1,	j=1 or j=-1	. . .
i=t,	j=0	
i=-t,	j=0	

The number of solutions is, $B(0) = 1$

$$B(t) = 2 + 4(t-1) + 2 = 4t \text{ for } t \geq 1.$$

To compute A(t), we sum B(t) and obtain

$$A(t) = \sum_{j=0}^{t} B(j) = 1 + \sum_{j=1}^{t} 4(j) = 1 + 4 \sum_{j=1}^{t} j = 1 + 4 \frac{t(t+1)}{2}$$

$$= 1 + 2t^2 + 2t = 2t^2 + 2t + 1$$

The rate at which A(t) the number of repeaters which receive the message by time t grows as $A'(t) = 4t + 2$ which is linear in t.

B. **The quantity $N_j^d(d + 2k)$, $k = 0, 1, 2, \ldots$** .

To compute the number of copies of the message received at a repeater with coordinates (d,j) at time $d + 2k$, $k = 0, 1, 2,\ldots$ we first draw some diagrams. Due to symmetry it suffices to examine only the first quadrant.
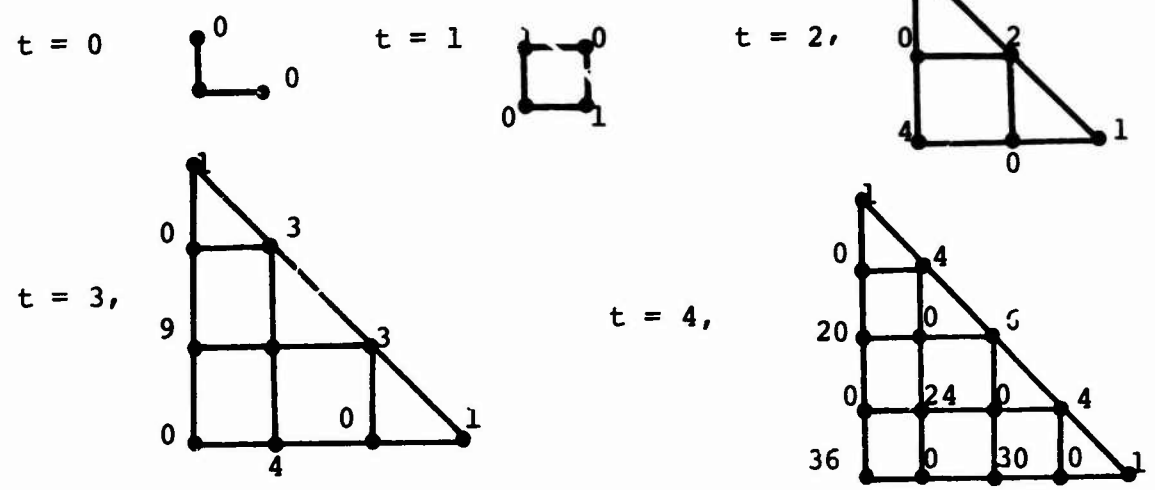
It seems clear from the diagram that a repeater with coordinates (d,j) will receive at $t=d$ the number of messages which is given by the binomial coefficient $\binom{d}{j}$. From the diagram we note the relationship of the outer edge to the dth now of a Pascal triangle:

$$
\begin{array}{c}
1 \\
1\ 1 \\
1\ 2\ 1 \\
1\ 3\ 3\ 1 \\
1\ 4\ 6\ 4\ 1 \\
1\ 5\ 10\ 10\ 5\ 1
\end{array}
$$
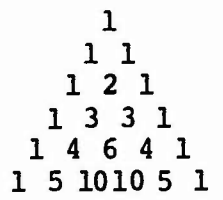
This result is also apparent from an argument based on the number of paths of a message from $(0,0)$ to (d,j). The number of messages received at a repeater with coordinates (d,j) is given by the number of paths from $(0,0)$ to (d,j) which is obviously $\binom{d}{j}$. To determine a general formula for $N_j^d(d + 2k)$ for $k > 1$, we can write and solve the appropriate difference equation.

$$N_j^d(d + 2k) = N_{j-1}^{d-1}(d + 2k-1) + N_{j+1}^{d+1}(d + 2k-1)$$
$$+ N_j^{d+1}(d + 2k-1) + N_j^{d-1}(d + 2k-1)$$

for $k = 1, 2, \ldots,; d = 0, 1, 2, \ldots,; j = 0, 1, 2, \ldots, d$.

The initial conditions are

$$N_j^d(t) = 0 \quad \text{if } t < d,$$

$$N_j^d(d) = \binom{d}{j}.$$

The solution to this equation is given by $N_j^d(d + 2k) = \binom{d + 2k}{k} \binom{d + 2k}{k+j}$

To check its validity note that the initial conditions are satisfied, and apply the well known definition of binomial coefficients;

$$\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}, \qquad n \geq 1 \quad j \geq 1.$$

$$N_{j-1}^{d-1}(d-1 +2k) + N_{j+1}^{d+1}(d+1+2(k-1)) + N_j^{d+1}(d+1+2(k-1)) + N_j^{d-1}(d-1+2k)$$

$$= \binom{d+2k-1}{k}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k}\binom{d+2k-1}{k+j}$$

$$= \binom{d+2k-1}{k}[\binom{d+2k-1}{k+j-1}+\binom{d+2k-1}{k+j}] + \binom{d+2k-1}{k-1}[\binom{d+2k-1}{k+j}+\binom{d+2k-1}{k+j-1}]$$

$$= \binom{d+2k-1}{k}\binom{d+2k}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k}{k+j}$$

$$= \binom{d+2k}{k+j}[\binom{d+2k-1}{k}+\binom{d+2k-1}{k-1}] = \binom{d+2k}{k+j}\binom{d+2k}{k}$$

For k very large with respect to d we can use a stirling approximation to note the $N_j^d(d+2k) \sim 2^{4k}$ i.e. grows as 2^{4k}.

To summarize:

a) $B(t) = 4t$, $t \geqslant 1$ $B(0) = 1$,

b) $A(t) = 2 t^2 + 2t+1$ $t \geqslant 0$,

c) $N_j^d (d+2k) = \binom{d+2k}{k+j} \binom{d+2k}{k} \sim 2^{4k}$ for large k .

11.2. A Fixed Number of Messages Originate at the Ground Station and Subject to Non-Capture

The model of message flow _from_ the ground station out to re-
peaters can be extended to allow the possibility of erasure or
non-capture of messages.

We assume that at each point of time t, τ messages are being
generated outward from the origin. Of messages accepted at each
repeater, a fixed proportion k_o are addressed to that repeater and
hence are not repeated.   We study the distributions of the number
of messages received and accepted at each repeater at each point
in time, assuming an infinite net.

Recall $X_{(i,j)}^{(1)}(t)$ = number of messages arriving at a repeater
with coordinates (i,j) at time t with Mode 1 capture.

$X_{(i,j)}^{A,(1)}(t)$ = number of messages accepted at a repeater with
coordinates (i,j) at time t in Mode 1 capture. In Mode 2 capture,
we use the same notation except that (1) as a superscript is re-
place by a (2).

A.   In Mode One:

In Mode 1 capture, the relationship between arriving and
accepted messages is described by the transfer function:

$$P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k,m)-j} (-1)^\nu \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \qquad (46)$$

when $j \leq \min(k,m)$, and zero otherwise.

We can study $X_{(ij)}^{(1)}(t)$ and $X_{(ij)}^{A,(1)}(t)$ recursively.

At the origin for each $t = 0, 1, 2, \ldots;$ $X_{(0,0)}^{(1)}(t) = \tau$, a fixed constant. Furthermore:

$$P\{X_{(0,0)}^{(1),A}(t) = w\} = P_{\tau,w} = \frac{\binom{m}{w}}{m^\tau} \sum_{\nu=0}^{\min(\tau,m)-w} (-1)^\nu \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} \qquad (47)$$

If we assume $\tau \leq m$;

$$P\{X_{(0,0)}^{(1),A}(t) = w\} = \begin{cases} \dfrac{\binom{m}{w}}{m^\tau} \displaystyle\sum_{\nu=0}^{\tau-w} (-1)^\nu \binom{m-w}{\nu} \dfrac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} & \text{if } w \leq \tau \\[4mm] 0 & \text{if } w > \tau. \end{cases} \qquad (48)$$

At coordinates $(1,1)$, $(1,0)$ the distribution of $X_{(1,0)}(t)$ and $X_{(1,1)}(t)$ are identical, hence we write only $X_{(1,0)}^{(1)}(t)$.

We have:

$$X_{(1,0)}^{(1)}(t) = \begin{cases} X_{(0,0)}^{(1),A}(t-1) & t = 1, 2, \ldots; \\[3mm] 0 & \text{if } t = 0. \end{cases} \qquad (49)$$

For the acceptance at $t = 1, 2, \ldots;$

$$P\{X_{(1,0)}^{A,(1)}(t) = j\} = \sum_{k=j}^{\tau} P_{kj} \cdot P\{X_{(1,0)}^{(1)}(t) = k\} \qquad j = 0, 1, 2, \ldots;$$

or recursively:

$$P\{X_{(1,0)}^{A,(1)}(t) = j\} = \begin{cases} \displaystyle\sum_{k=j}^{\tau} P_{kj} \cdot P\{X_{(0,0)}^{(1),A}(t-1) = k\} & \text{if } j \leq \tau; \\[4mm] 0 & \text{otherwise.} \end{cases} \qquad (50)$$

11.93

Equation(50) is recursively solvable since $P\{X_{(0,0)}^{(1),A}(t-1) = k\}$ is given by (48)and P_{kj} is given by (46).

Now more generally, at a repeater at distance d with coordinates (d,j); $j \neq 0,d$. We have for d = 2, 3, ...;

$$X_{(d,j)}^{(1)}(t) = (1-k_0) \left[X_{(d-1,j-1)}^{(1),A}(t-1) + X_{(d-1,j)}^{(1),A}(t-1) \right]. \tag{51}$$

The acceptances are given by:

$$P\{X_{(d,j)}^{A,(1)}(t, = r\} = \sum_{k=r}^{(2m)} P_{k,r} \cdot P\{X_{(d,j)}^{(1)}(t) = k\}; \tag{52}$$

where P_{kr} is given by (1) and $P\{X_{(d,j)}^{(1)}(t) = k\}$ can be computed recursively from (51) using the notion of isodesic line joint densities. The equations for mode 2 analysis are identical except that P_{kj} is replaced by P_{kj}^{*}.

When $j = 0$ or d, i.e. the repeater is on the axis at distance d a simpler analysis unfolds. Since the random variables $X_{(d,0)}(t)$ and $X_{(d,d)}(t)$ have the same probability distribution, we write equations only for $X_{(d,0)}(t)$; $d \geq 2$ since $X_{(0,0)}(t)$ and $X_{(1,0)}(t)$ have already been determined.

$$X_{(d,0)}^{(1)}(t) = (1-k_0) X_{(d-1,0)}^{(1),A}(t-1); \qquad t = d, d + 1,...; \tag{53}$$
$$d = 2, 3, ...$$

For the acceptances;

$$P\{X_{(d,0)}^{(1),A}(t) = j\} = \sum_{k=j}^{\infty} P_{kj} P\{X_{(d,0)}^{(1)}(t) = k\}, \qquad = 0, 1, 2, ..., m \tag{54}$$

The equations (46)through (54)can be used with computer generated data to study message arrivals and acceptances at each repeater.

In particular, we can write equations such as 46-54 for the closed net under consideration and obtain numerical data for flow from the ground station.

Let $X_{(i,j)}(t)$ be the number of messages received at (i,j) at time t and $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t. We assume $X_{(o,o)}(t)=J$ a fixed constant for all time points. As in the <u>inward</u> model, $Y_{(i,j)}(t)$ is obtained from $X_{(i,j)}(t)$ by randomizing over either mode 1 or mode 2 slotting. When performing the calculations on the computer, we assumed a finite grid of 61 repeaters as earlier. However, now a repeater repeats messages to those repeaters which are one unit of distance further from the origin or ground station. Thus for example a repeater in a quadrant repeats to its two further neighbors, while a repeater on the axis repeats to the one repeater which is one unit further.

The specific equations used for the first quadrant calculations follow, we assume that $\frac{1}{61}$ of those messages accepted are addressed to each repeater and hence not repeated. The calculations were carried out in each of the two slotting modes.

Step 1.   Set $X_{(o,o)}(t) = J = 80$, $t = 1, 2,...,35$.

Step 2.   Compute $Y_{(o,o)}(t)$ by randomizing over the transfer distribution in each of two modes.

Step 3.   Compute $X_{(1,1)}(t)$ and $X_{(1,2)}(t)$ from:

$$X_{(1,1)}(t) = X_{(1,2)}(t) = \frac{60}{61} \, Y_{(o,o)}(t-1).$$

Step 4.   Compute $Y_{(1,1)}(t)$ and $Y_{(1,2)}(t)$ in each of two modes.

Step 5.   For $t = 2, 3, 4, \ldots, 37$; Compute $X_{(2,1)}(t)$, $X_{(2,2)}(t)$

and $X_{(2,3)}$ from:

$$X_{(2,1)}(t) = \frac{60}{61} Y_{(1,1)}(t-1)$$

$$X_{(2,2)}(t) = \frac{60}{61} (Y_{(1,1)}(t-1) + Y_{(1,2)}(t-1))$$

$$X_{(2,3)}(t) = \frac{60}{61} Y_{(1,2)}(t-1).$$

Step 6.   Compute $Y_{(2,1)}(t)$, $Y_{(2,2)}(t)$ and $Y_{(2,3)}(t)$ by randomizing in each slotting mode.

Step 7.   Compute $X_{(3,2)}(t)$, $X_{(3,3)}(t)$, $X_{(3,4)}(t)$ from:

$$X_{(3,2)}(t) = \frac{60}{61}\left[Y_{(2,1)}(t-1) + Y_{(2,2)}(t-1)\right]$$

$$X_{(3,3)}(t) = \frac{60}{61}\left[Y_{(2,2)}(t-1) + Y_{(2,3)}(t-1)\right]$$

$$X_{(3,4)}(t) = \frac{60}{61} Y_{(2,3)}(t-1).$$

Step 8.   Compute $Y_{(3,2)}(t)$, $Y_{(3,3)}(t)$, $Y_{(3,4)}(t)$ by randomizing in mode 1 and mode 2.

Step 9.   Compute $X_{(4,3)}(t)$ and $Y_{(4,4)}(t)$ by randomizing.

Step 10.   Compute $X_{(5,4)}(t)$ from:

$$X_{(5,4)}(t) = \frac{60}{61}\left[Y_{(4,3)}(t-1) + Y_{(4,4)}(t-1)\right].$$

Step 11.   Compute $Y_{(5,4)}(t)$ by randomizing and print out X's and Y's for $t = 1, 2, \ldots, 40$. The numerical data is summarized in Table 26 and the accompanying Figure 10.

11.3   Messages Coming Outward From the Origin

It is assumed that at each point in time eighty (80) messages originate at the origin (ground station). The messages are repeated outwards to the various repeaters. At each point in time each repeater sends all but a fixed proportion of its accepted message to each neighboring repeater, one unit of distance further from the ground station. The fixed proportion not repeated is 1/61 of the number of accepted messages, which are assumed to be addressed to the given repeater. The number of slots is fixed at 100.

In table 47 we summarize the result of this calculation by giving the average number of messages accepted and arriving as a function of distance and mode. The numerical data is displayed graphically in figure 22 .

DISTANCE	MODE 1 ARRIVING	ACCEPTED	MODE 2 ARRIVING	ACCEPTED
0	80	33	80	52
1	32	21	51	36
2	27	19	47	33
3	32	22	57	41
4	46	30	87	58
5	58	31	114	66

TABLE 47

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MODE 2, ARRIVALS

MODE 2, ACCEPTANCES

MODE 1, ARRIVALS

MODE 1, ACCEPTANCES
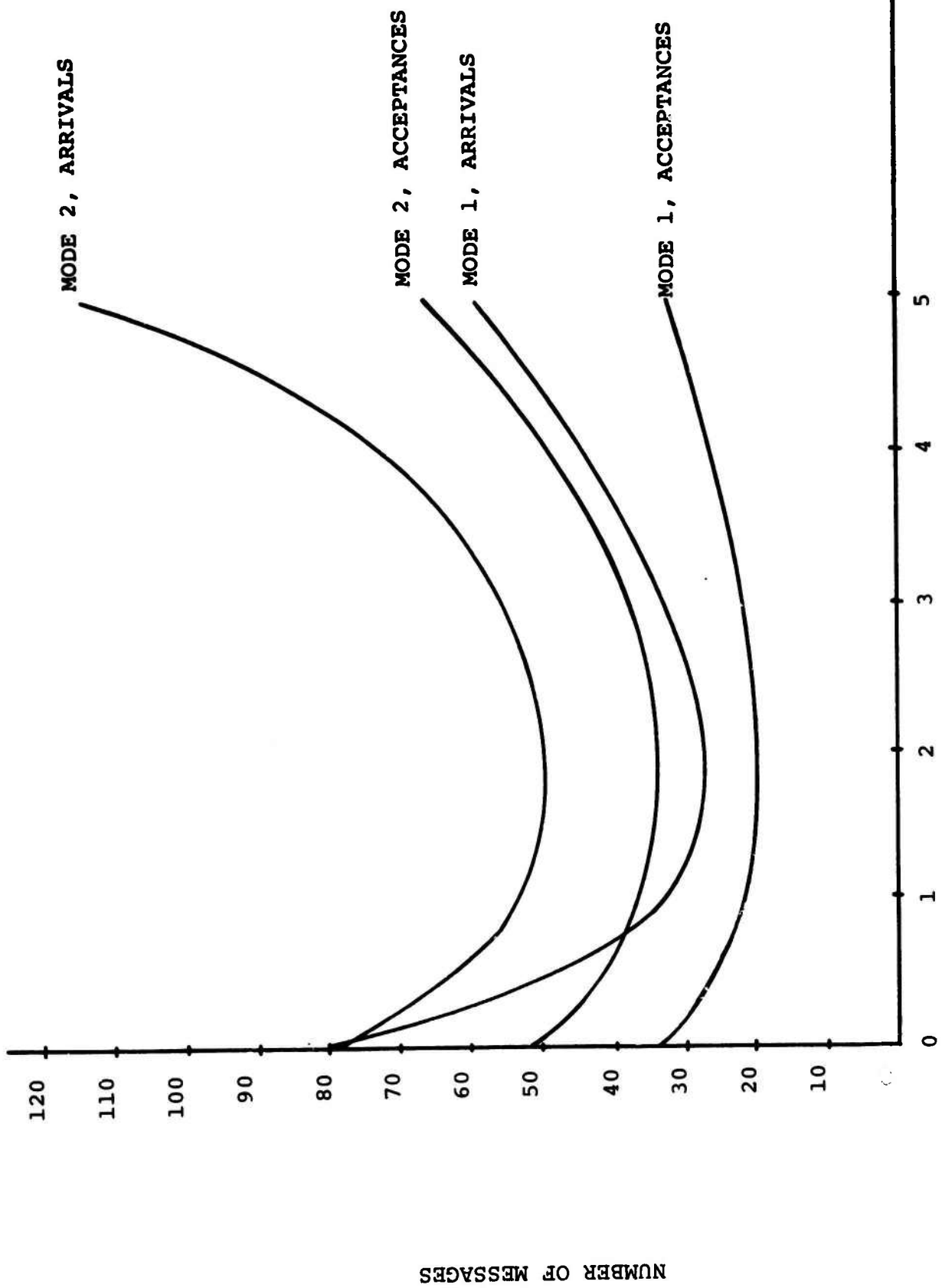
DISTANCE FROM ORIGIN

FIGURE 22

NUMBER OF MESSAGES

11.98

12. OTHER MODELS

A number of models other than the "basic model" were considered. The results for quantities of interest were obtained in closed forms under the assumption of an infinite grid. One example of such a model was described in   Section 3   for messages repeated only toward the ground station. Of course that was part of our "basic model". In the process of developing the results of   Section 3   , we assumed that a single message originated at each repeater at each point in time and multiplied the results by the mean, τ, to obtain average flows. We will now "justify" that calculation and study uninhibited passage of messages in each direction in an infinite grid. Our new assumptions are:

1) At each point in time, starting at $t=0$, messages originate at each repeater   according to a Poisson distribution with mean λ.

2) The arrivals (originations) at each repeater   are independent over time and different repeaters.

The probability that exactly j new messages originate at any time point at any repeater is

$$\frac{e^{-\lambda} \lambda^{j}}{j!} \ , \quad j = 0, 1, 2, \ldots .$$

We compute formulas for;

a) $N_{o}(t)$ = average number of messages which arrive at the origin at time t. Since all repeaters   are statistically identical there is no loss in generality in studying message flow at the origin.

b) $N_o'(t)$ = Average number of distinct messages which arrive at the origin at time t for the first time.

c) Ieff(t)=Inefficiency of the network defined

by;

$$Ieff(t) = \frac{N_o(t)}{N_o'(t)} = \frac{\text{average number of messages}}{\text{average number of new messages for the 1st time.}}$$

This is a measure of inefficiency since the larger Ieff, the more inefficient the system.

The actual number of messages which arrive at the origin at time t is a random variable. In fact it is a sum of a large number (when t is fairly large) of independent random variables. The summand random variables can loosely be described as the contribution to message flow at the origin arising out of some number of messages originating at each repeater at each point in time.

To compute $N_o(t)$ we can sum up all the contributions. This is interesting but tedious. A simpler method is to compute $X_o(t)$ which we define as the number of messages arriving at the origin at time t in a deterministic model obtained by assuming that at each point in time, at each repeater, exactly one new message originates. It will then follow that

$$N_o(t) = \lambda X_o(t).$$

Similarly, if we define $X_o'(t)$ to be the number of distinct messages that arrive at the origin at time t in the deterministic model it will follow that

$$N_o'(t) = \lambda X_o'(t).$$

We indicate an armwaving proof of the first assertion. The quantity $N_o(t)$ is a sum of average contribution to the flow at

the origin at time t, as a result of messages originating at
repeaters less than t   units in distance and times earlier
than t. The average contribution from each repeater is
a _constant_ (not with time) at each fixed time point and
repeater, multiplied by λ, the average generation rate.
(the constant is given by the calculations in section II
and depends on the coordinates (d,j) and time. Thus λ factors
from the sum and $N_o(t)$ is λ multiplied by the total flow resulting
from a single message originating at each repeater at each
point in time.

   We now make the specific assumption.

   At each point in time and at each repeater, a single new
message is originated.

   Under this model to compute $X_o(t)$ and hence $N_o(t)$ is trivial.
To fix ideas we depict the situation at three time points

t = 0
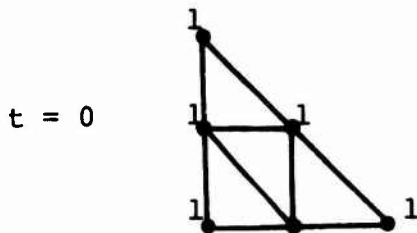
At time zero one message originates at each repeater, hence the
message flow is $X_o(0)=1$.

t=1

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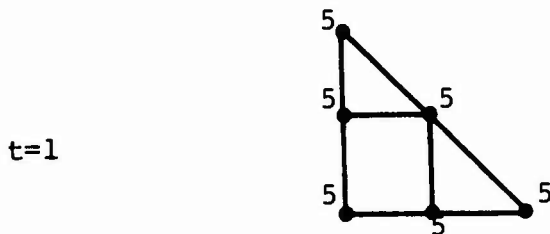
At time 1 each repeater receives 5 messages, one from each of four nearest neighbors and one new message.

$$t = 2,$$


```
21
21 ──── 21
21 │ │
21 │____│ 21
```
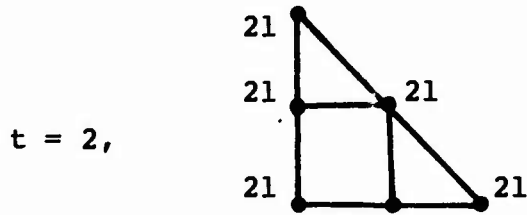
At time $t = 2$ each repeater receives 5 messages from each of its four nearest neighbors and one new message for a total of 21.

To compute $X_o(t)$ in general we note that each repeater is statistically identical in terms of message flow. Hence,

$$X_o(t) = 4X_o(t-1) + 1 \qquad t \geq 1$$

$$X_o(0) = 1.$$

This difference equation is trivial to solve and hence,

$$X_o(t) = \frac{4^{t+1}-1}{3} \qquad t \geq 0.$$

Thus

$$N_o(t) = \frac{\lambda}{3}(4^{t+1}-1)$$

To compute $N_o'(t)$ we consider the same deterministic model and compute $X_o'(t)$, the number of distinct messages which arrive at the origin for the first time at time t. It is easy to compute $X_o'(t)$ from the following table by summing contributions.

Time of Origination	Distance from Origin	Number of Messages
o	t	4t
1	t-1	4(t-1)
2	t-2	4(t-2)
$\vdots$	$\vdots$	$\vdots$
t-1	1	4
t	0	1

The first column is the time the message first appeared in the system if it is received by the origin for the first time at time t. The second column indicates the distance from the origin that the message originated. The third column indicates the number of message originated at that time and distance which are received at the origin at time t. Thus,

$$X'_o(t) = 4t + 4(t-1) + .. + 4 + 1 = \sum_{j=1}^{t} 4j + 1$$
$$= 2t^2 + 2t + 1, \quad t \geq 0 .$$

Thus,

$$N'_o(t) = \lambda(2t^2 + 2t + 1)$$

The inefficiency of this "undamped" network is

$$I_{eff}(t) = \frac{\frac{\lambda}{3}(4^{t+1} - 1)}{\lambda(2t^2 + 2t + 1)} = \frac{4^{t+1} - 1}{3(2t^2 + 2t + 1)} \sim \frac{4^{t+1}}{6t^2}$$

The inefficiency grows rapidly with time for this undamped system.

We can now put restrictions on the operating characteristics
of the repeaters and message flow.

12.1. No Message Can Be Transmitted More Than k Times

In this mode it is assumed that each message has a counting
feature whereby each time the message is repeated   the counter
is updated by one unit. When the counter reaches the number k
the message is no longer repeated   and disappears from the system.

In this mode we compute;

a) $N_o(t)$ = average number of messages received by the origin
        at time t.

b) $N_o'(t)$ = average number of distinct messages received
        at the origin for the first time at time t.

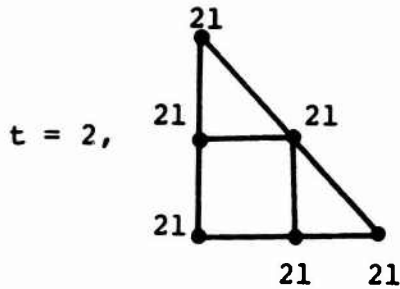$$\text{Ieff}(t) = \frac{N_o(t)}{N_o'(t)}$$

Once again by the argument presented in the previous section
it suffices to consider the deterministic model and to compute
$X_o(t)$ and $X_o'(t)$. To fix ideas we diagram the first 6 time points,
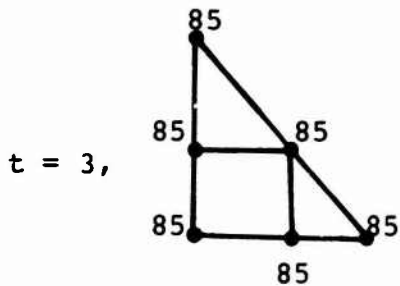inherently assuming $k \geq 5$.

t = 0,
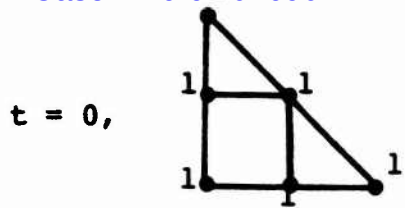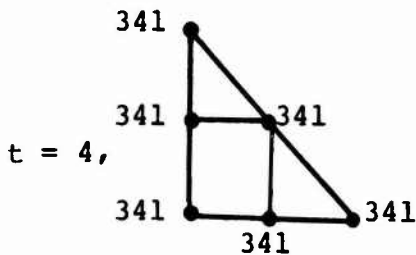
one message at each repeater,
all of age zero

t = 1,
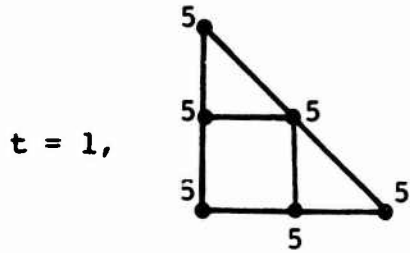
five messages each repeater:
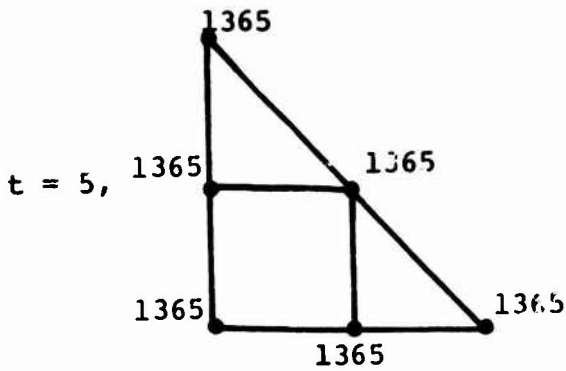one of age zero
four of age one

t = 2,

21 messages at each repeater

1 of age zero
4 of age one
16 of age two

t = 3,

85 messages at each repeater

64 of age three
16 of age two
4 of age one
1 of age zero

t = 4,

341 messages at each repeater

256 of age four
64 of age three
16 of age two
4 of age one
1 of age zero

11.105

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1365 Messages	
1024	of age 5
256	of age 4
64	of age 3
16	of age 2
4	of age 1
1	of age 0
1365	TOTAL

Let $X_o^{>k}(t)$ be the number of messages arriving at the origin at time t whose age is less than k. That is, those that will be transmitted. Since the flow at all repeaters is statistically identical:

$$\dot{X}_o(t) = 4X_o^{>k}(t-1) + 1 , \qquad t \geq 1 .$$

The number of messages received at the origin at time t is four times the number that will be transmitted by any of the four nearest neighbors plus one new one. Once again this is trivial difference equation to solve in k. We obtain as a solution:

$$X_o(t) = \frac{4^{k+1}-1}{3} \qquad t \geq k,$$

$$X_o(t) = \frac{4^{t+1}-1}{3} \qquad t \leq k.$$

Hence by the arguments above;

$$N_o(t) = \begin{cases} \frac{\lambda[4^{k+1}-1]}{3} & t \geq k \\ \\ \frac{\lambda[4^{t+1}-1]}{3} & t < k, \end{cases}$$

.. similar analysis shows that,

$$N_o'(t) = \begin{cases} 2k^2 + 2k + 1, & t \geq k, \\ 2t^2 + 2t + 1 & t \leq k. \end{cases}$$

Thus:

$$Ieff(t) = \frac{\frac{\lambda}{3}[4^{k+1}-1]}{\lambda(2k^2 + 2t + 1)} \qquad t \geq k.$$

$$= \frac{4^{k+1}-1}{3(2k^2 + 2k +1)} \sim \frac{4^{k+1}}{6k^2} = \frac{2\cdot4^k}{3k^2}.$$

In tabular form for k=1, 2, 3, 4, 5 the inefficiencies are given

Ieff	1	1.62	3.40	8.3	22.4	....
k	1	2	3	4	5	....

12.2 If the Same Message Arrives From Different Sources Only one Transmission is Made.

In this mode of operation a repeater has the ability to compare messages which arrive at the same instant. We may consider this mode to be a "memory" type system of length of time "one unit" or instantaneous. We seek to compute $N_o(t)$ and $N_o'(t)$. Again we consider the deterministic model. To fix ideas let us examine some early time points and compute multiplicities. All repeaters in this case also have statistically identical flows.
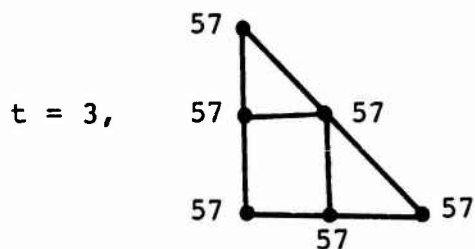

footer: 11.107
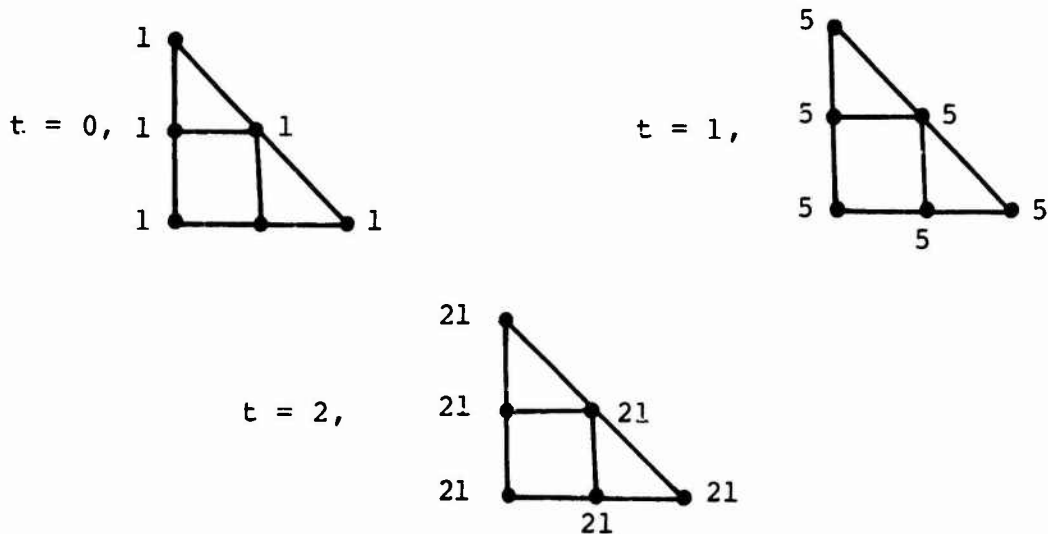
At time t = 2 all repeaters receive 21 messages,and they are statistically identical. Of the 21 total messages received at the origin only 14 are distinct.

We can partition this total using a table as follows:

Origin of Message	Time of Origin	Number Distinct
(0,0)	t=2	1
(1,1), (1,0)	t=1	4
(2,0),(2,1), 2,2)	t=0	8
(0,0)	t=0	1

The total distinct is 14,

Hence the diagram for t=3 is,

t = 3,

of the 57 = 4·14 +1, one is new and 14 came from each of four nearest neighbors.

The same method can be used to compute $X_0(2t)$ and $X_0(2t-1)$ in general. Let $X_0^d(2t)$ and $X_0^d(2t-1)$ be the number of distinct messages received at the origin at time $(2t)$ and $(2t-1)$ respectively. Clearly, these quantities satisfy;

$$X_0(2t) = 4X_0^d(2t-1) +1 \quad \text{and}$$
$$X_0(2t+1) = 4X_0^d(2t) +1.$$

Thus it suffices to compute $X_0^d(2t)$ and $X_0^d(2t-1)$.

To compute $x_o^d(2t)$ and $x_o^d(2t-1)$ we decompose each as follows:

$$x_o^d(2t) = x_o^{d,1}(2t) + x_o^{d,2}(2t) + x_o^{d,3}(2t) + \ldots + x_o^{d,t+1}(2t),$$

$$x_o^d(2t-1) = x_o^{d,1}(2t-1) + x_o^{d,2}(2t-1) + \ldots + x_o^{d,t}(2t-1).$$

Where $x_o^{d,v}(t)$ = number of distinct messages received at the origin at time t for the vth time. The quantities $x_o^{d,v}(t)$ are computed by the following table, which essentially decomposes $x_o^{d,v}(t)$ by distance and time of origination of each message contributing to $x_o^{d,v}(t)$. For $x_o^d(2t-1)$;

1st time —

time of origin.	0	1	$\cdots$	2t-2	2t-1
dist. at origin	2t-1	2t-2	$\cdots$	1	0

2nd time —

time of origin.	0	1	$\cdots$	2t-4	2t-3
dist. of origin.	2t-3	2t-4	$\cdots$	1	0

$\cdots$

(t-1)th time —

Time of origin.	0	1	2	3
dist. at origin.	3	2	1	0

tth time —

time of origin.	0	1
dist at origin.	1	0

The grand sum is;

$$x_o^{d,1} = 1 + 4\sum_{j=1}^{2t-1}(2t-j) \; , \quad x_o^{d,2} = 1 + 4\sum_{j=3}^{2t-1}(2t-j), \ldots, \quad x_o^{d,t} = 4\sum_{j=2t-1}^{2t-1}(2t-j)$$

since the number of messages originating at distance d is 4d as we have seen earlier. Therefore,

$$x_o^d(2t-1) = t + 4\sum_{k=1}^{t}\sum_{j=2k-1}^{2t-1}(2t-j)$$

After summing we obtain:

$$x_o^d(2t-1) = \frac{t(4t+1)(2t+1)}{3},$$

By a similar analysis we can show that

$$x_o^d(2t) = \frac{(4t+3)(2t+1)(t+1)}{3},$$

Thus,

$$X_o(2t) = 4 \cdot x_o^d(2t-1) + 1 = \frac{4t(4t+1)(2t+1)}{3} + 1 \quad , \quad \text{or}$$

$$X_o(2t) = \frac{(4t+3)(8t^2+1)}{3} .$$

Similarly,

$$X_o(2t+1) = 4 \cdot x_o^d(2t) + 1$$

$$= \frac{4(4t+3)(2t+1)(t+1)}{3} + 1$$

$$= \frac{32t^3+72t^2+52t+15}{3} .$$

Hence:

$$N_o(2t) = \frac{\lambda(4t+3)(8t^2+1)}{3} ,$$

$$N_o(2t+1) = \frac{\lambda(32t^3+72t^2+52t+15)}{3} ,$$

$$N_o^1(2t) = \lambda(2t^2+2t+1) = \lambda(8t^2+4t+1) .$$

The efficiency of this mode is:

$$Eff(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3} .$$

12.3 A Repeater Never Transmits The Same Message More Than Once Except Upon Initial Reception

This mode of operation implies infinite but not instantaneous memory in the repeater. We analyze the deterministic case with no loss of generality.

Pictorially the first few time points appear as follows;
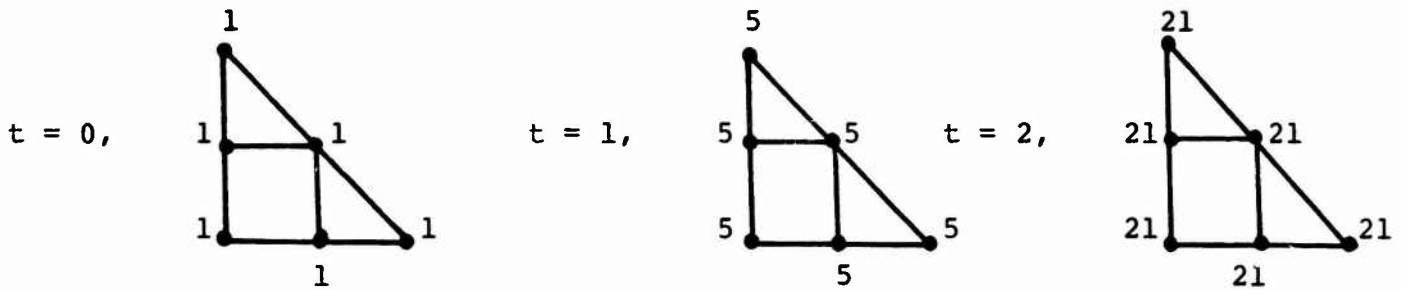
Of the 21 messages received at t=2 there are 17 which are received for the first time. These can be enumerated by point and time of origin: 1 new one at (t=2,d=0), 4 at (t=1, d=1), 12 at (t=0, d=2). Thus there will be 69 messages received at each repeater at t=3.

In general let:

$X_o(t)$ be the number of messages received at the origin at time t, and $X_o^1(t)$ be the number of messages received at the origin for the first time at time t. Then,

$$X_o(t) = 4 X_o^1(t-1) + 1 .$$

If a message was received for the first time at any repeater at time t-1 it originated at distance t-1 at time 0, or at d=t-2 at time 1 or, ..., at distance 0 at time t-1. Summing these by their appropriate multiplicities we obtain,

$$X_o^1(t-1) = 4 \sum_{k=2}^{t} \sum_{j=0}^{t-k} \binom{t-k+1}{0} \binom{t-k+1}{j} + 1 \, ,$$

or,

$$X_o^1(t-1) = 2^{t+2} - 4t - 3 \qquad \text{and} \, ,$$

$$X_o(t) = 2^{t+4} - 16t - 11 \, .$$

Similarly;

$$N_o(t) = \lambda (2^{t+4} - 16t - 11) \quad \text{and as before}$$

$$N_o^1(t) = \lambda (2t^2 + 2t + 1)$$

The efficiency of this mode is:

$$\text{Eff} = \frac{N_o(t)}{N_o^1(t)} = \frac{2^{t+4} - 16t - 11}{2t^2 + 2t + 1} \sim \frac{2^{t+3}}{t^2} \, .$$

12.4   Mixture of Modes 12.2 and 12.3.

In this mixed mode a repeater has infinite memory for comparison of messages and instant memory. The result (operational) of this mixed mode is that when multiple reception of the same message is received only one transmission is made. Furthermore if a message is received for the second time it is not transmitted at all. The result of this combined mode is to reduce the transmission in the instantaneous mode to only messages received for the first time.

Therefore:

$$X_o(t) = 4X_o^{',d}(t-1) + 1$$

where

$X_o^{',d}(t)$ is the number of distinct messages received at the origin for the first time at time t. This quantity has been previously computed to be,

$$X_o^{',d}(t-1) = 2(t-1)^2 + 2(t-1) + 1 = 2t^2 - 2t - 1.$$

It follows that,

$$X_o(t) = 8t^2 - 8t - 3, \text{ and}$$

$$N_o(t) = \lambda(8t^2 - 8t - 3).$$

The efficiency of this mixed mode is seen to be

$$Eff(t) = \frac{\lambda(8t^2-8t-3)}{\lambda(2t^2+2t+1)} \quad \frac{8t^2-8t-3}{2t^2+2t+1} \sim 4 \quad .$$

12.5   Mixture of Modes 12.1 and 12.2

In this mixture of modes no message can be transmitted more than k-times and each repeater has instantaneous memory but not finite memory. This mode also provides an upper bound to the case where each repeater has zero capture and messages are dropped after k transmissions.

A little analysis will show that for $t \leq k$ there is no change in the message flow from the instantaneous memory case. For $t \geq k$ the exact same analysis as in the instantaneous memory only case shows that the formulas are exactly those except that t should be replaced by k.

Therefore in this mixed mode, for memory of k.

$$X_o(2t) = \frac{(2k+3)(8k^2+1)}{3}, \quad t \geq [\frac{k}{2}]$$

$$= \frac{(2t+3)(8t^2+1)}{3}, \quad t \leq [\frac{k}{2}]$$

$$X_o(2t+1) = \frac{32k^3+72k^2+52k+15}{3} \qquad t \geq k$$

$$= \frac{32t^3+72t^2+52t+15}{3} \qquad t \leq k.$$

Furthermore,

$$N_o(2t) = X_o(2t),$$

$$N_o(2t+1) = X_o(2t+1).$$

We arrive at the same result with t replaced by k.
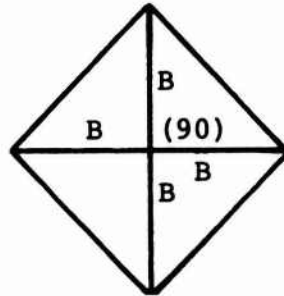
$$IEff(2t) = \frac{(4k+3)(4k^2+1)}{3(8k^2+4k+1)} \sim \frac{4}{3} k.$$

In tabular form;

IEff	1.33	2.67	4.0	5.3	6.67	8.0
k	1	2	3	4	5	6

12.6   A Closed Boundary Model

In this part of the model it is assumed that each repeater is less than B units from the origin. As in earlier cases, the units are measured in steps. The region is now closed and appears as follows:

Furthermore, in the initial stages of this model, it is assumed that a single message originates at the origin at time t=0. All message flow is generated as a result of this single message. Unfortunately, this model causes a loss of symmetries which facilitated the ease of obtaining closed form solutions for message flow in the previous models. However, some closed form analysis can be obtained. In particular it is not difficult to obtain an algorithm which will supply a complete analysis of message flow at any point in time on any repeater or station.

Let $N^d_{B,j}(2t+d)$ be the number of messages received at a station or repeater which is d-units from the origin and j-units from the y-axis at time $(2t+d)$ where $d=0, 1, 2,\ldots,B-1$, $j=0, 1, 2,\ldots B$.

The equations for $N_{B,j}^d (2t+d)$ are identical to the equations for the open boundary case when $0 \leq t \leq B-d$ since the closing of the boundary does not affect message flow at a repeater at distance B-d until t=B+d. Therefore;

$$N_{B,j}^d (2t+d) = \binom{2t+d}{t}\binom{2t+d}{t+j}; \quad t \leq \frac{B}{2}.$$

We can use this equation to compute message flow at all time points up to and including t=B. To compute message flow after time B, we can work backwards from the boundary and successively compute message flow at each station or repeater for any time point using the equation we will develop. First however, it is helpful to examine a particular case with a diagram and compare the closed boundary with the open boundary. We let B=4.

Closed Boundary at B=4   Open Boundary

The closing of the boundary will not affect any repeaters or stations. until t=6.

11.116

t=4

t=5

t=6

t=7

t=8
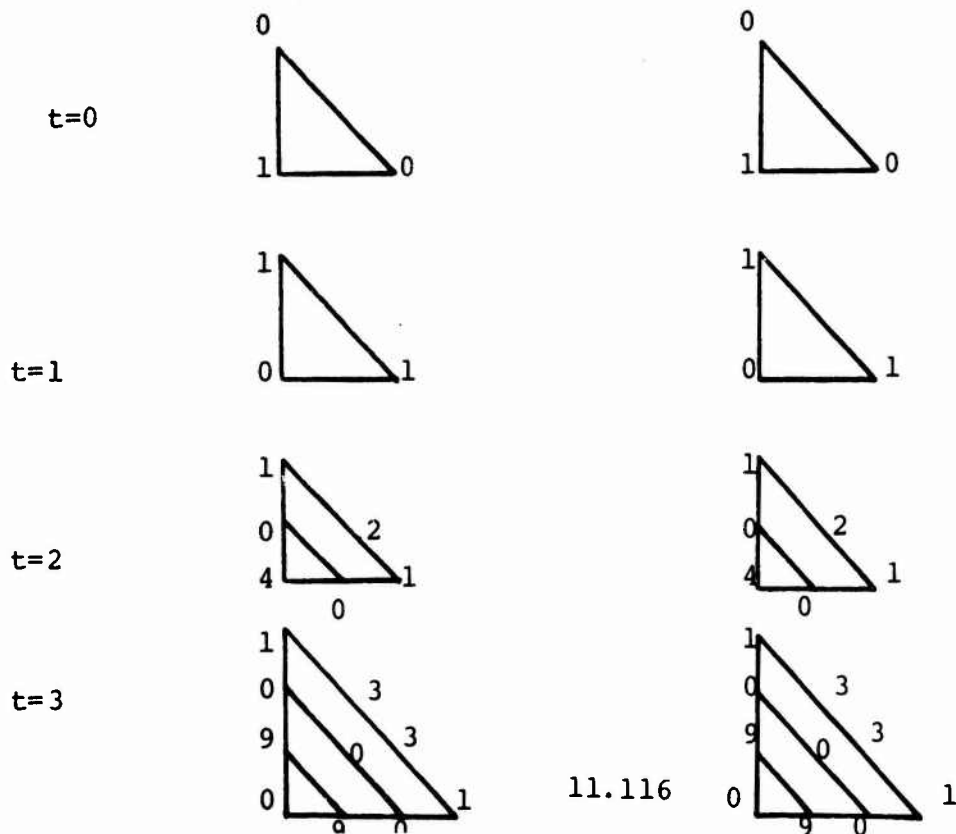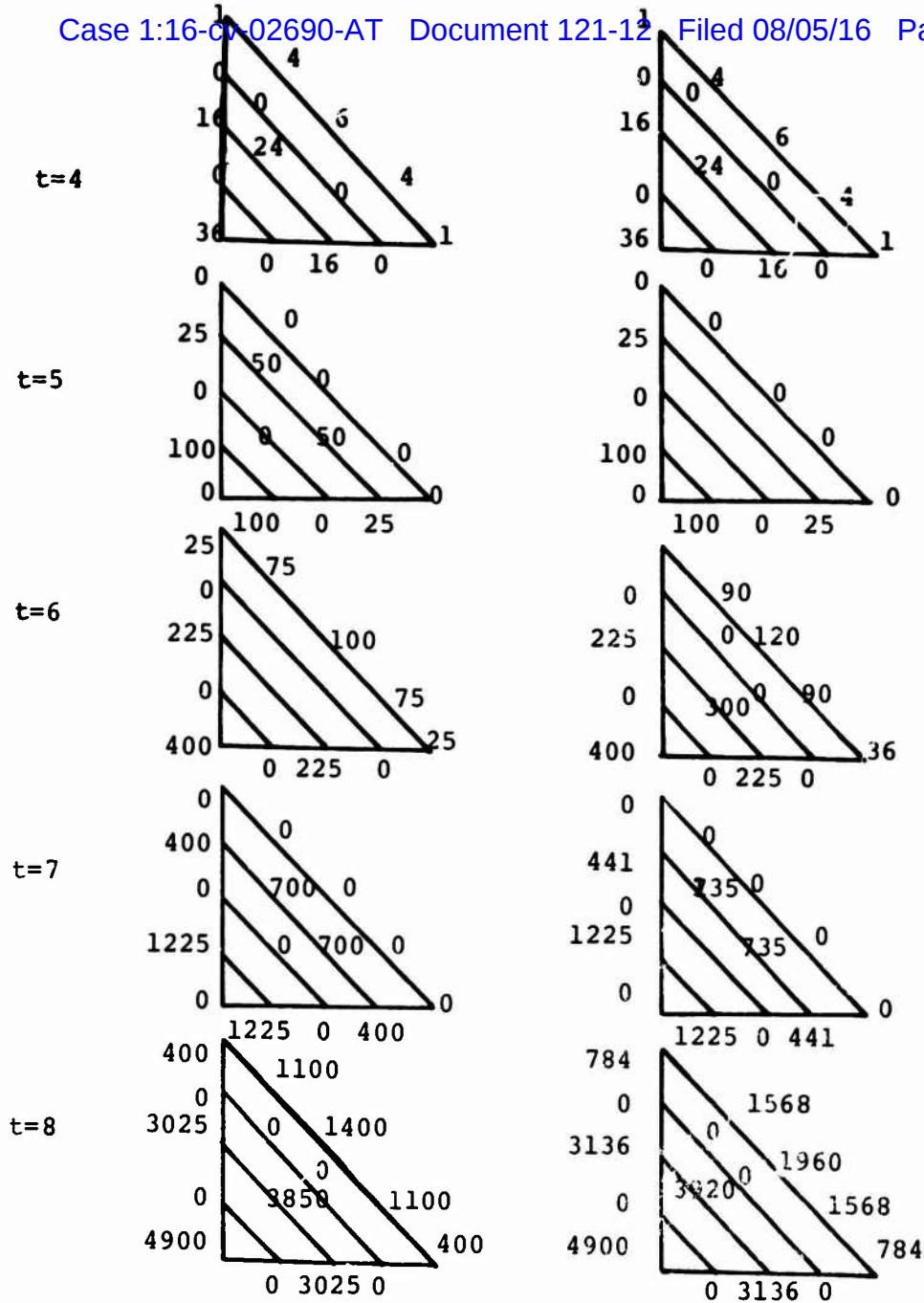
It is obvious how to continue for as long as one wishes to obtain the message flow. Since all message flow can be computed by the formulas of the preceeding section for $t \leq B$, the message flow with closed boundary can be computed by backward iteration for all times larger than B. The initial conditions at time t=B are given by the following relations,

At time t=B

$$N^{d}_{B,j}(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise.} \end{cases}$$

11.117

since $N^d_{B,j}(B)$ is zero unless B and d are both odd or even numbers.

At time t=B+1

$N^B_{B,j}(B+1)=0$, i.e. no messages at the boundary,

$$N^d_{B,j}(B+1) = \begin{cases} \binom{B+1}{k}\binom{B+1}{k+j} & \text{if } B=2k+d-1, \\ 0 & \text{otherwise.} \end{cases}$$

At time t=B+2

On the boundary:

$$N^B_{B,0}(B+2) = N^{B-1}_{B,0}(B+1) = \binom{B+1}{1}^2 = \binom{B+1}{B}^2 = N^B_{B,B}(B+2).$$

$$N^B_{B,j}(B+2) = N^{B-1}_{B,j}(B+1) + N^{B-1}_{B,j-1}(B+1)$$

$$= \binom{B+1}{B}\binom{B+2}{j+1} = N^B_{B,B-j}(B+2)$$

$$\text{for } j=1, 2,\ldots,B-1.$$

for d < B

$$N^d_{B,j}(B+2) = \begin{cases} \binom{B+2}{k}\binom{B+2}{k+j} & \text{if } B=2k+d-2, \\ 0 & \text{otherwise} \end{cases} \qquad j=0, 1, 2,\ldots,d.$$

At time t=B+3

On the boundary

$$N^B_{B,j}(B+3) = 0: \qquad j=0, 1, 2,\ldots,B.$$

At distance d=B-1

$$N^{B-1}_{B,0}(B+3) = N^B_{B,0}(B+2) + N^{B-2}_{B,0}(B+2) + 2N^B_{B,1}(B+2)$$

$$= [\binom{B+1}{B}+\binom{B+2}{B}]^2 = N^{B-1}_{B,B-1}(B+3).$$

11.118

$$N^{B-1}_{B,j}(B+3) = N^{B-2}_{B,j-1}(B+2) + N^{B}_{B,j+1}(B+2) + N^{B-2}_{B,j}(B+2)$$

$$+ N^{B}_{B,j}(B+2)$$

$$= \binom{B+3}{j+2} [\binom{B+1}{B} + \binom{B+2}{B}] = N^{B-1}_{B,B-1-j}(B+3).$$

$$j=1, 2,\ldots,B-2.$$

At distance $d < B-1$

$$N^{d}_{B,j}(B+3) = \begin{cases} \binom{B+3}{k}\binom{B+3}{k+j} & B=2k+d-3 \\ 0 & \text{otherwise} \end{cases} \qquad j=0, 1,\ldots,d.$$

At time $t=B+4$

On the boundary

$$N^{B}_{B,0}(B+4) = N^{B-1}_{B,0}(B+3) = [\binom{B+1}{B}+\binom{B+2}{B}]^2 = N^{B}_{B,B}(B+4).$$

$$N^{B}_{B,1}(B+4) = N^{B-1}_{B,0}(B+3) + N^{B-1}_{B,1}(B+3)$$

(Note that $N^{B}_{B,1}(B+4)$ must be computed separately since $N^{B-1}_{B,0}(B+3)$ and $N^{B-1}_{B,j}(B+3)$ for $j>1$ do <u>not</u> share a common formula, i.e. are not special cases of a general formula.)

$$N^{B}_{B,1}(B+4) = N^{B}_{B,B-1}(B+4) = [\binom{B+1}{B}+\binom{B+2}{B}][\binom{B+1}{B}+\binom{B+2}{B}+\binom{B+3}{B}]j$$

$$N^{B}_{B,j}(B+4) = N^{B-1}_{B,j-1}(B+3) + N^{B-1}_{B,j}(B+3)$$

(Note the formula is the same for $j=1$ as above.)

$$= [\binom{B+1}{B}\binom{B+2}{B}][\binom{B+4}{j+2}] = N^{B}_{B,B-j}(B+4)$$

$$j=2,\ldots,B-2.$$

At distance $d=B-2$

$$N^{B-2}_{B,0}(B+4) = N^{B-3}_{B,0}(B+3) + N^{B-1}_{B,0}(B+3) + 2 N^{B-1}_{B,1}(B+3)$$

$$= [\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}]^2 = N^{B-2}_{B,B-2}(B+4)$$

$$N_{B,j}^{B-2}(B+4) = N_{B,j-1}^{B-3}(B+3) + N_{B,j+1}^{B-1}(B+3) + N_{B,j}^{B-1}(B+3)$$

$$+ N_{B,j}^{B-3}(B+3).$$

$$= \binom{B+4}{j+3}[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}]$$

$$= N_{B,B-j-2}^{B-2}(B+4).$$

for $d < B-2$

$$N_{B,j}^{d}(B+4) = \begin{cases} \binom{B+4}{k}\binom{B+4}{k+j} & \text{if } B=2k+d-4 \\ 0 & \text{otherwise} \end{cases}$$

At time t=B+5

at distance d=B-1

$$N_{B,0}^{B-1}(B+5) = N_{B,0}^{B-2}(B+4) + N_{B,0}^{B}(B+4) + 2\, N_{B,1}^{B}(B+4)$$

$$= [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]^2 = N_{B,B-1}^{B-1}(B+5)$$

$$N_{B,1}^{B-1}(B+5) = N_{B,0}^{B-2}(B+4) + N_{B,2}^{B}(B+4) + N_{B,1}^{B-2}(B+4)$$

$$+ N_{B,1}^{B}(B+4)$$

$$= [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]$$

$$\cdot [\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}] = N_{B,B-1}^{B-1}(B+5).$$

$$N_{B,j}^{B-1}(B+5) = N_{B,j-1}^{B-2}(B+4) + N_{B,j+1}^{B}(B+4) + N_{B,j}^{B-2}(B+4)$$

$$+ N_{B,j}^{B}(B+4)$$

$$= \binom{B+5}{j+3}[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]$$

$$= N_{B,B-j-1}^{B-1}(B+5).$$

$$j=2,\ldots,B-3.$$

<u>At distance B-3</u>

$$N_{B,0}^{B-3}(B+5) = N_{B,0}^{B-4}(B+4) + N_{B,0}^{B-2}(B+4) + 2N_{B,1}^{B-2}(B+4)$$

$$= [\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-3}^{B-3}(B+5).$$

$$N_{B,j}^{B-3}(B+5) = N_{B,j-1}^{B-4}(B+4) + N_{B,j+1}^{B-2}(B+4)$$

$$+N_{B,j}^{B-4}(B+4) + N_{B,j}^{B-2}(B+4)$$

$$= \binom{B+5}{j+4}[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N_{B,B-j-3}^{B-3}(B+5) \qquad j=1, 2,\ldots,B-4.$$

<u>for d < B-3</u>

$$N_{B,j}^{d}(B+5) = \begin{cases} \binom{B+5}{k}\binom{B+5}{k+j} & \text{if } B = 2k+d-5 \\ \\ 0 & \text{otherwise.} \end{cases}$$
$$j=0, 1, 2,\ldots,d.$$

We will continue for three more time points to get some idea as to how the solution behaves with time. We will omit the general equations and give only the results since it is clear that the expressions become too cumbersome for easy comprehension.

<u>For time t=B+6</u>

at the boundary

$$N_{B,0}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]^2 = N_{B,B}^{B}(B+6)$$

$$N_{B,1}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][3\binom{B+1}{B} + 3\binom{B+2}{B}+2\binom{B+3}{B}+\binom{B+4}{B}]$$

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$$N_{B,2}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]$$

$$N_{B,j}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+6}{J+3}]$$

$$j=3, 4,\ldots,B-3.$$

At distance $d=B-2$

$$N_{B,0}^{B-2}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-2}^{B-2}(B+6)$$

$$N_{B,1}^{B-2}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]$$

$$\cdot [\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}] = N_{B,B-3}^{B-2}(B+6)$$

$$N_{B,j}^{B-2}(B+6) = \binom{B+6}{j+4}[3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}] = N_{B,B-2-j}^{B-2}(B+6)$$

$$j=2,\ldots,B-4.$$

At distance $d=B-4$

$$N_{B,0}^{B-4}(B+6) = [\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]^2 = N_{B,B-4}^{B-4}(B+6)$$

$$N_{B,j}^{B-4}(B+6) = \binom{B+6}{j+5}[\binom{B+1}{B} + \binom{B+2}{B} + \cdots + \binom{B+5}{B}]^2 = N_{B,B-4-j}^{B-4}(B+6)$$

$$j=1, 2,\ldots,B-5.$$

For $d < B-4$

$$N_{B,j}^{d}(B+6) = \begin{cases} \binom{B+6}{k}\binom{B+6}{k+j} & \text{if } B=2k+d-6 \\ 0 & \text{otherwise.} \end{cases}$$

At time $t=B+7$

At distance $d=B-1$

$$N_{B,0}^{B-1}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-1}^{B-1}(B+7).$$

$$N_{B,1}^{B-1}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{3} + \binom{B+4}{B}]$$

$$\cdot\ [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]$$

$$= N_{B,B-2}^{B-1}(B+7).$$

$$N_{B,2}^{B-1}(B+7) = \left(\sum_{j=1}^{6}\binom{B+j}{B}\right)\ [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N_{B,B-3}^{B-1}(B+7).$$

$$N_{B,j}^{B-1}(B+7) = \binom{B+7}{j+4}[5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}].$$

$$j=3,\ldots,B-4.$$

At distance d=B-3

$$N_{B,0}^{B-3}(B+7) = [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]^2$$

$$N_{B,1}^{B-3}(B+7) = \left[\sum_{j=1}^{6}\binom{B+j}{B}\right][4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]$$

$$N_{B,j}^{B-3}(B+7) = \binom{B+7}{j+5}[4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]$$

$$j=2,\ldots,B-5.$$

At distance B-5

$$N_{B,0}^{B-5}(B+7) = \left[\sum_{j=1}^{6}\binom{B+j}{j}\right]^2 = N_{B,B-5}^{B-5}(B+7)$$

$$N_{B,j}^{B-5}(B+7) = \left[\sum_{j=1}^{6}\binom{B+j}{j}\right]\binom{B+7}{j+6}.\quad j=1,\ 2,\ldots,B-6.$$

At d < B-5

$$N_{B,j}^{d}(B+7) = \begin{cases} \binom{B+7}{k}\binom{B+7}{k+j} & \text{if } B=2k+d-7 \\ 0 & \text{otherwise} \end{cases}$$

At time t=B+8

at the boundary

$$N_{B,0}^{B}(B+8) = N_{B,B}^{B}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]^2$$

$$N_{B,1}^{B}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+2}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N_{B,2}^{B}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot [5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N_{B,3}^{B}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$N_{B,j}^{B}(B+8) = \binom{B+8}{B}[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$j=4,\ldots,B-4.$$

At distance d=B-2

$$N_{B,0}^{B-2}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]^2$$

$$N_{B,1}^{B-2}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$\cdot [5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N_{B,2}^{B-2}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N_{B,j}^{B-2}(B+8) = \binom{B+8}{B}[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

At distance d=B-4

$$N_{B,0}^{B-4}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]^2$$

$$= N_{B,B-4}^{B-4}(B+8)$$

$$N_{B,1}^{B-4}(B+8) = [\sum_{k=1}^{7}\binom{B+k}{B}][5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N_{B,j}^{B-4}(B+8) = \binom{B+8}{j+6}[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$j=2,\ldots,B-6.$$

At distance d < B-4

$$N_j^d(B+8) = \begin{cases} \binom{B+8}{k}\binom{B+8}{k+j} & \text{if } B=2k+d-8 \\ \\ 0 & \text{otherwise.} \end{cases}$$

This completes an analysis of the first eight time points past the time to the boundary. The expressions are quite unwieldy. However, the algorithm is clear and can compute message distribution throughout the grid at any time point. We write down the general set of equations and then indicate a "closed form" combinatorial method to make "sense" out of the unwieldy expressions.

The General Equations

The initial conditions for t=B are:

At time t=B: Initial Conditions

$$N_{B,j}^d(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise} \end{cases}$$

The General Equations

Cn the boundary $k \geq 1$

1. $N_{B,0}^{B}(B+2k) = N_{B,B}^{B}(B+2k) = N_{B,0}^{B-1}(B+2k-1)$.

2. $N_{B,j}^{B}(B+2k) = N_{B,j}^{B-1}(B+2k-1) + N_{B,j-1}^{B-1}(B+2k-1)$

$$j=1, 2,\ldots,B-1.$$

Note that:

$$N_{B,j}^{B}(B+2k) = N_{B,B-j}^{B}(B+2k), \qquad j=0,\ldots,B.$$

Along the axes $0 < d < B$, $k=1, 2,\ldots$.

$$N_{B,0}^{d}(B+2k) = N_{B,0}^{d+1}(B+2k-1) + N_{B,0}^{d-1}(B+2k-1)$$

$$+ 2N_{B,1}^{d+1}(B+2k-1)$$

$$= N_{B,d}^{d}(B+2k)$$

Off the axes $0 < d < B$, $0 < j < d$

$$N_{B,j}^{d}(B+2k) = N_{B,j}^{d+1}(B+2k-1) + N_{B,j}^{d-1}(B+2k-1)$$

$$+ N_{B,j-1}^{d-1}(B+2k-1) + N_{B,j+1}^{d+1}(B+2k-1).$$

At the origin:

$$N_{B,0}^{0}(B+2k) = 4N_{B,1}^{1}(B+2k-1).$$

This completes the general equations.

A Conjecture on Solutions to the General Equation

An examination of the ccefficients of the $\binom{B+k}{B}$ expressions

in the first eight time points indicates that the coefficients

are the same as the rows in the following infinite sequence of tableaus.

T_1

j=1	2	3	4	5	6
k=1 1					
k=2 1	1				
k=3 2	2	1			
k=4 5	5	3	1		
k=5 14	14	9	4	1	
k=6 42	42	28	14	5	1
.	.	.	.	.	.
.	.	.	.	.	.

T_2

k	j	1	2	3	4	5	6	7
1		0	1					
2		1	1	1				
3		3	3	2	1			
4		9	9	6	3	1		
5		28	28	19	10	4	1	
6		90	90	62	34	15	5	1
		.	.	.	.	.	.	.
		.	.	.	.	.	.	.

T_3

k=1	0	0	1					
2	1	1	1	1				
3	4	4	3	2	1			
4	14	14	10	6	3	1		
5	48	48	34	20	10	4	1	
6	165	165	116	69	35	15	5	1
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.

T_4

k=1	0	0	0	1					
2	1	1	1	1	1				
3	5	5	4	3	2	1			
4	20	20	15	10	6	3	1		
5	75	75	55	35	20	10	4	1	
6	275	275	200	125	70	35	15	5	1
	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.

The tableaus through T_4 cover all the coefficients which arise up to and including t=B+8. It is possible to give a solution for $S_{kj}(T_\nu)$ which is the jth element in the kth row of the tableau T_ν, $\nu = 1, 2,...$ However, it is perhaps not obvious how these tableaus are generated. The jth term of the kth row of any tableau is obtained by summing the last $(k+\nu-j)$ terms in the previous row. That is, the third term in the fourth row of T_2 is obtained by summing the last three terms of the third row. In mathematical form the equation for the tableaus are:

Initial Condition $S_{1,j}(T_\nu) = \begin{cases} 0, j<\nu, \\ 1, j=\nu \\ 0, j>\nu. \end{cases}$

$$S_{k,1}(T_\nu) = S_{k,2}(T_\nu) = \sum_{j=1}^{k+\nu-2} S_{k-1,j}(T_\nu).$$

$$S_{kj}(T_\nu) = \sum_{w=j-1}^{k+\nu-2} S_{k,w}(T_\nu), \qquad k > 1$$

It is clear that the between tableau equations are:

$$S_{k,j}(T_\nu) = S_{k,j}(T_{\nu+1}) - S_{k-1,j}(T_{\nu+2}) \ , \ k > 1, \ \nu = 1, \ 2, \ldots$$

$$S_{k,j}(T_2) = S_{k+1,j}(T_1) - S_{k,j}(T_1).$$

Thus, to solve these equations it suffices to give a general formula for $S_{k,j}(T_1)$. The other terms can then be computed recursively. It is easy to check that the solution for T_1 is given by,

$$S_{k,j}(T_1) = \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad \begin{matrix} j=1, \ 2,\ldots,k \\ k=1, \ 2,\ldots. \end{matrix}$$

Then by recursion:

$$S_{k,j}(T_2) = \frac{j}{k+1}\binom{2k-j+1}{k} - \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad \begin{matrix} k=1, \ 2,\ldots. \\ j=1, \ 2,\ldots,k+1. \end{matrix}$$

$$S_{k,j}(T_3) = \frac{j}{k+2}\binom{2k-j+3}{k} + \frac{2j}{k+1}\binom{2k-j+1}{k}$$

$$\vdots$$

To apply these results we take a particular example to conjecture:

$$N_{B,0}^B(B+2k) = \left[\sum_{j=1}^{k} S_{k,j}(T_1)\binom{B+j}{B} \right]^2 \qquad , \qquad k \geq 1$$

$$= \left[\sum_{j=1}^{k} \frac{j}{k}\binom{2k-j-1}{k-1}\binom{B+j}{B} \right]^2$$

11.128

This conjecture holds for time points k=1, 2, 3, 4.

$$N_{B,0}^{B-1}(B+2k+1) = [\sum_{j=1}^{k+1} S_{k+1,j}(T_1)\binom{B+j}{B}]^2$$

This conjecture holds for all k=1, 2, 3, and satisfies the general equations.

13   SUMMARY OF RESULTS OF THEORETICAL MODELS

Recall that;

$N_o(t)$ = expected number of messages received at the origin at time t.

$N'_o(t)$ = expected number of distinct messages received at the origin for the first time at time t.

$$Eff(t) = \frac{N_o(t)}{N'_o(t)}.$$

These symbols are in the model where there is   Poisson input at each   repeater   at each time point.   In all modes;

$$N'_o(t) = (2t^2 + 2t + 1).$$

A.   Deterministic   The first model analyzed is the effect of a single message originally at the origin at time zero.   No other messages ever enter the system.   The propagation of this message can be measured by the following three quantities.

1) $B(t)$ — the number of   repeaters   which receive the message for the 1st time at time t.

$$\boxed{B(t) = 4t \text{ for } t \geq 1, \quad B(0) = 1}$$

2) $A(t)$ = the number of   repeaters   which have received the message by time t,

$$\boxed{A(t) = 2t^2 + 2t + 1.}$$

3. $N_j^d(t)$ = number of copies of the message received at time t at a   repeater   with coordinates (d,j).

$$N_j^d(t) = 0, \quad \text{if} \quad t < d$$

$$N_j^d(d) = \binom{d}{j} \quad j=0, 1,\ldots,d; \quad d=0, 1,\ldots\ldots,$$

$$N_j^d(d+2k+1) = 0 \quad k=0, 1, 2,\ldots \quad ,$$

$$N_j^d(d+2k) = \binom{d+2k}{k}\binom{d+2k}{k+j} \quad k=0, 1,\ldots \quad .$$

The quantity $N_j^d(d+2k)$ can be more generally interpreted as the number of copies of a message received at a repeater which is at distance d and horizontal distance j from the originating repeater, after d+2k time units.

B.   Poisson Input Results

1.   With no restrictions imposed on the operation of any repeater;

$$N_o(t) = \frac{\lambda}{3}(4^{t+1}-1)$$

$$N_o'(t) = \lambda(2t^2+2t+1)$$

$$Eff(t) = \frac{4^{t+1}-1}{3(2t^2+2t+1)} \sim \frac{4^{t+1}}{6t^2}$$

2.   **No Message Can Be Transmitted More Than k-Times**

$$N_o(t) = \frac{\lambda}{3}[4^{k+1}-1]$$

$$N_o'(t) = \lambda[2k^2+2k+1]$$

$$Eff(t) = \frac{4^{k+1}-1}{3(2k^2+2k+1)}$$

Eff	1	1.60	3.40	8.3	22.4
k	1	2	3	4	5

11.131

3. If the Same Message Arrives From Different Sources Only One Transmission is Made

This mode of operation assumes that a repeater   can compare all messages arriving at the same time and transmit only one copy of duplicate messages. It is inherently assumed in this mode that there is only instantaneous memory (which we may call 0-memory)

$$N_O(2t) = \lambda \frac{4t+3)(8t^2+1)}{3}$$

$$N_O'(2t) = \lambda(8t^2+4t+1)$$

$$IEff(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3}$$

This mode produces a substantial reduction in duplicate traffic.

4. All Messages Are Transmitted in the Direction of a Fixed Ground Station

In this mode it is assumed that repeater   "knows"   where the message should be received.

$$N_O(t) = \lambda \; 2^{t+3}+4t-7)$$

$$N_O'(t) = \lambda(2t^2+2t+1)$$

$$IEff(t) = \frac{2^{t=3}+4t-7}{2t^2+2t+1} \sim \frac{2^{t+2}}{t^2}$$

This mode is better than no mode, but holds little promise by itself.

5. A Repeater   Never Transmits The Same Message More Than Once, Except Upon Initial Reception

In this mode a repeater has "infinite" memory but <u>not</u> instantaneous memory.

$$N_O(t) = \lambda (2^{t+4} - 16t - 11)$$

$$N_O'(t) = \lambda (2t^2 + 2t + 1)$$

$$I_{Eff}(t) = \frac{2^{t+4} - 16t - 11}{2t^2 + 2t + 1} \sim \frac{2^{t+3}}{t^2}$$

It appears that infinite memory without instant memory does not "damp" the message flow significantly: in fact it is worse than "directionality" in the repeater.

6.   Infinite Memory Plus Instant Memory

If we combine the two modes of operation

$$N_O(t) = \lambda (8t^2 - 8t - 3)$$

$$N_O'(t) = \lambda (2t^2 + 2t + 1)$$

$$I_{Eff}(t) = \frac{8t^2 - 8t - 3}{2t^2 + 2t + 1} \sim 4$$

The effect of combining the instant memory feature with the instantaneous memory feature is to reduce the message flow in the instantaneous mode by a factor of $t/3$ which is significant.

7.   Instant Memory Plus no Message Transmitted More Than k-Times

In this mixed mode the analysis shows that all results in the instant memory case are the same except that the flow becomes independent of time after $t \geq k$. The inefficiency is

$$I_{Eff}(t) \sim \frac{4}{3}k$$

I_{Eff}	1.33	2.67	4	5.33	6.67	8
k	1	2	3	4	5	6

CHAPTER 12

TIME AND SPACE CAPTURE IN SPREAD SPECTRUM RANDOM ACCESS

I.   INTRODUCTION AND SUMMARY

When a receiving station for packet data communication is being accessed in a random access mode, the use of spread spectrum communication offers the possibility of time capture. That is, a packet may be distinguished and received correctly even if contending packets overlap the transmission as long as the signal strength of the contending packets is not too great. Moreover, if multiple receivers are available at the receiving station, spread spectrum coding can be used to allow the reception of several distinct packets being transmitted with overlap on a single channel.

When the transmitters are widely distributed, geometric or power capture is possible [Roberts; 1972]. With or without spread spectrum, if a competing signal is much weaker (further away), than the desired signal, there is no interference. Both types of capture can give rise to performance superior to that predicted by a simple unslotted ALOHA model. On the other hand, power capture gives rise to bias against the more distant transmitters because the close in transmitters overpower the ones further away. Thus, the probability $q(r)$ of a successful transmission at a distance r from the station will decrease as r increases and the number of retransmissions increase as well as delay.

The purpose of this chapter is to characterize the probability of successful transmission as a function of:

1.   distance from the receiver, r,

2.   the multiplicity, M, of receivers available at the receiving location,

3.   the packet arrival rate,

4.   the time bandwidth product K (extent of spectrum
spreading),

5.   the required signal to noise ratio SN of the
receivers,

6.   the exponent α in the inverse power law (s/r^{α})
assumed for the transmission power of a transmitter
at distance r from the receiver.

The situation we consider is that of a circular field of
transmitters of radius R accessing a receiving station in the
center with M receivers where R is the range of the receivers.
Arriving traffic is assumed to have a uniform rate per unit area
in the field and the arrivals plus retransmissions are assumed
to be Poisson distributed.

Particularly important for the design of a Packet Radio Sys-
tem is the analysis of multiple receiver stations. Adding extra
receivers to the Packet Radio Station adds throughput and shortens
delay. This is a very attractive approach because the modifi-
cation is simple and is applied only at the station. No modifica-
tions are required to the terminals, the repeaters, or their
geographical location. Finally, the enhancement is applied directly
where it is needed at the station bottleneck where all the infor-
mation flow of the system must eventually pass. The simulation
results show that multiple receivers will in fact increase perfor-
mance substantially; moreover, very few additional receivers are
required to realize most of the advantage (usually only one addi-
tional is required). For addition of receivers to be effective,
two prerequisites must be satisfied: (i) the traffic rate must
be quite high; otherwise, the chance of several messages arriving
simultaneously will be small and (ii) the spread spectrum factor
K must be sufficiently large compared to the required signal to
noise ratio SN; otherwise, even if there are sufficient receivers

to receive a number of simultaneous messages the interference created by the messages will cause none of them to be received correctly.

The q(r) function derived here will also be very useful in the location of repeaters to provide coverage for terminals. The simplest models require that each possible terminal location be "covered" by some number of repeaters in order to guarantee reliable communication. This type of model assumes a binary characterization of transmitter-repeater communication: either communication is possible or it is not. The q(r) function could be used to reflect the fact that a far away receiver does not provide as good service as a nearby one.

In Section 2, the model and the assumptions it is based upon are introduced. In Section 3, analysis is used to predict the qualitative behavior of the q(r) function especially with respect to limiting values of parameters. It was found, for example, that to a user at far distances, $r \to R$, the system performs as in unslotted ALOHA. That is, a transmission arriving at time t_o gets through if and only if no other transmission starts in the interval $[t_o-T, t_o+T]$ where T is a packet transmission time. On the other hand, for $r \to o$ for the one receiver case performance approximates slotted ALOHA in the sense that only packets preceeding, $t_o-T \leq t \leq t_o$, can interfere. Explicit bounds on the benefits of adding multiple receivers are also given. In Section 4, simulation is used to verify and extend these results. Finally, the simulation program is documented in an Appendix.

The first analysis of the geometric effect of capture for a circular field of packet transmitters accessing a receiver in a random access mode was [Roberts; 1972]. Roberts made (among others) the following assumptions:

  1.   the probability q of a packet being received
  correctly for a given transmission or retransmission
  is:

(a)   independent of distance r from the
receivers,

(b)   independent of whether the transmission is
the first or a retransmission,

2.   the probability of more than one contending packet
is negligible,

3.   noise is negligible compared to competing signal
strengths.

Kleinrock and Lam [1972] improved on these results by relaxing
assumption (1 b) to allow different q's for the initial transmission
and for retransmissions.   John Leung [1973] considered the same
problem where the signal power, instead of being represented as a
deterministic inverse square of distance, had a Rayleigh distribu-
tion to represent the effects of multipath.   One consequence of
this is that the receiver has unlimited range.   Leung also essen-
tially assumes (1) and (2) (see in particular, eq. 7, where $G(r)$
is apparently assumed proportional to r; i.e., that the rate of
arrivals plus retransmissions per unit area is independent of r.

Fralick [1972] also does an analysis of the field of terminals
situation, extending Roberts' results for the use of spread slotted
spectrum, including propagation delays.   He assumes (1), (2), and
(3).

Abramson   [1973] considers a central receiver in an infinite
field of transmitters and determines the "Sisyphus distance" R_s
which is the radius beyond which the expected number of retrans-
missions is infinite.   He assumes (2) and (3).   Thus, the critical
distance R_s is defined by competing signals while the R we use is
defined by noise considerations.   All the above models assumed
one receiver, M=1.

The model for spread spectrum reception is similar to those
described in [Kaiser; 1973, p. 2], [McGuire; 1973], and [Fralick;
1973a].   Fralick [1973a] considers several models of spread spectrum

12.4

random access the one closest to ours is the one using "Synch Preamble." In particular, our lower bound for q(o) is based on his analysis of this case. Also, Fralick pointed out an error in normalization in Section 2 that appeared in an earlier version of this note. For simplicity, we assume that the surface wave devices for encoding the spread spectrum code are programable and that the chip code varies from bit to bit in a pseudo-random manner so that competing signals appear more nearly like noise; this, for example, avoids many of the difficulties pointed out by Fralick [1973]. Programmable surface wave devices are discussed in [Staples and Claiborne; 1973] and [LaRosa; 1973].

2. MODEL AND ASSUMPTIONS

A central receiving station is receiving messages from an infinite number of terminals within a radius R where R is defined to be the range of the station; that is, the distance at which a terminal can just be heard. The terminals send packets according to a Poisson distribution with arrival rate density ρ packets per unit time per unit area. The principal objective is to determine the grade of service for terminals as a function of distance from the station. The grade of service $q(r)$ is defined to be the probability that a message sent by a terminal at radius r from the station will be received by the station on any one transmission (first and subsequent retransmissions are assumed to have the same probability of success [Kleinrock & Lam; 1972].)

Having $q(r)$ we can also define $G(r) = S(r)/q(r)$ to be the arrival plus retransmission rate density at radius r where $S(r)$ is the arrival rate density at radius r given by $2\pi r \rho$.

For our purposes, the receiving station can be in one of $M+1$ states; o receivers busy, 1 receiver busy, ..., M receivers busy. We will denote these states as $S_0, S_1, \ldots, S_M$. In state S_i (i<M), i receivers are busy and one of M-i remaining is awaiting a synchronization code from the beginning of some packet. If the receiver achieves synchronization with a new packet, the receiver is captured ($S_i \rightarrow S_i+1$) and is now busy (in a receiving state) and remains in that state for one packet transmission time, T. The new packet is correctly received if the total power of contending signals does not become too high during the transmission time. The receiver is busy for the full time T in either case. When all the receivers are busy, state S_M, all arriving packets are lost until one of the receivers becomes free. We specifically ignore the possibility of false alarms; that is, the receiver going from free to busy on the basis of noise. Thus, a packet arriving at time t is assumed to be received correctly by the station if the

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following three conditions are satisfied:

1.   a receiver is free at t.

2.   the signal strength is sufficiently stronger than ambient noise and other signals arriving in (T-t,t) so that the receiver achieves synchronization.

3.   signals arriving in (t, t+T) are not strong enough to drown out the signal after synchronization is achieved.

In our model of the receiver, we assume that a signal can be heard if the desired signal energy is sufficiently greater than the competing energy which consists of two elements: ambient noise and the other undesired signals. We are allowed to use spectrum spreading to increase discrimination between signal and the competing energy.

To be more specific, assume we divide each bit into a coded sequence of K chips. Further, suppose the receiver works by integrating the received signal correlated with the chip code. If the integral of the signal amplitude over one chip time is A, then, if there is correlation with the chip code, the received energy in one bit is $K^2A^2 = s/r^\alpha$ for a signal at distance r. If the signal is uncorrelated with the chip code, the received signal is the integral of a sum of K binomially distributed signals of amplitude A which for K reasonably large approaches a Gaussian distribution with mean o and variance $KA^2 = s/Kr^\alpha$. Thus, received signals in synchronization with the receiver deliver after correlation a factor of K more power than competing signals of the same strength which are uncorrelated with the receiver.

If we let N be the ambient noise per bit, the signal to noise energy ratio is:

$$\frac{\dfrac{s}{r_o^\alpha}}{\dfrac{1}{K} \left(\sum \dfrac{s}{r_i^\alpha}\right) + N} \geq SN \tag{1}$$

which we require to be no less than SN for reliable reception where r_o is the radius of the desired signal which is assumed to be in synch with the receiver's chip code and r_i, $i \neq o$ correspond to competing signals not in synch which effectively look like noise added to the ambient noise N.

The range R is determined by:

$$\frac{\dfrac{s}{R^\alpha}}{N} = SN, \text{ or}$$

$$R = \left(\frac{1}{N} \ \frac{s}{SN}\right) \ 1/\alpha \tag{2}$$

The parameters we wish to study are SN, α, and K; so, by judicious choice of units, we may normalize the units of power and distance, so that $s = 1$ and $R = 1$. This results in the decision criterion:

$$\frac{K \dfrac{1}{r_o^\alpha}}{\sum \dfrac{1}{r_i^\alpha} + \left(\dfrac{K}{SN}\right)} \geq SN \tag{3}$$

$$0 \leq r_i \leq 1 \quad i = 0, 1, \ldots$$

See [Kaiser; 1973] for a similar model.

3. PRELIMINARY ANALYSIS

The exact analysis of the probability, $q(r)$, of a message getting through on any given transmission from a radius r seems most difficult; however, we can get quite a good qualitative idea of $q(r)$ by analysis. This analysis will be supplemented by simulation result: in the next section.

Since the signal power is monotone decreasing with increasing radius and the message arrivals are independent, it is clear that $q(r)$ is monotone non-increasing. A message being transmitted, starting at time t from the critical radius $R = 1$, will be success-ful if, and only if, there is no other transmission in the interval $(t-T, t+T)$ since any additional signal will prevent reception. But this is exactly the situation for unslotted ALOHA. Thus, if G is the rate of message arrivals and retransmissions for the entire circular area of R radius from the receiver:

$$q(R) = e^{-2\ GT}. \tag{4}$$

Unfortunately, we do not yet know G. Clearly from (4) and the fact that $q(r)$ is monotone $G \leq G_u$ where G_u is the arrival plus retransmission rate for an unslotted ALOHA system; i.e., G_u is the solution of

$$S = G_u\ e^{-2G_u T}$$

where $S = \pi R^2 \rho$ is the total arrival rate. We can generalize (4) somewhat. For, even if $r < R = 1$, if r is sufficiently large, any other transmission can prevent it from being received. Thus, suppose we are considering a packet from r_o contending with a packet arriving from $r_1 = R = 1$, the weakest possible. The message from r_o can be heard whenever,

$$\frac{K \dfrac{1}{r_o^\alpha}}{1 + \dfrac{K}{SN}} \geqq SN$$

or

$$r_o \leqq \left(\frac{1}{1 + \dfrac{SN}{K}}\right)^{1/\alpha} = \bar{r} \; ;$$

On the other hand, if $r_o \geq \bar{r}$, any other message is sufficient to block reception. Therefore, we have,

$$q(r) = e^{-2GT} \geqq e^{-2G_u T}$$

$$\text{for } \bar{r} \leqq r \leqq 1. \tag{5}$$

As $r \to o$, the signal power becomes infinite, so that the probability of successful transmission depends only on whether a receiver is free or not. For the case where $M=1$ and with perfect time capture, Fralick [1973a] has shown that the probability the receiver is free is $\frac{1}{1+G}$. Since there isn't perfect time capture in our model, the probability the receiver is free is greater than that. So we have:

$$g(o) \geqq \frac{1}{1+G}.$$

Given there are one or more transmissions in $(t-T, t)$ the probability the receiver is captured is greater than the probability that one specific transmission, say the first in the interval, captures the receiver which is, in turn, greater than the probability that that specific transmission is received which is S/G on the average.

Thus,

q(o) = 1 - (Prob. of transmission in (t-T, t) times the
probability receiver is captured by one of the transmissions)

$$\leq 1 - (1 - e^{-GT}) \frac{S}{G}$$

yielding

$$\frac{1}{1+G} \leq q(o) \leq 1 - (1-e^{-GT}) \frac{S}{G} \tag{6}$$

If we add additional receivers we get improved performance
for small r, but not for $\bar{r} \leq r \leq 1$. In order for additional
receivers to be helpful, there must be sufficient traffic to keep
them busy but not so much that the interference caused by non-
synchronized messages overwhelms the desired one. We first de-
termine the maximum number of receivers which can all be correctly
receiving at the same time. This clearly happens when all signals
are at the same radius and that radius approaches zero. For n
receivers at radius ϵ with $\epsilon \to o$, (3) has the limit

$$\frac{K}{n} \geq SN$$

Thus, we see that

Maximum number of receivers $\leq$ K/SN $\tag{7}$

For example, if the spread spectrum factor K were 100 and the
required signal to noise power SN were 20, then at most, five
receivers could be simultaneously receiving correctly. This in-
dicates that the number of repeaters which can profitably be added
is small.

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Another factor affecting the utilization of multiple receivers is the amount of traffic in the system. Suppose the gross traffic including retransmissions is a Poisson Process with arrival rate G per second. Then, the number of messages being received at a given time t_0 is simply the messages which arrived in the interval (t_0-T, T) where T is the transmission time. The probability $P(k)$ of k active messages is then given by the Poisson distribution:

$$P(GT;k) = \frac{(GT)^k e^{-GT}}{k!} \qquad k = 0, 1, \ldots \qquad (8)$$

We define

$$Q(GT;k) = \sum_{i=o}^{k} P(GT;i)$$

For an M-receiver system, a new arrival at r=o will be received if less than M messages arrive in the previous T seconds (although sometimes a receiver will be free even if M or more arrived in the interval); thus,

$$q(o) \geqq Q(GT, M-1).$$

Similarly for $K \to \infty$ we have

$$q(r) \equiv \frac{S}{G} \geqq Q(GT, M-1).$$

We do not know G but we can define $\bar{G}$ to be the solution of

$$S = G \, Q(GT, M-1)$$

yielding the bound $\frac{S}{G} \geqq \frac{S}{\bar{G}}$ in the case $K \to \infty$ and $q(o) \geqq \frac{S}{\bar{G}}$ in general.

To get an upper bound, we consider a hypothetical system with perfect capture in which messages turn themselves off if they do not capture a receiver. In other words, a message is received correctly if and only if there are less than M receivers which are in the process of correctly receiving a message. For such a system,

$$q(r) \equiv \frac{S}{G} = Q \ (ST, \ M-1)$$

where we assume the packets received correctly follow a Poisson process with arrival rate S. Since messages do not turn themselves off, our performance is worse and for real systems we have,

$$q(r) \overset{\leq}{=} Q(ST, \ M-1) \tag{9}$$

For M=1 we obtain,

$$q(r) \overset{\leq}{=} e^{-ST} \tag{10}$$

Since a finite K will only make the system behave still worse, (9) and (10) also hold for finite K.

It is important to note that almost all these results depend on the processes involved being Poisson. There are three processes of interest the arrivals to the system, the starting times of first transmissions and retransmissions, and the starting times of successful transmissions. The first and third processes have arrival rates S in equilibrium while the second process has rate G. It is reasonable to assume the first process is Poisson, and by being sufficiently clever with the retransmission scheme used, the second process can probably be made Poisson to an arbitrarily close approximation, but there is no way the third process can be Poisson. For example, the probability of M+1 successful receptions

starting in an interval of length T is zero for the third process and positive for a Poisson process. The Poisson approximation has been shown to be accurate for lower traffic rates and one receiver but for the high traffic rates examined here and with multiple receivers discrepancies can be expected.

For example, safer bounds for (9) and (10) respectively might be,

$$(9') \qquad q(r) \leqq \frac{Q(ST,\ M-1)}{Q(ST,\ M)}$$

and for $M=1$

$$(10') \qquad q(r) \leqq \frac{1}{1 + ST}$$

In the simulations of the next section only the second process is assumed Poisson.

Now let us summarize our knowledge. In general we have:

(i) $q(r_1) = q(r_2)$ for $o \leqq r_1 \leqq r_2 \leqq 1$

(ii) $e^{-2G_u T} \leqq e^{-2GT} \leqq q(r)$ for $o \leqq r \leqq 1$ where G_u satisfies

$S = G_u e^{-G_u}$

(iii) $q(r) = e^{-2GT}$ for $\bar{r} \leq r \leq 1$ where $\bar{r} = \dfrac{1}{1 + \frac{SN}{K}} \quad 1/\alpha$

(iv) $q(r) \leqq Q(ST,\ M-1)$

(v) $q(o) \geqq \dfrac{S}{G}$ where $\bar{G}$ satisfies $S = GQ(GT,\ M-1)$

(vi) $q(o) = \dfrac{1}{1 + G}$ if $K \to \infty$ and $q(o) \geq \dfrac{1}{1 + G}$ for finite K.

(vii) $q(o) \overset{\leq}{=} 1 - (1-e^{-GT}) \dfrac{S}{G}$

Finally, the maximum number of receivers which can be correctly receiving messages at the same time is less than or equal to K/M.
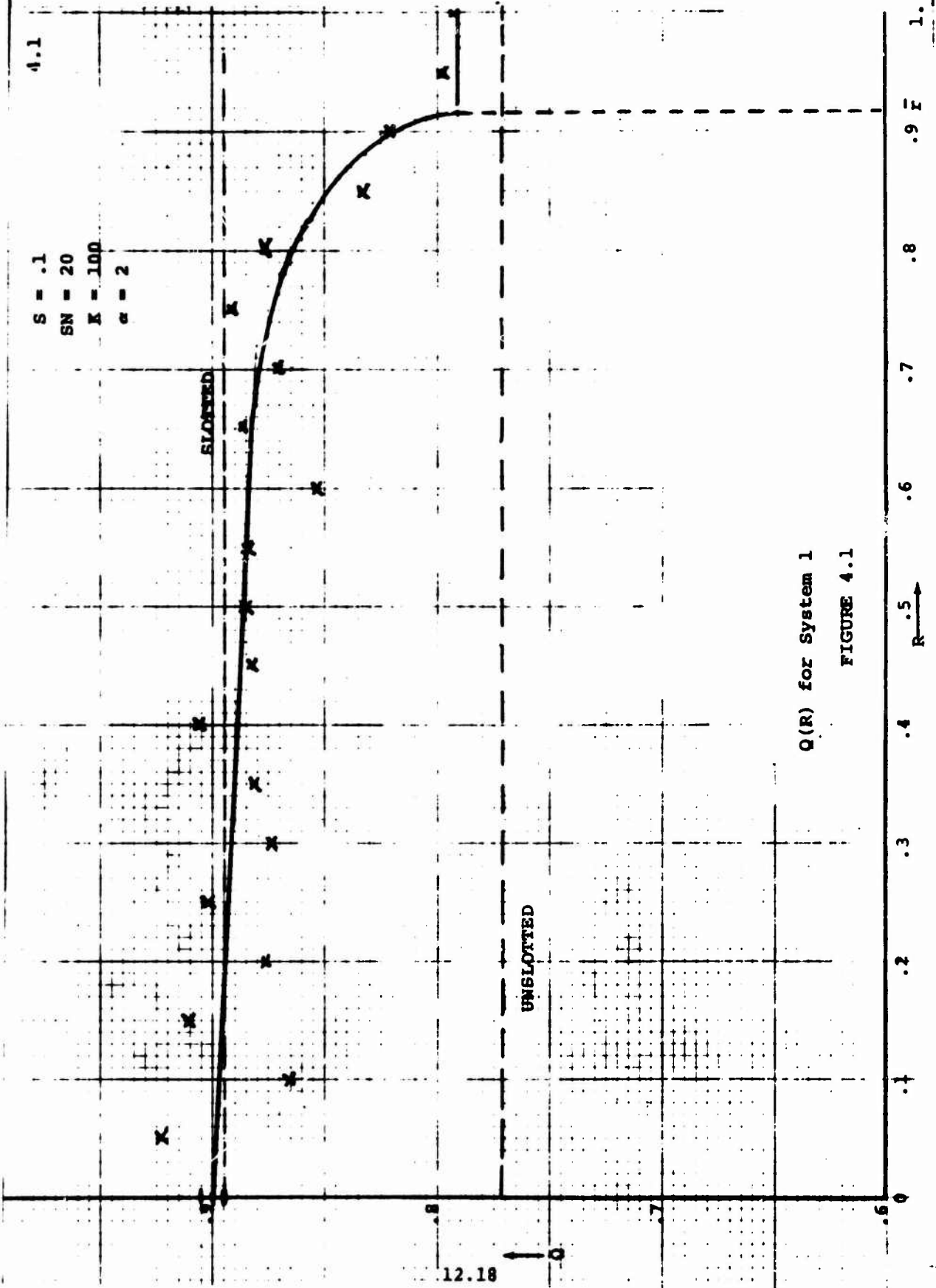
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4. A SIMULATION APPROACH

Simulation was resorted to in order to verify the relations derived in Section III and to estimate more detailed attributes of the system in particular to estimate G which is required for several of the estimates in Section III. Details of the simulation method are given in the Appendix. In general, the estimates of G were quite stable, with apparent accuracies of better than 1%. The functional form of $q(r)$ was not as successfully simulated; however, the simulations were consistent with the approximations and bounds of Section III within sampling error.

The nominal system simulated had S = .1, SN = 20, K = 100, M = 1, and α = 2; T was everywhere taken to be 1. Table   1 summarizes the systems simulated. Figures   1 -   7 are $q(r)$ curves for the first seven systems simulated. Figure   8 shows the change in $q(r)$ obtained by adding a second receiver (Systems 2 and 10); changes for adding more than one receiver were negligible in all cases. Figure   9 illustrates the dependence of G on K for fixed S; while Figure   10 shows the relation between G and S for K = 100 and K = 1000.

SYSTEM NUMBER	ARRIVAL RATE S	SIGNAL TO NOISE SN	SPREAD SPECTRUM FACTOR K	POWER EXPONENT α	CRITICAL RADIUS $\bar{r}$	MEASURED RATE G	NUMBER OF RECEIVERS M
1	.1	20	100	2	.913	.1170	1
2	.2	20	100	2	.913	.2920	1
3	.1	10	100	2	.953	.1152	1
4	.1	20	10	2	.577	.1255	1
5	.1	20	1000	2	.990	.1126	1
6	.1	20	1	2	.223	.1290	1
7	.1	20	100	4	.955	.1166	1
8	.1	20	100	2	.913	.1087	2
9	.1	20	100	2	.913	.1085	3
10	.2	20	100	2	.913	.2414	2
11	.2	20	100	2	.913	.2397	3
12	.2	20	100	2	.913	.2397	5
13	.333	20	1000	2	.990	.5503	1
14	.333	20	1000	2	.990	.3718	2
15	.333	20	1000	2	.990	.3589	3

TABLE 1
Simulation Runs

12.17

$Q(R)$ for System 1

FIGURE 4.1

B = .2
SN = 20
K = 100
α = 2

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Q(R) for System 2

FIGURE 4.2

R →

12.19
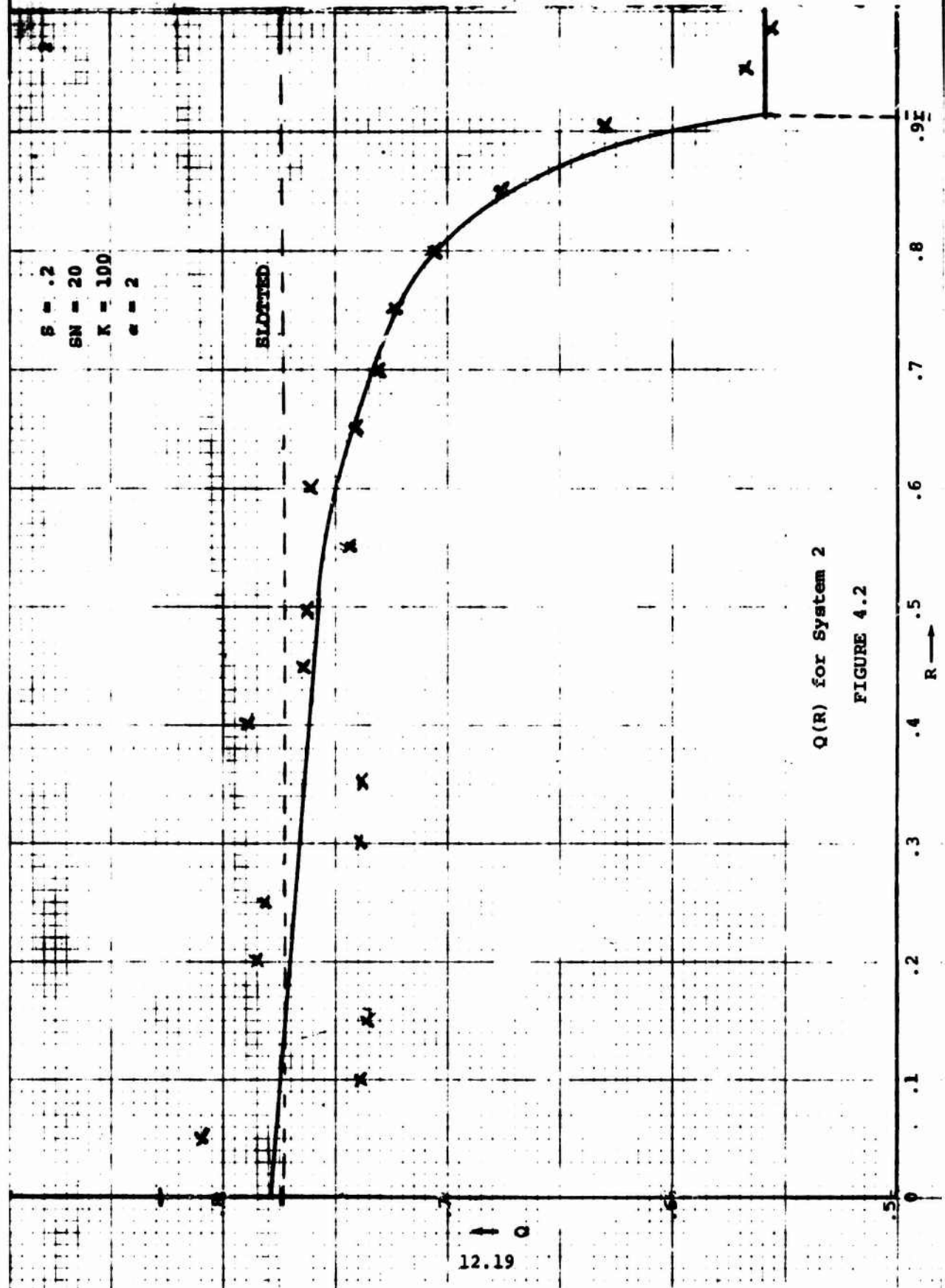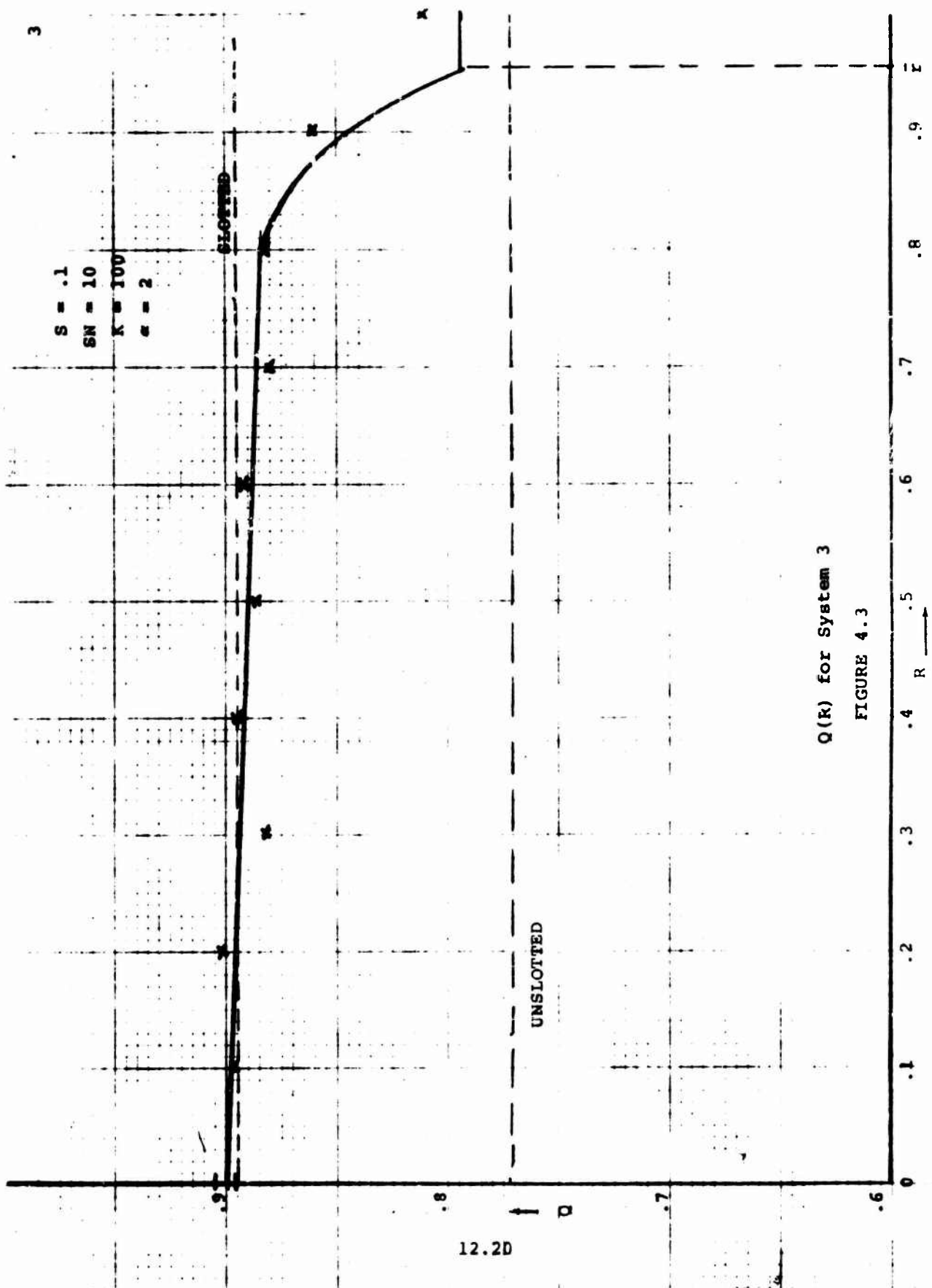
$Q(R)$ for System 3

FIGURE 4.3

$Q(R)$ for System 4

FIGURE 4.4

S = .1
SN = 20
K = 10
α = 2

SLOTTED

UNSLOTTED

12.21
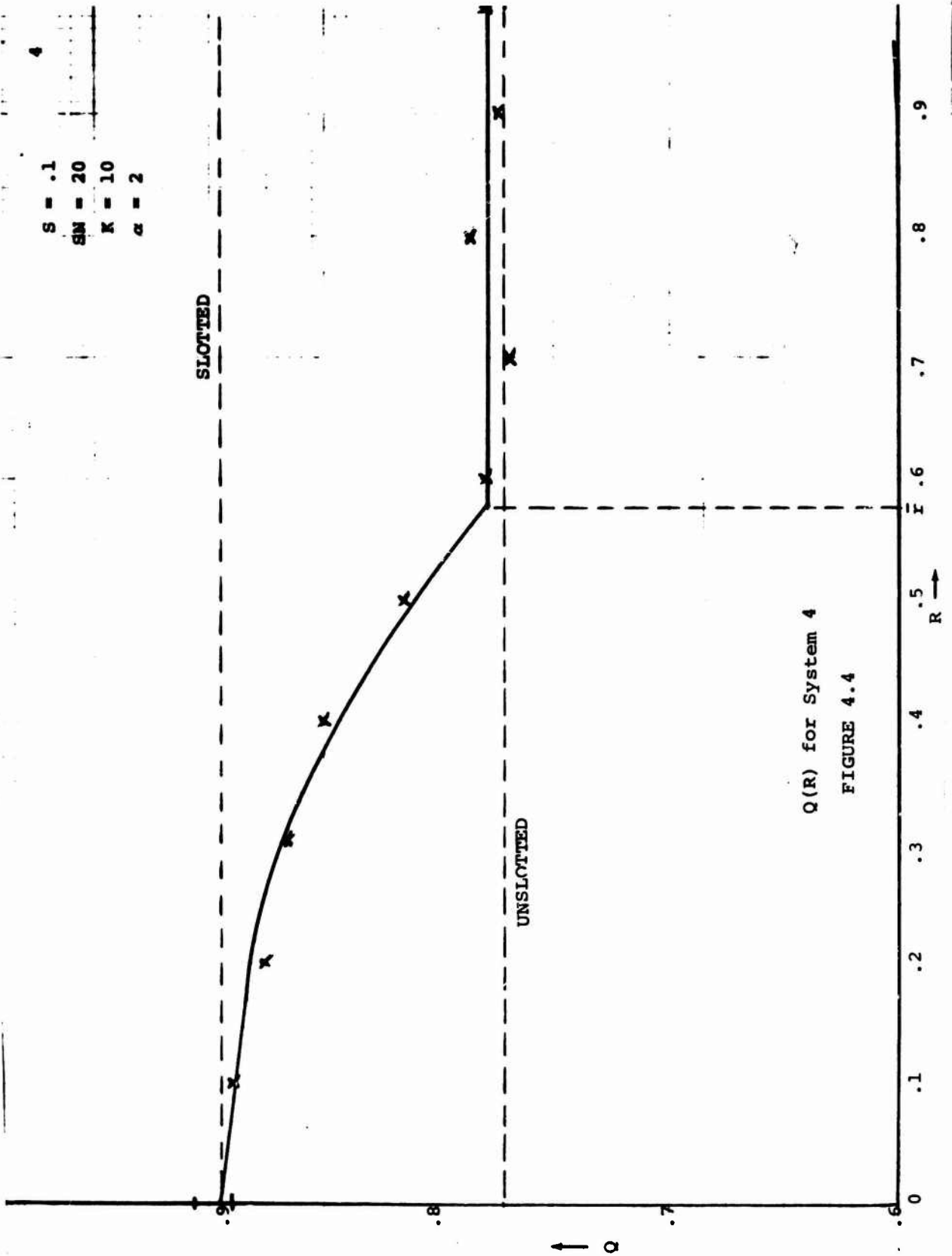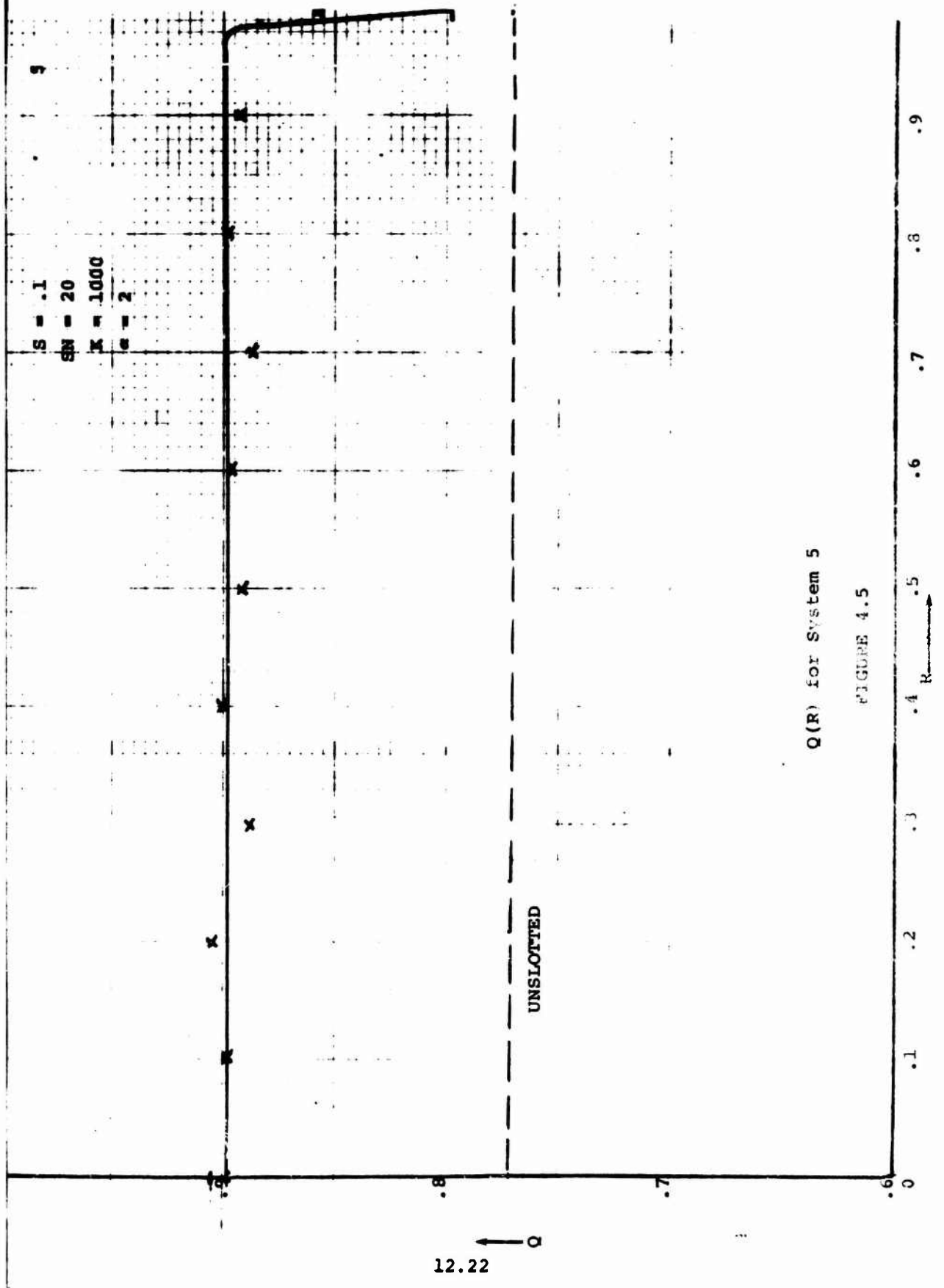
$Q(R)$ for System 5

FIGURE 4.5

UNSLOTTED

12.22

S = .1
SN = 20
K = 1
q = 2

SLOTTED

UNSLOTTED

Q(R) for System 6

FIGURE 4.6

R ⟶

Q ⟵

12.23

$s = .1$
$SN = 20$
$K = 100$
$a = 4$

SLOTTED

UNSLOTTED
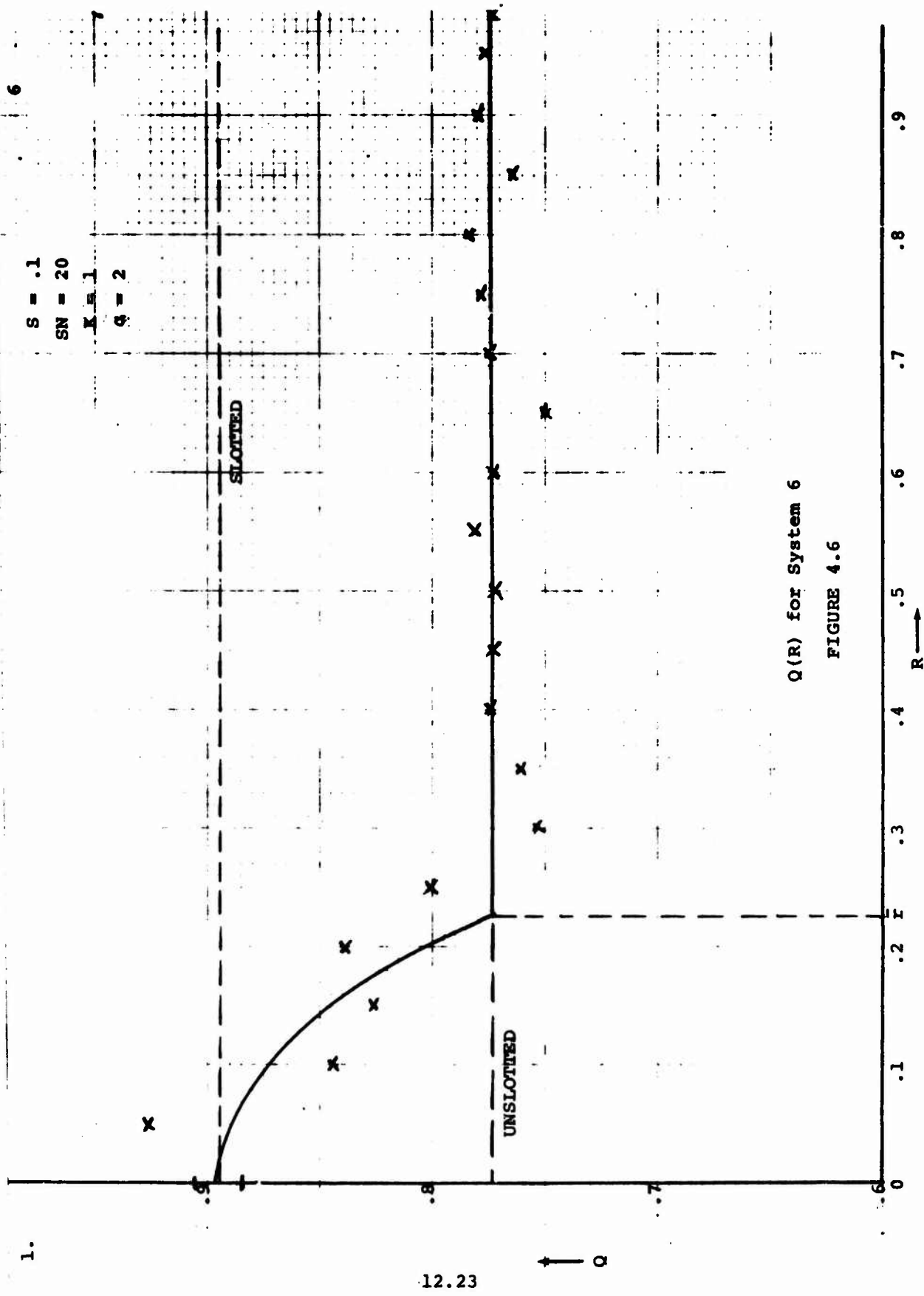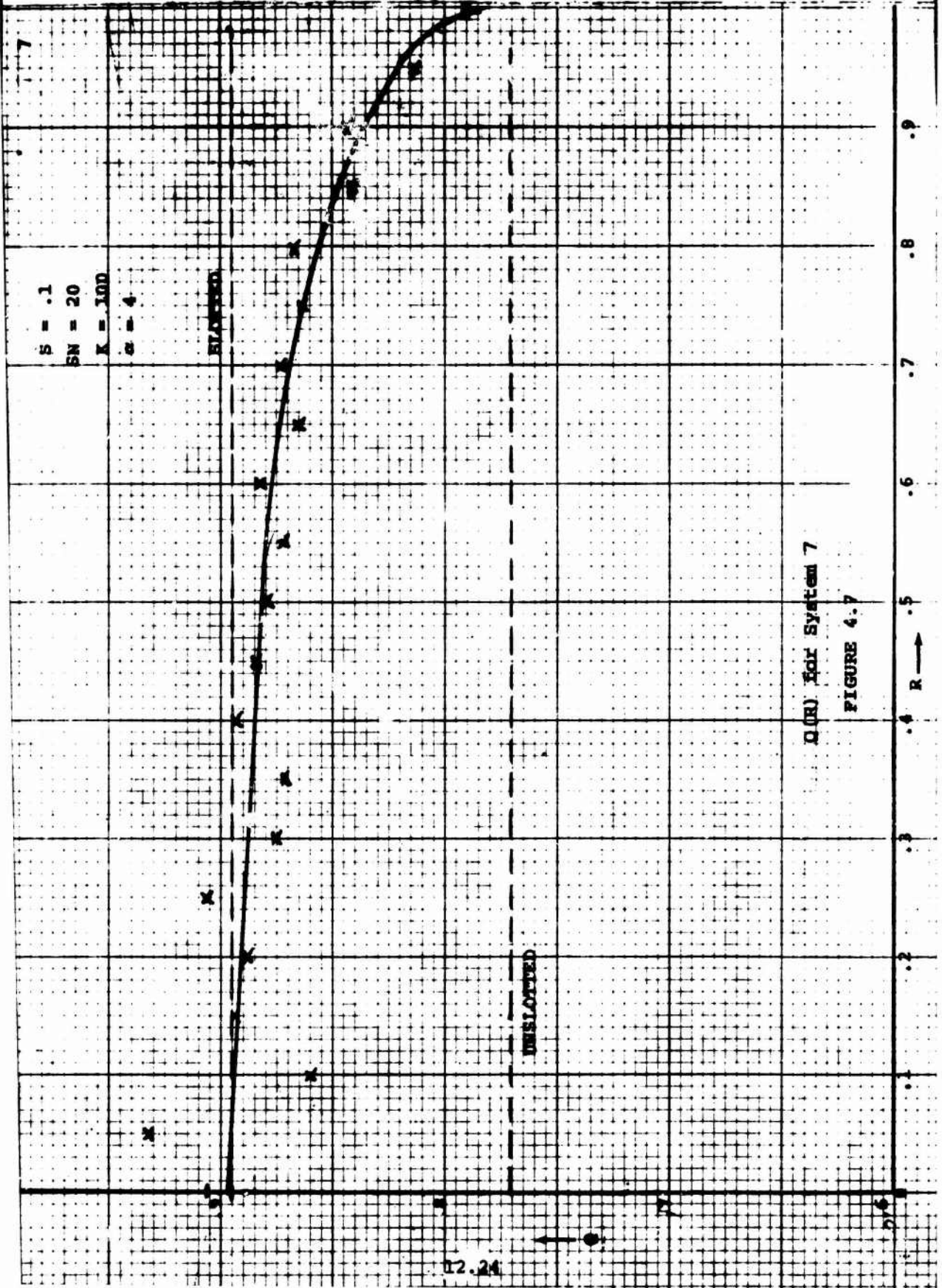
$Q(R)$ for System 7

FIGURE 4.7

$R \longrightarrow$
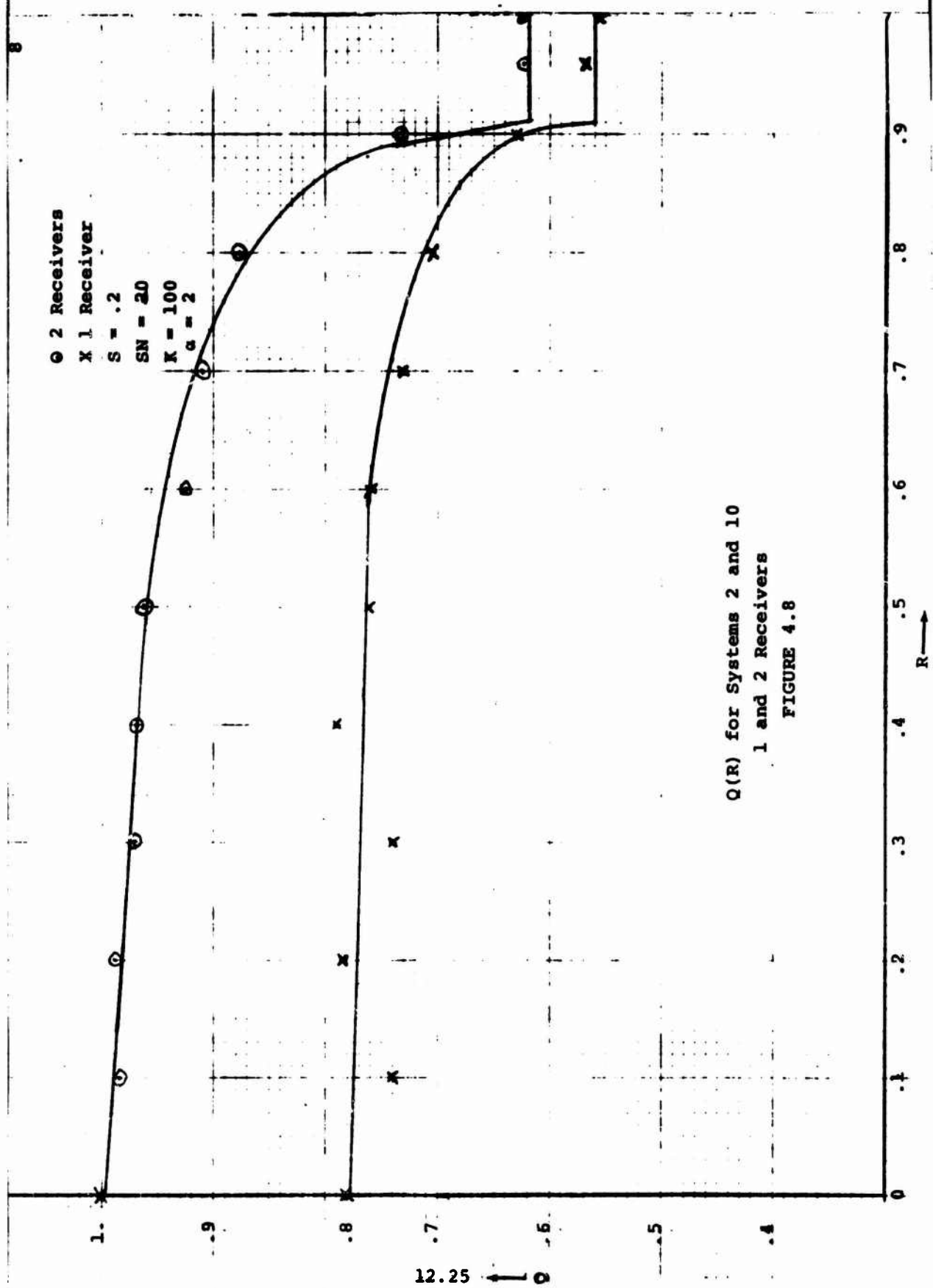
Q(R) for Systems 2 and 10
1 and 2 Receivers

FIGURE 4.8

⊙ 2 Receivers
X 1 Receiver
S = .2
SN = 20
K = 100
α = 2

S = .1
SN = 20
α = 2
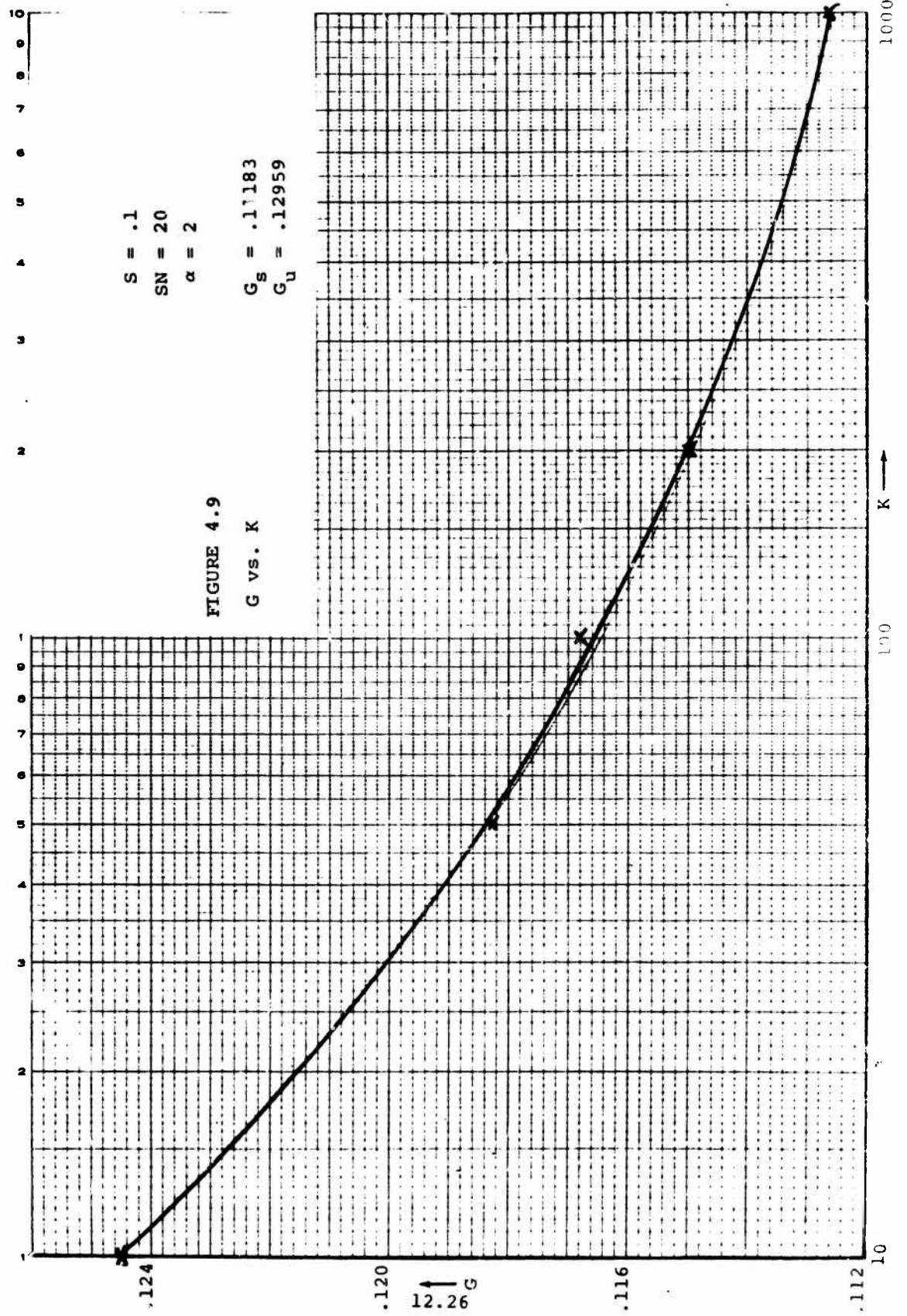
G_s = .11183
G_u = .12959

FIGURE 4.9

G vs. K

FIGURE 4.10

G vs. S

12.27
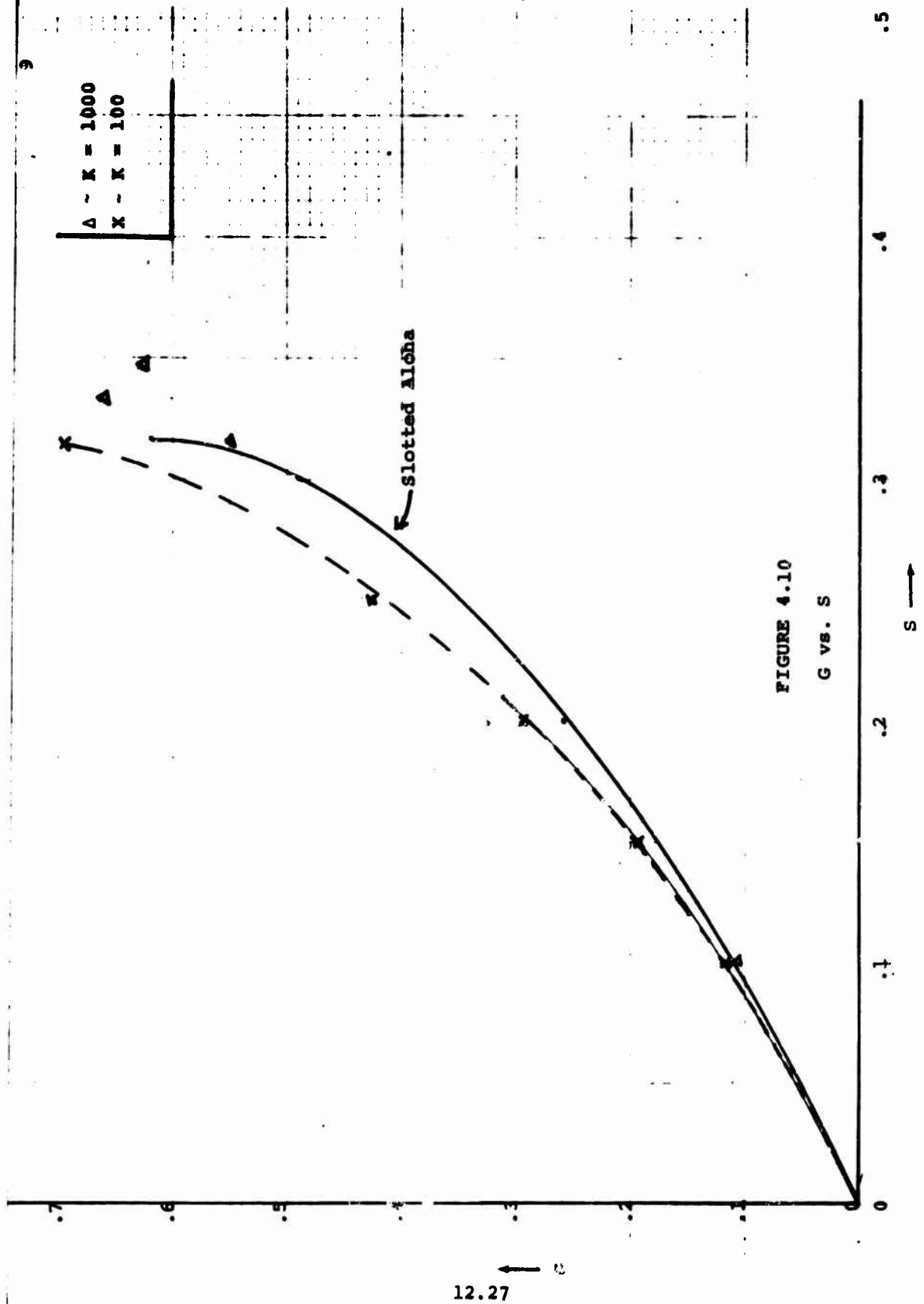
5. APPENDIX: METHOD OF SIMULATION

The radius from the station (R=o) to R=1 is divided into ND intervals. Associated with each interval, I, is a ring of width 1/ND and area $2 \pi I/ND$. Thus, each ring is assigned a weight (proportional to radius) of arrivals in an array RDN, so that the arrival rate in the ring is RDN(I) * SIGMA; Q(I) is the current estimate of the probability a transmission from the Ith ring is successful. RDL(I) is defined proportional to RDN(I)/Q(I), so that the estimated arrival plus retransmission rate in the Ith interval is RDL(I) * GAMMA, where SIGMA~S and GAMMA~G.

Initially, we take GAMMA = SIGMA and RDL = RDN. The general step is to generate NS Poisson arrivals with arrival rate GAMMA. The transmissions are assigned radii according to the distribution RDL and the corresponding signal powers calculated in an array S. In an array ST, the instantaneous power is calculated for each time at which a transmission is initiated. Finally, in an array SMX the maximum power over the T seconds subsequent to each start of transmission is calculated.

To calculate the number of successful transmissions, the transmissions are considered in order. An indicator keeps track of how many receivers are busy. For a given transmission, the program first determines which interval from 1 to ND the transmission originates from; if it is in the Ith for example, then RBX(I) is bumped. Then the program makes the following tests to see if the transmission is successful:

1.   Using the ST array, the program determines if the receiver can hear the beginning of the transmission. If so we go to Step 2. If not, we go on to the next transmission.

12.28

2.   Is a receiver free? If one isn't, go on to
the next transmission; if one is, mark the receiver
busy for the next T seconds and go on to Step 3.

3.   Can the entire transmission get through without
being drowned out by subsequent competing transmissions?
This is determined using the array SMX. If this test
is passed, the transmission is assumed successful and
a counter SBX(I) is bumped.

After all the transmissions are processed, and improved
estimate for Q(I) given by

$$Q(I) = SBX(I)/RBX(I) \qquad\qquad (A.1)$$

is obtained and this is repeated several times until the distri-
bution settles down. It turns out that this process was very
unstable, at least in the estimate for Q(I). The estimates for
GAMMA were quite easy to obtain and reliable, but there were rather
wide fluctuations in the Q(I) between iterations or when using
different seeds for the random number generator. The reason is
apparently because the power for originations near the origin
become unboundedly large and hence have disproportionate influence,
while the number of such events is quite small so the resulting
variance is rather large. This was ameliorated to some extent by
using exponential smoothing; that is, replacing (A.1) with

Q(I) : = RLX * Q(I) + (1-RLX) SBX(I)/RBX(I) where
$0 \le RLX < 1$.

CHAPTER 13

PACKET DATA COMMUNICATIONS ON MATV AND CATV SYSTEMS: A FEASIBILITY STUDY

1.   INTRODUCTION

In 1970, the ARPA network was still written about in terms
of goals to be reached.

> "For many years, small groups of computers have
> been interconnected in various ways. Only re-
> cently, however, has the interaction of computers
> and communications become an important topic in
> its own right. In 1968, after considerable pre-
> liminary investigation and discussion, the
> Advanced Research Projects Agency of the Depart-
> ment of Defense (ARPA) embarked on the implemen-
> tation of a new kind of nationwide computer in-
> terconnection known as the ARPA Network. This
> network will initially interconnect many dis-
> similar computers at ten ARPA-supported research
> centers with 50-kilobit common-carrier circuits.
> The network may be extended to include many
> other locations and circuits of higher bandwidth.
>   The primary goal of the ARPA project is to
> permit persons and programs at one research
> center to access data and use interactively pro-
> grams that exist and run in other computers of
> the network. This goal may represent a major
> step down the path taken by computer time-sharing,
> in the sense that the computer resources of the
> various research centers are thus pooled and
> directly accessible to the entire community of
> network participants." [F.E. Heart, et.al., 1970]

Now these goals have been completely achieved with even
further developments largely solidified. For example, TIPS
(Terminal Interface Processors) to connect terminals to the ARPANET
are a reality as are VDH (Very Distant Host) connections. In fact,
in 1972 the result of the ARPA network development led to the con-
clusive statement that its performance is superior to other existing
methods·   Extensions to personal hand held radio terminals, are
in the offing using a random access mode as begun in the University

Network Analysis Corporation

of Hawaii. The merits of the ALOHA packet mode have been thoroughly investigated with a conclusion:

> "...packet technology is far superior to circuit technology, even on the simplest radio transmission level, so long as the ratio of peak bandwidth to average bandwidth is large. Most likely, the only feasible way to design a useful and economically attractive personal terminal is through some type of packet communication technology. Otherwise one is restricted to uselessly small numbers of terminals on one channel. This result may also apply to many other important developments, only to be discovered as the technology of packet communication is further developed." [Roberts, 72]

Both as an adjunct to the interactive packet radio mode and as an extension of the ARPANET, this report describes the use of MATV and CATV coaxial cable systems for local distribution of packet data.

We first explain the need for a local distribution medium such as an MATV and CATV System, to augment a packet radio system in urban and suburban areas. We next investigate the properties of the coaxial cable systems in order to evaluate their data handling capability. To demonstrate the validity of our conclusions, actual designs are discussed for the existing CATV system in the Metropolitan Boston area. Specifications are given for all required digital equipment, and finally, test procedures are given. The overall conclusion of the study is that MATV and CATV interactive packet systems would form an excellent local distribution medium.

2.   THE IN-BUILDING PROBLEM

In urban areas two difficulties impede the reception of radio signals in buildings:

   1.   The attenuation of signals in passing through building walls; and

   2.   The reception of multipath signals due to reflections off buildings.

In a study of the in-building reception problem, C. H. Vandament of Collins Radio concludes,

> "...signal strength environment on a city street will probably need to be 25 to 30 db higher than that previously considered if direct radiation into buildings is to be considered (i.e. no building distribution system employing amplification). This is quite unattractive since this implies either excessive transmit power and/or quite close repeater spacing." [Vandament, 1973]

Both of these problems can be avoided while still retaining the main idea of using ALOHA random access multiplexing. Instead of operating in an over-the-air broadcast transmission mode, data can be sent over the existing wideband coaxial cable facilities of master antenna (MATV) and cable television (CATV) systems. Recent FCC rulings require that all new CATV systems have two way capability. penetrations of 40-60% of U.S. homes is projected by the end of the decade [Sloan, 1971]. Moreover, most of the major new office buildings will have wide-band communication channels built in. For example, in the New York World Trade Center, there is a system of switched wideband communication channels. The user can select 60KHz audio channels or 15 MHz video channels and has an individual wideband cable connection to the central switch [Friedlander, 1972]. By 1980, "the wired city" will be close to reality.

Vandament, after considering radiation, telephone wires, power cables, and other special wiring schemes for the in-building problem, comes to a similar conclusion regarding the merits of coaxial cable transmission:

> "...Every building will require some analysis to determine which technique is required to deliver a useable signal to a terminal inside.

> If the building has windows and is relatively
> close to a system repeater, no special techniques
> will be required. At greater distances, a
> simple repeater with directional antennas
> focused on specific buildings will deliver
> the signals to interior users by radiation.
> Buildings which are effectively shielded
> must have a simple, dedicated repeater to
> receive a signal and pipe it into the building
> over conductors of one type or another...High
> grade coaxial cables solve that problem nicely,
> but this answer would probably be prohibitively
> expensive for the packet radio scenario of
> general distribution throughout every impor-
> tant building in the U.S. ...Where such cables
> exist, they do offer an attractive solution to
> the in-building distribution problem."
> [Vandament, 1973]

In the next five sections, we will show the technical merits
of MATV systems for solution of the in-building problem and
CATV systems for local distribution in high density suburban and
urban areas. To do this, we first describe the properties of
typical modem MATV and CATV systems in Sections 3 and 4 respectively.

3.   A TYPICAL MATV SYSTEM

A master antenna television system as shown in Figure 3.1 serves
a concentration of television sets such as in an apartment build-
ing, hotel, or motel. The main purpose of the MATV system, as
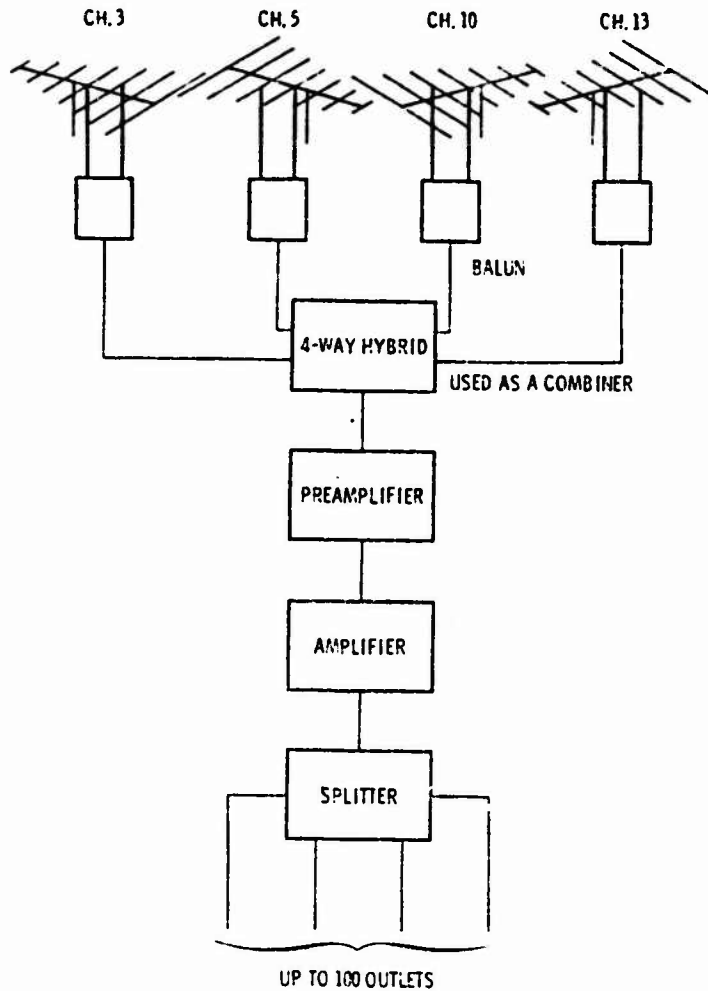
Figure 3.1   Typical Hotel or Apartment Building MATV System

shown in Figures 3.2, 3.3, 3.4, is to provide a usable signal to
a large number of television sets fed by a local distribution net-
work.
     A number of television sets connected to the same antenna
system without a signal amplifier would not provide any of the sets
with a strong enough signal to produce good pictures. An all-channel
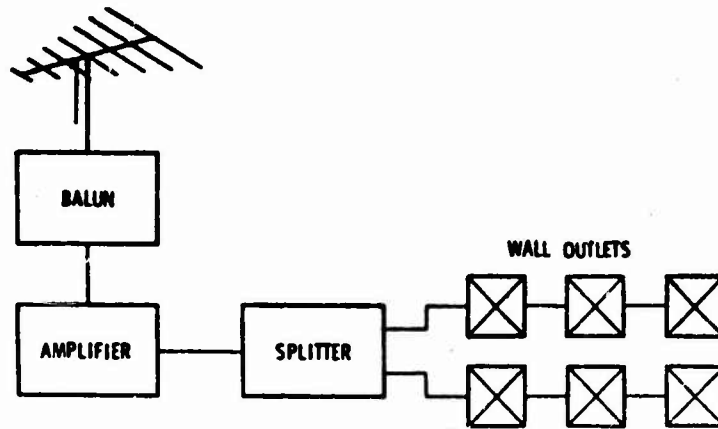
Figure 3.2   Typical Motel MATV System
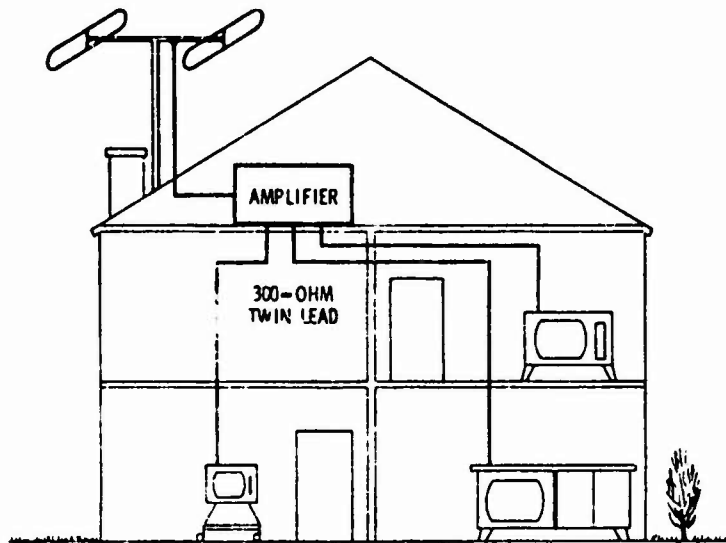
Figure 3.3   Typical Home MATV System

master antenna amplifier is connected to one antenna (sometimes
more) which provides across-the-band amplification of all tele-
vision signals in the VHF band and the f-m broadcast band.
     Some MATV systems employ more than one amplifier. A separate
single-channel amplifier may be used as shown in Figure 3.5, to
provide greater amplification of a single channel. Generally,
a separate antenna is used with a single-channel amplifier.

13.6
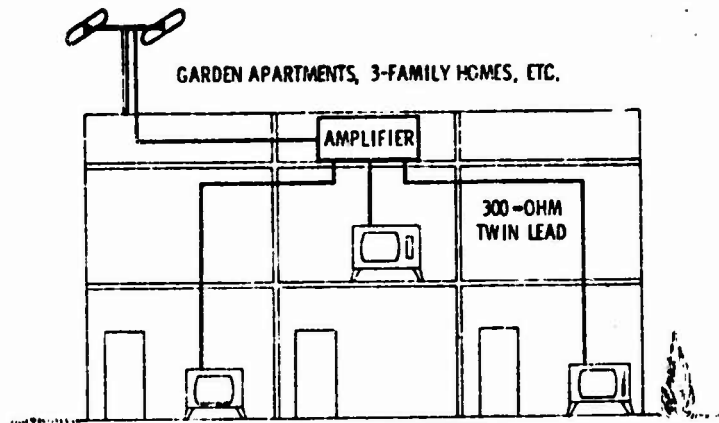
Figure 3.4   MATV System for Multiple Dwelling

For reception of UHF television stations, a UHF-to-VHF translator is required.
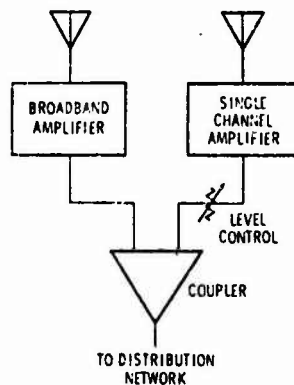
Figure 3.5

Use of Single Channel and Broadband Amplifiers for Receiving Distant Stations

For MATV systems with more than 100 outlets, further amplification may be required, and in a large office building cascades of two or three amplifiers might be expected.   Never-

theless, compared to CATV systems - to be described in the next
section - modern MATV systems are relatively uncomplicated media
for data transmission. Signal to noise ratios of better than
43 db are reasonable; there are few environmental problems, since
the system is i oors; there is minimal temperature variation;
and amplifier cascades are low since the wide band capabilities
of 0.5 inch coaxial cable are used. In a CATV system, the signal
at the building is received from a cable distribution system
rather than from antennas. However, the in-building part of the
system is essentially unchanged from that used for an MATV system.

4.   A TYPICAL CATV SYSTEM

CATV systems are an historical outgrowth of MATV and community antenna television systems. CATV systems perform roughly the same function for an entire town that a MATV system performs for a building, namely the distribution of TV singals to many terminals from a central reception area. Although the basic engineering strategies are the same for MATV and CATV systems, the CATV system is different in one crucial aspect; since it is larger, as many as thirty amplifiers may have to be cascaded to deliver a TV signal to the most distant terminal in the system. Therefore, the system design requirements are stringent; very high quality amplifiers and off the air reception equipment are essential. In order to motivate our proposal of data options and techniques for CATV systems, a detailed description of these systems is in order.

CATV systems in the U.S.A. are almost universally tree structured networks of coaxial cables installed for the distribution of broadcast type television signals from a central receiving station, called the "head end," to home type television receivers. Different television signals, which may be received at a central site or relayed over long distances by microwave systems, are processed at the head end and frequency division multiplexed onto coaxial cable for distribution. Coaxial cables now in use are universally 75 ohm inpedance types, usually of seamless aluminum sheathed construction, foam polyethylene dielectric, and solid copper or copper clad aluminum center conductor. The coaxial cables range in size from 0.75 inch outer diameter for "main trunk cables," through 0.5 inch size down to a 0.412 inch size for "local distribution." The service drop lines to the houses are usually flexible cables of about 0.25 inch diameter. The useful frequency range includes the VHF television band, 54-216 MHz, and broadband transistorized amplifiers are installed with equalizers to compensate for cable losses. Practical systems are aligned to be unity gain networks

with amplifiers spaced about 20 db at the highest transmitted
frequency.

Cable losses range from about 1 db/100' at 220 MHz for the
0.75 inch size cable to about 5 db/100' for the flexible service
drop cables. Power division at multiple cable junctions and taps
into subscribers' homes are accomplished through hybrid and direc-
tional couplers. All system components are carefully matched to
75 ohms to minimize internal signal reflections within the system.

System amplifiers are subject to rigorous linearity specifi-
cations. Amplifier overloads manifest themselves as cross-modula-
tions between channels and as undesired second and third order
intermodulation and harmonic products. Amplifier operating levels
are bounded on the lower side by system signal to noise ratio ob-
jectives which are about 40 db over a 4 MHz band for reasonable
acceptable performance as a television distribution system. A
typical system will have amplifier inputs at about +10 dbmv and
outputs at about +30 dbmv. System operating levels are controlled
by automatic gain control circuits driven by pilot carriers and
thermal compensation devices.

More recent cable systems have been built using a "hub"
principal in which tree structured networks originate from a number
of "hubs" throughout the community serviced (see Figure 4.1). The
"hubs" may contain equipment for more elaborate control of signal
levels and it may be possible to perform some special switching
functions such as the interconnection of sub-trunks for special
purposes.
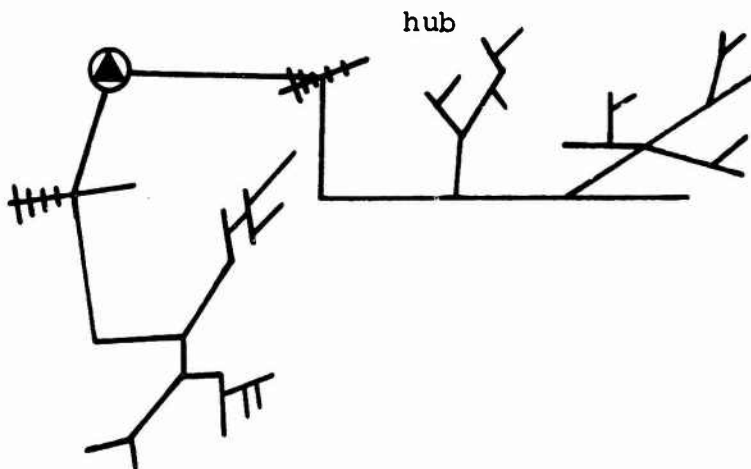
Figure 4.1 Hub System

13.10

Two-Way CATV Configurations:

FCC regulations now require that new CATV systems must have two-way capability. Practically speaking, this does not mean that all new systems are two-way systems, but rather that amplifier units are installed with forward amplifier modules in place and with distances between amplifiers constrained so that at some future date reverse amplifier modules can be installed for two-way operation. However, a number of actual fully two-way systems are presently being built and the number is increasing rapidly. Most present two-way systems use the configuration in Figure 4.2. Filters at each end of the station separate low (L) and high (H) frequencies and direct them to amplifiers usually referred to as "downstream" from the head end and "upstream" toward the head end.

A number of possible "two-way" configurations are shown in Figures 4.2, 4.3, and 4.4 [Jerrold, 1971]. The final choice between single cable/two-way, multi-cable two-way, and multi-cable without two-way filters will probably be made on the basis of marketing opportunities for special services.

There are no government regulations as to minimum specifications for a system. Hence, we will base our discussion on the characteristics of a representative two-way system, the Boston complex. This system is being built in about 10 stages, one of which is already installed and the final phase of which is to be completed within a year. At its completion, Boston will be one of the largest systems in existence in the U.S. The Boston system uses the "feederbacker" configuration shown in Figure 4.4 with the frequencies assigned to the upstream and downstream paths specifically indicated.

Data transmission on CATV systems will generally have to be fitted into space not being used to TV channels or for pilot frequencies. Hence, it is best to first describe the frequency allocation for video signals. TV channels are allocated six Megahertz
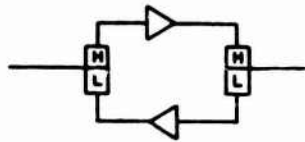
FIGURE 4.2
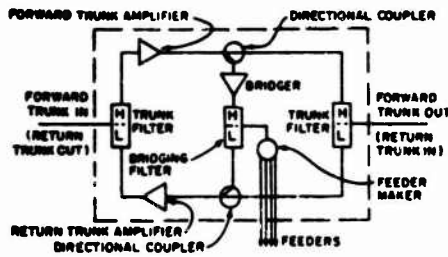
FIGURE 4.3   Two-way CATV Repeater (with feeders).
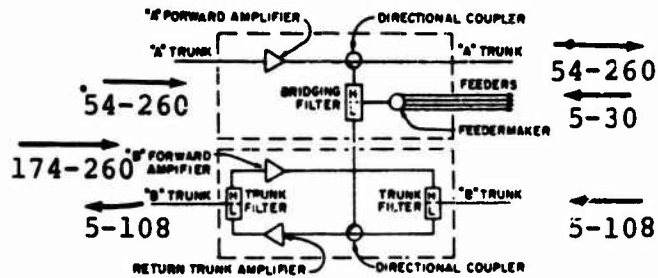
FIGURE 4.4   Dual Trunk/Single Feeder Station (Boston Configuration)

bandwiths.   Broadcasted TV channels are in the Lo-VHF range 54
MHz-88MHz, the Hi-VHF range 174 MHz-216 MHz, and the UHF range
470 MHz-890MHz.   The TV frequency allocations on cable are
different.   They partition the 54-300 MHzspectrum as follows:

Sub-VHF	5-54 MHz
Lo-VHF	54-88 MHz
Mid-band	88-174 MHz
Hi-VHF	174-216 MHz
Super-band	216-300 MHz

There is still discussion going on as to whether the mid-band
should be used within a cable system because of danger of interfer-
ence to aircraft navigation in case of signal leakage out of the
cable.   Data might be squeezed into bands not used by TV signals.
Some space is available below Channel 2 at 48-54 MHz.   The space
between Channel 4 and Channel 5 (72-76 Mhz) might be used for low
level data signals.   High signal levels in this area could cause
harmful picture interference on some TV sets.

A more likely situation is that two 6 MHz video channels - one
upstream and one downstream - would be set aside for data trans-
mission.   The simple fact is that 21 to 30 channels are available on
a single cable system and 42 to 60 channels on a dual cable system.
These channels have not all yet been pre-empted by video transmission.

5.   DATA OPTIONS ON MATV AND CATV SYSTEMS

Based on the details of MATV and CATV systems, we can demon-
strate the merits of in-building and local distribution on MATV and
CATV systems and describe detailed data options for experimentation
and practical implementation.

Most two-way systems are being developed with an upstream
channel designed to permit input from virtually any location in
the network. The result is a large number of noise sources being
fed upstream toward a common source. Individual cable television
amplifiers usually have a noise figure of about 10db for a 6 MHz
channel. Cascading amplifiers can increase effective system noise
figure by 30db or more. Nevertheless, we shall see that system
specifications on signal-to-noise ratio for CATV systems are
stringent enough so that packets can be sent with existing analog
repeaters, and no digital repeaters, such that bit rate error
probabilities are negligible.

For example, in Boston the worst signal-to-thermal noise ratio
is limited to 43db and the worst cross-modulation to signal ratio
is limited to -47db. System operators may want to limit data
channel carriers to a level of 10 to 20db below TV operating
levels in order to minimize additional loading due to the data
channel carriers [Switzer, 1972]. The cable operator, at least
for the time being, is making his living by providing a maximum
number of downstream television channels and will accept data
channels only on a non-interfering basis.

Accepting these restrictions, in the worst case, we would be
limited to 23db signal to thermal noise ratio and -27db cross-
modulation to signal ratio. Let us consider both of these sets
or restrictions to determine the resulting CATV system performance
for random access packet transmission.

From the calculations in Appendix A, we have the error rates
shown in Table 5.1. The calculations are performed for a FSK
system with incoherent detection to determine a lower bound for
system performance.

13.14

TABLE 5.1
ERROR RATES FOR FSK

System Label	Type of Specification	(N_C/S)	(S/N_r)	m	Pe
A	Boston Specs.	-47db	43db	2	$1/2\, e^{-5,148} \approx 1/2 \times 10^{-2,239}$
B	Boston Specs.	-47db	43db	4	$3/4\, e^{-571} \approx 3/4 \times 10^{-248}$
C	Boston Specs.	-47db	43db	8	$7/8\, e^{-104} \approx 7/8 \times 10^{-45}$
D	Boston Specs. - degraded by 20db	-27db	23db	2	$1/2\, e^{-51} \approx .5 \times 10^{-22}$

It is well known that for effecti· signal-to-noise ratios above 20db there is a threshold effect for error probabilities. This is born out by the negligible error rates in Table 5.1. Even for the degraded specifications the error rate is low enough for the most stringent practical data requirements.

At a rate of 10^6 pulses/second the FSK signal will occupy the 6MHz bandwidth [Switzer, 1972] with negligible intermodulation into TV channels [Schwartz et al., 1966].

In Figure 5.1 we plot the maximum number of active terminals for the systems in Table 5.1 as calculated in Appendix B. The curves labeled A,B,C, and D correspond to the slotted systems in Table 5.1 labe¹ ` B,C, and D. The lines labeled A',B',C', and D', are for t. ·sponding unslotted systems.

We can now ·...ne Figure 5.1 to determine system performance under some typical data transmission requirements. For a data rate of 40 bits/second per terminals, a single trunk can handle 900 terminals with a slotted ALOHA system (1 Megabit/sec) with an error rate of 10^{-22} at a signal to noise ratio degraded by 20db. The average number of TV sets per trunk in Boston is approximately 27,000. Hence, the simplest modulation scheme will handle one third of all terminals in Boston as active terminals. At 100Kbits/second, the system will handle 900 active terminals.
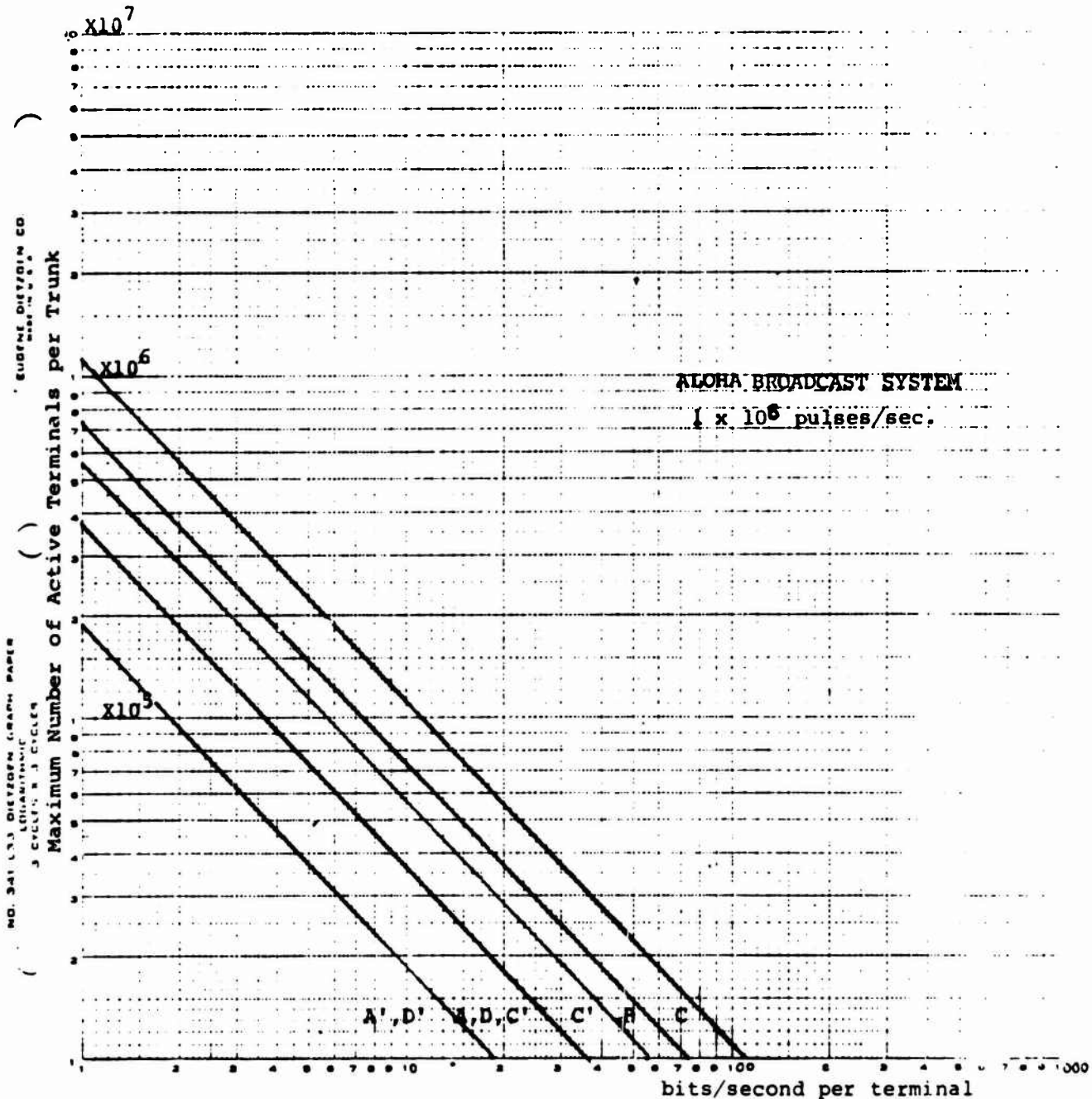
FIGURE 5.1
NUMBER OF TERMINALS PER TRUNK VS.BITS/SEC. PER TERMINAL

We will also consider the introduction of a number of basic data processing options into the CATV system in order to make our data transmission system adaptable to a variety of traffic requirements while satisfying the system constraints.

We will use the terminology of the cable TV industry in describing the direciton of signal flow. Signals traveling from the head end toward terminals will be said to be directed in the "forward" direction on a "forward" link and signals traveling from terminals toward the head end will be said to be directed in a "reverse" direction on a "reverse" link. A convenient synonym for "forward" will be "downstream", and for "reverse", will be "upstream". To install the device to be described in the forward and reverse channels simple diplex and triplex filters can be used. More detailed specifications for the filters and the data devices are given in Appendix E.

CARRIER FREQUENCY CONVERSION:

In the simplest version of a data system, two carrier frequencies are used; one for forward transmission from the head end to the terminals, and one for reverse transmission from the terminals to head end. Let us call these angular frequencies ω_f and ω_r respectively. The next simplest option is to use frequency converters at a small selected set of points in the system. <u>In the forward direction, the converter converts from ω'_f to ω_f and in the reverse direction, it converts from ω_r</u> to ω'_r. The net result is that the terminals still receive and transmit at the frequencies ω_f and ω_r. However, in the trunk between the converters and the head end, there are four frequencies in use, ω_r, ω'_r, ω_f and ω'_f so that in these trunks twice the traffic can be handled.

The converter and the other devices to be described in this section are shown schematically in Figure 5.2.

The advantages of this scheme are:

    a) All terminals are identical.

    b) The converters also act as digital repeaters to reshape signals.

    c) The capacity of the system is increased since two channels are available in each direction.
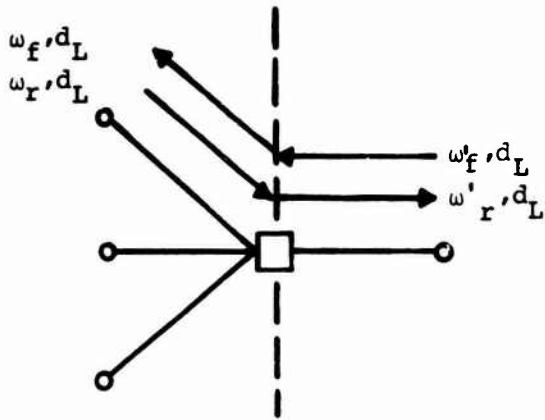
(a) converter
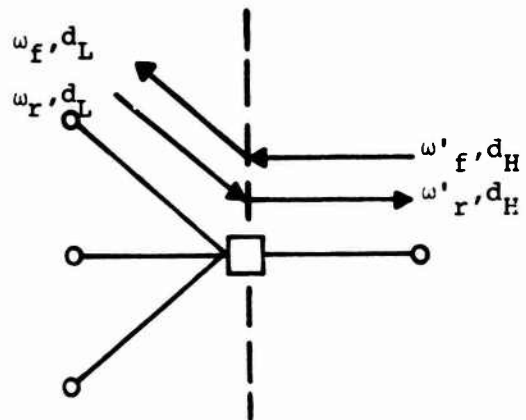
(b) compressor
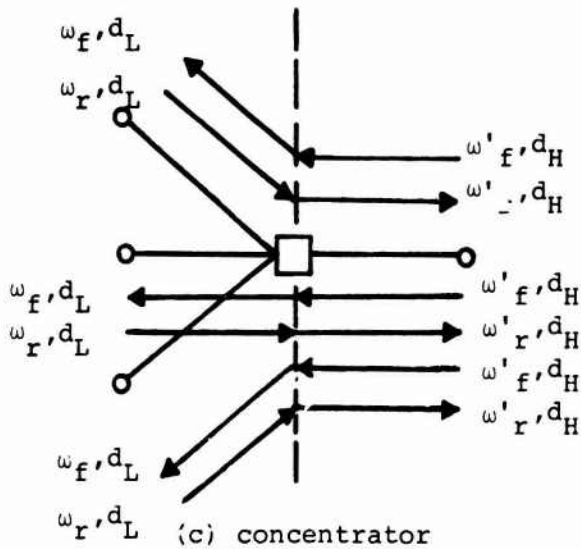
(c) concentrator
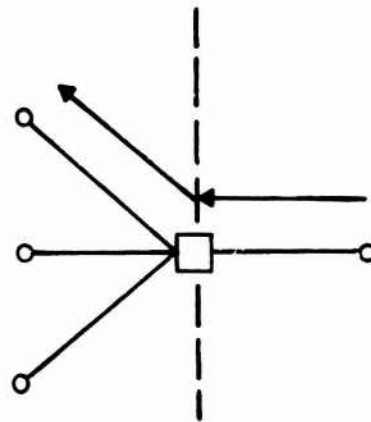
(d) forward router
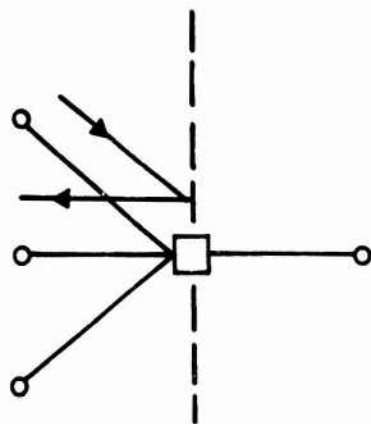
(e) local router

Figure 5.2
Device schematics; devices indicated by squares. Carrier frequencies ω_r, ω'_r, reverse; ω_f, ω'_f, forward. High data rate d_H; Low data rate, d_L.

13.18

ROUTING:

In the interactive packet system, there is no requirement for routing since the basic premise is that all receivers listen to all messages that reach them and merely select the ones addressed to them. Nevertheless, we will consider the addition of some primitive low cost routing schemes to study their effect on system capacity.

In the central transmission mode there is routing needed in the reverse direction since all messages reach the head end along the unique paths from the originating terminal. In the forward direction the signals at frequency ω'_f are blocked by filters at the converters and yields a simple form of routing.

In fact, at any section of trunk not requiring signals at ω'_f filters can be added to block ω'_f. Adding these filters does not increase system capacity but may be useful if the frequency ω'_f can be used for local signaling when not being used for data transmission.

At any junction containing a converter, digital routers <u>must be added to send messages at ω_f down the trunk to which they are addressed rather than all trunks</u>. Such a router may be added at any other point in the system as well. This is called "<u>forward</u> <u>routing</u>" and can increase system capacity. Forward routing requires a digital router which can read and interpret message addresses.

Let us now consider these system options in the presence of local traffic. The option of frequency conversion is unaffected and performs in exactly the same manner as in the central transmission mode. However, an extra routing option is available for local trans- mission. In particular, <u>if two terminals are on the same trunk, then the message between them can be intercepted and routed at a routing station rather than travel all the way to the head end. Such routing is called "local routing"</u>. Local routing reduces the traffic on the main trunk.

COMPRESSION:

As the next more complicated option, the data rate as well as the carrier frequency is changed at a converter, i.e., a compressor can be used. The advantages of this arrangement are:

A. All terminals operate at low data rate.

B. On heavily used lines near the head end a higher data rate, say one megabit/second, can be used to increase the number of potential active users or decrease the delay.

C. Even though a section of the trunk can carry high data rate traffic at carrier frequencies ω'_r anand ω'_f other users can still use the system at the low data rates at ω_r and ω_f. Thus, the number of compressors required is small.

D. At the low data rate, signal distortion is kept to a minimum.

CONCENTRATION:

Finally, the compressor at junctions may be replaced by a concentrator. That is, messages arriving simultaneously on two or more links in the reverse direciton are buffered and sent out sequentially at the higher data rate. This essentially makes the system downstream from the concentrator appear to operate at the higher data rate and hence increases the system capacity even further.

With the use of converters, compressors , concentrators and routers, we have a highly flexible interactive packet data system which obviously meets most of the system requirements of capatibility with the CATV system, the packet radio system and the population. In particular, the system has the following characteristics:

A. The number of active users that may have access to the system can be readily controlled by varying the number of converters, compressors , concentrators and routers.

B. The transmission rates at every terminal are at the low data rate of 100 kilobits/second.

C. Terminal equipment is inexpensive because modulation takes place at the low data rate.

D. All terminals receive and transmit at the same frequencies and data rates.

FREQUENCY DIVISION MULTIPLEXING:

In case the data rate is limited by the head end mini-computer, an available option is to frequency division multiplex several 100Kbits/sec. channels, each of which is processed by a separate head end mini-computer.

The assignment of these options in an optimal fashion requires detailed expressions for the traffic in the links. Formulae for the traffic are given in Appendix C and are used to give an augmented design for the Boston system in Appendix D.

6.   FUNCTION OF EXPERIMENTAL DIGITAL SYSTEM DESIGN

The previous considerations indicate the desirability of developing a system of MATV and/or CATV lines for inter- and intrabuilding communication to and from packet data terminals. A picture of the system is shown in Figure 6.1.

In order to communicate through the cable system, each terminal will be connected to an interface and modem. A minicomputer will be connected to the cable system at its head end by means of a similar modem and modem-to-computer interface. The minicomputer is used as a test device to:

A.   Receive and generate basic traffic to establish the viability of the system configuration.

B.   Provide message loading to determine limits of system performance.

C.   Demonstrate the functional capabilities required for the head end processor.

D.   Determine the performance required to communicate with external facilities.
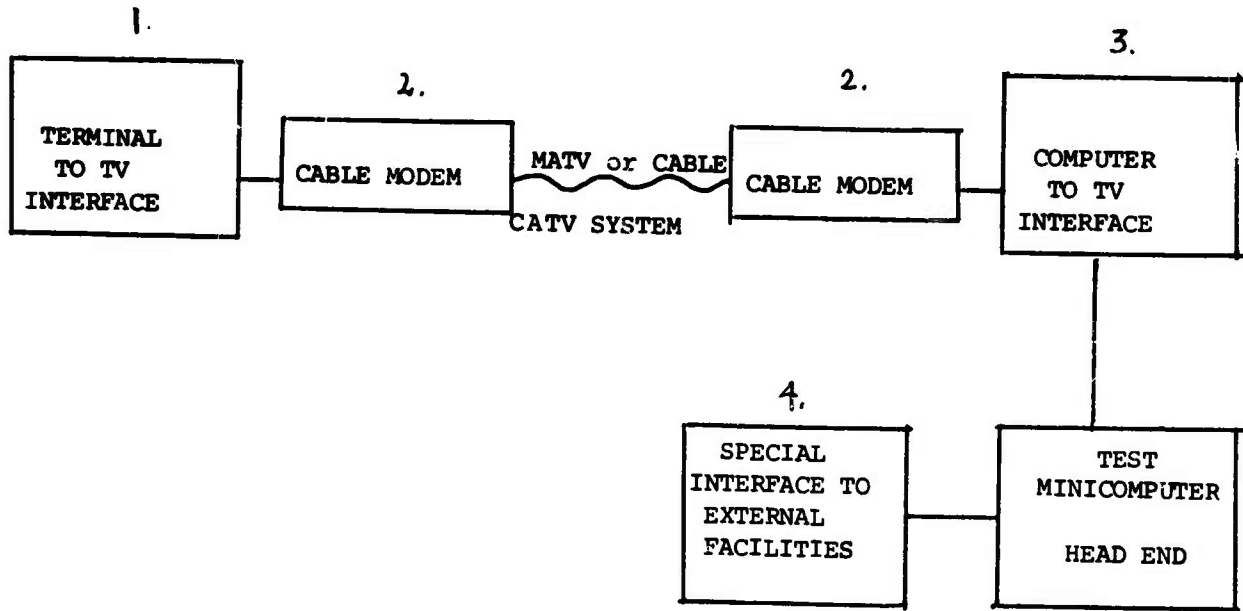
FIGURE 6.1   SYSTEM DIAGRAM

13.23

7.   **SYSTEM CONSTRAINTS**

In specifying hardware, a dominant consideration is the fact that the interactive packet cable transmission system must interface with a CATV system and a packet radio transmission system, among others, and an existing population of unsophisticated users. Thus, any hardware innovation must satisfy the following three classes of system constraints.

Interface With CATV System:

Two Way Options:

The data transmission system must be readily adaptable to a wide variety of existing CATV system designs and two-way options.

Data Rates:

The data signals must not cause visible interference with video signals.

Installation:

If auxiliary data equipment is to be added to the CATV system, it must satisfy the following requirements:

- It can be installed with only minor changes in the CATV system.

- It need be installed in only a small number of locations.

- It can be installed rapidly in early hours of the morning to prevent interference with TV service.

Low Cost:

To maximize the marginal utility of data distribution over the CATV system, any equipment introduced must be inexpensive. Depending upon the data rates and bandwidths involved, the radio and CATV systems can be arranged to share some modules, at the baseband or IF frequency range.

13.24

Interface With Population:

Population Density Variations:

Standard transmission configuration options must be available for systems of various sizes, population densities and percent of active users. Because of the huge number of potential users, all terminal equipment must be simple and inexpensive.

Unsophisticated Users:

To minimize user interaction with the system operating mode, all terminal equipment must be the same for each location; it must use the same frequencies and data rates; and it must have no options for equipment modification by the user.

8.   COMPONENTS OF EXPERIMENTAL SYSTEM

With the overall system plan described in Section 6 and the specifications described in Section 7, we can now develop a detailed description of each system component. These components are examined in depth since the system under consideration must be compatible with both CATV and data transmission technologies. Systems similar to the one proposed have never been designed or built. Hence, major system components must be designed to establish the difficulty of meeting technical requirements and to ultimately establish a cost for each component.

A future report will study in detail the cost/performance tradeoffs of this system versus other local transmission methods involving other technologies such as packet radio, polled multidrop lines, dial up, and other communication techniques.

13.26

8.1   MATV AND CATV SYSTEMS

The digital MATV and CATV study should be carried out in two phases. Phase one will develop data transmission techniques for a locally built MATV system. In the second phase, this technology will be transferred to an existing CATV system. The advantages of this mode of operation are the following:

- The MATV system will be local and hence there would be no initial problems of equipment transportation.

- There are no initial restrictions on TV interference or picture quality.

- There are no initial restrictions on carrier frequency or bandwidth.

- The initial system can be varied in structure and performance to approximate the degradation in any CATV system.

- A final advantage of performing the tests on high quality, well built MATV system is that it would completely prove out the viability of using MATV systems to solve the in-building distribution problem. Although MATV equipment is generally poorer quality than CATV systems, there should be only minor technical problems with the local MATV system since cascades are low and environmental conditions are good. The local system will also provide a standard to which older, poorly built MATV systems must be raised for data transmission capabilities.

The MATV system should meet the following specifications:

- The system will have a separate antenna and pre-amplifier for each channel, so the levels for each channel can be controlled independently.

13.27

- All available off-the-air channels will be received.

- The system will cover the full VHF and UHF range.

- The head end amplifiers will be driven at full output capability of about 55 dbmV in order to test significant noise and cross modulation figures.

- The system will be two-way so that at any tap a signal modulated by digital data can be inserted and received at any other tap. This will be done by feeding the signal back to the head end and redistributing it through an amplifier after conversion.

- The system will contain at least one extender amplifier on a leg of at least 1500 feet to simulate paths in a large building.

- The system will contain converters so that signals can be converted from a sub-VHF channel to a mid band channel.

A block diagram for the system is shown in Figure 8.1.1 and a bill of materials using Jerrold equipment is given in Table 8.1.1.

Once the equipment and techniques are perfected for the MATV data system, they must be tested and modified under environmental operating conditions on an actual CATV system. CATV systems use much higher quality equipment than do MATV systems, but have much longer cascades. Since the test CATV system should be an operating system, there will be certain restrictions on test operations.

The requirements on the data transmission are as follows:

- The data signal bandwidth be limited to an available 6 MHZ TV channel. The placement of the signal is still being investigated. If operation were to be at ~70 MHZ, the 4 MHZ guard band between 70 and 74 MHZ could be used.

- The data signal must not yield visible
  interference with TV service. A reason-
  able guarantee is achieved if the data
  signal is kept 20 db below the video
  transmission.
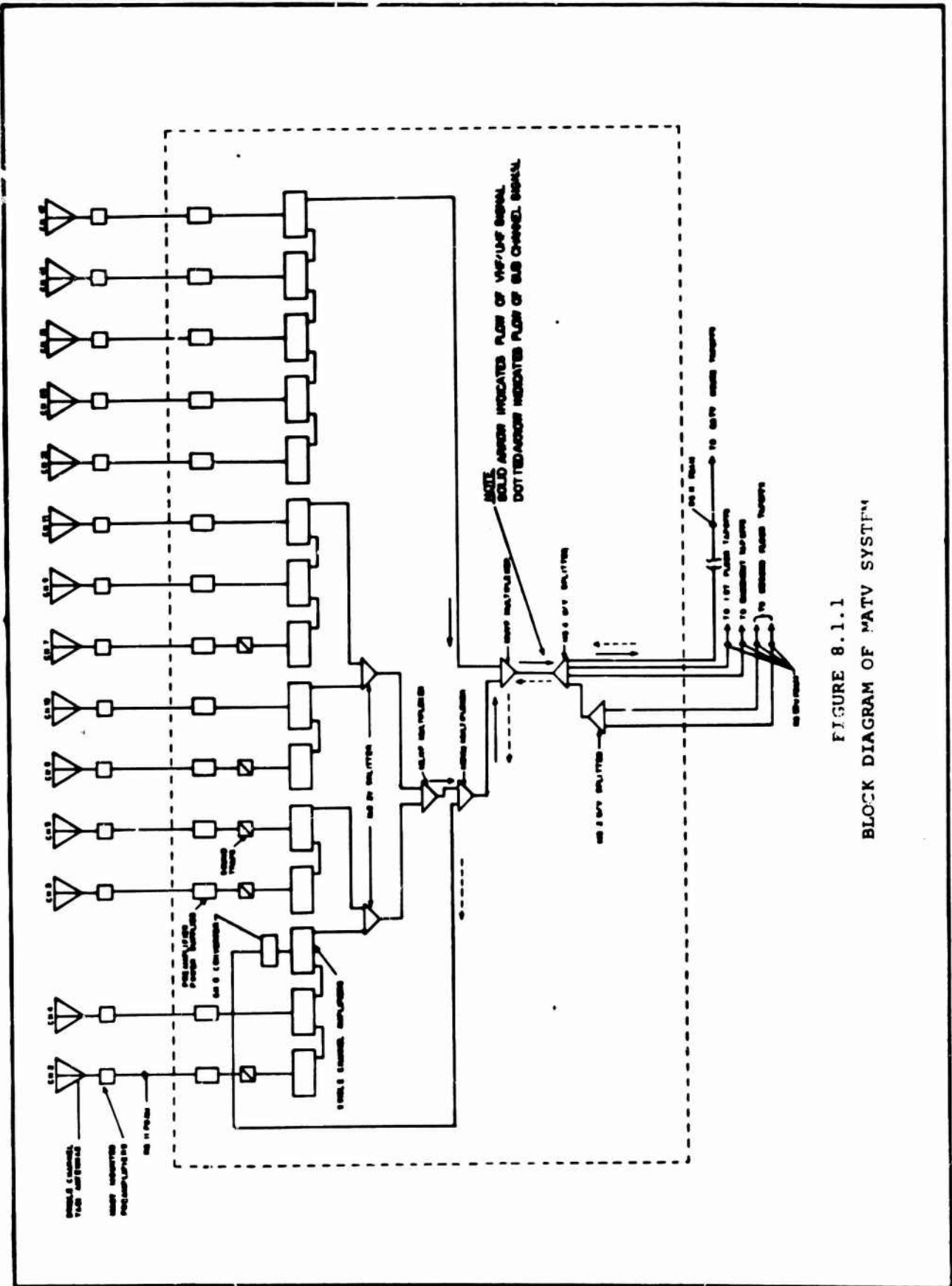
FIGURE 8.1.1

BLOCK DIAGRAM OF MATV SYSTEM

MATV SYSTEM BILL OF MATERIALS

QUANTITY	JERROLD CATALOG NUMBER	DESCRIPTION
4	J-Series	Lo band VHF antenna
5	J-Series	Hi band VHF antenna
5	J-275	UHF antenna
9	TPR	VHF preamplifiers
5	TPR (1443, 3968)	UHF preamplifier
2	PPS-8A	preamplifier power supply
10	THPM	VHF amplifier
5	UCA	UHF amplifier
3	TLB-2	trap filter
2	THB-2	trap filter
1	SCON-Sub-V	subchannel to VHF converter
2	1597	four-way splitter, UHF/VHF
2	1592B	two-way splitter, VHF
1	LHS-76	VHF multiplexer
1	FCO-320	UHF/VHF multiplexer
1	'CO47	subchannel - VHF, multiplexer
1	1596A	two-way splitter, VHF/UHF
38	UT-82	Subscriber taps
1	SPS-30/60A	power supply
1	--	rack and miscellaneous hardware
1	SLE-300-2W	CATV extender amplifier
4000 ft.	JA-500-cc-J	5 inch coaxial cable

TABLE 8.1.1

8.2   TERMINAL INTERFACE

A.   The interface can use a non-slotted ALOHA random access mode. The first transmission, as well as subsequent retransmissions, will be randomized over time.

B.   The interface will transmit and receive data to a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem or utilize modem provided clock signals along with modem data.

C.   The interface will provide and receive data for the interactive terminal at the rate required by the particular terminal. The exact rate chosen will be selected from the rates given below by means of simple plug changes or terminal block jumper wiring at the interface unit. The data rates possible are:

110	bits per second
134.5	bits per second
150	bits per second
300	bits per second
600	bits per second
1200	bits per second
2400	bits per second

The terminal input and output rates shall be independently selectable. In addition, it should be possible to control the transfer of data to and from the terminal by means of an externally supplied clock signal(s) which is below 2400 bits per second.

D.   The interface shall be capable of receiving or transmitting 5, 6, 7, and 8 bit characters. The number of bits per character shall be selectable independently for transmitter and receiver by means of plugs or jumper straps within the interface.

E. The interface shall be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

F. The following formats will be used for the TV/Terminal system:

1. Data Packets

HEADER	TEXT	CHECKSUM

2. Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D.	SOURCE I.D.	CONTROL CODES	MESSAGE NUMBER	TERMINAL RCVR RATE
40 bits	40 bits	8 bits	4 bits	4 bits

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation.

There are three possible means of identifying source and destination in the TV/Terminal Interface.

A. "Log in" to the interface via the terminal and establish the ID codes which the interface then uses in each header.

B. Use short, hard wired (or switch selectable) interface ID codes for header information. The user of the terminal "logs in" to the head end to identify fully.

C. Use full switch selectable ID codes on the interface into which the user sets his ID number. A likely choice is a Social Security number of 9 4-bit digits. No further identification of the user is required to the head end.

The first alternative is unattractive because the implementation of a dialogue capability between the terminal and the interface will be more costly and complicated than is warranted. A hardware design which allows (C), also allows (B) as an alternate later on with only

software changes while the reverse is not true. For this reason,
the initial test interfaces will be built to allow insertion of a
36 bit (9 4-bit digits) ID code for source and destination. If at
a later date it is desired to use a log in procedure at the head
end, the switches will be used to set in a smaller interface ID
code.

Since there are boundaries at 8 and 16 bits due to most mini-
computers which might be used at the head end of the CATV/MATV systems,
initial choice of 40 bit fields for both source and destination ID's
is recommended. An 8 bit control code field should also be adequate.
The control code field will contain information such as message ID
and acknowledge bits. The total header length is then 96 bits or six
16 bit words.

G.   The following description of the proposed operation of the
transmitter section of the TV/Terminal interface assumes that the
terminal has not been operated with the interface prior to this operation.

1.   The switches, jumpers, and plugs on or in the interface
should be set to conform to the parameters of the terminals.
This includes input and output data rates, character size, and
interfacing conventions such as EIA, current loop, etc.

2.   The initial parameters which are required for
operation with a given head end computer are set in via
switches, plugs, or jumpers. These parameters include
source and destination I.D. codes and special escape
characters which, upon receipt by the interface, cause
initiation of various functions (if any). As an example,
the character sequence which, when received from the
terminal, causes the interface to initiate message trans-
mission is one such parameter.

3.   The terminal should be connected to the inter-
face and modem, and A.C. power should be applied to all
units.

13.34

4. The terminal operator should compose the first
message and enter it on the terminal keyboard. The inter-
face will accept and buffer the message up to a maximum
number of characters. For interactive terminals, the
maximum number of characters will be 125, the maximum
line length. Depending on the terminal type and mode of
operation, the interface might provide data character
echoing to indicate correct message receipt to the
terminal operator.

5. Upon receipt of the escape character sequence
from the terminal (line terminator such as carriage re-
turn, line feed, or a similar sequence), or upon filling
of the interface buffer, a transmission would be initiated
from the interface, and the message would be transmitted
at 100K or 1M bits per second. Upon transmission of the
message, a timer would be started to time out the reception
of an acknowledgement from the message destination.

The timer will serve two functions: (a) time out the
reception of an acknowledgement (of the transmitted message)
from the head end for retransmission by the terminal inter-
face, and (b) randomize the starting time for the retrans-
mission. The fixed minimum delay before acknowledgement
should be slightly greater than 14 milliseconds. The
exact number will be chosen later and will reflect the ease
of generation of the hardware,and be based on a multiple
of an available clock rate.

The random interval of delay with respect to the
starting time for the retransmission will vary from
0 to $\approx$ 5 full packet intervals (0 to $\approx$ 64 milliseconds)
with an exact choice determined in the same manner as
for the fixed interval. The maximum random interval
will be divided into 1/2 millisecond sub-intervals as
the various transmission starting times, corresponding
to 128 possible starting delays. If the timer runs out

13.35

without the reception of an acknowledgement, the message is retransmitted. The transmitter retransmits a message N times, where N is selected between 0 and 15 by means of jumpers or a switch in the interface. If, after N retransmissions, no acknowledgement has been received, an indicator will be activated to notify the terminal operator of failure to communicate with the head end computer.

There will be a "Packet Acknowledged" light on the interface which allows the terminal operator to know if the packet he last sent has been acknowledged and tells him when he can start to send the next packet. In the case of an unacknowledged packet, the interface will refuse any new data from the terminal, and that state may be cleared by pressing a "reset" button on the interface.

6. As soon as an acknowledgement is received, the local copy of the transmitted message may be released and a new message accepted from the terminal for transmission. With this type of operation, only one message may be handed at a given time by the interface. Once a given message has been accepted for transmission by an interface, the terminal user must wait until either the message is sent and acknowledged or the message is cleared by the interface reset button.

7. A preliminary flow chart for operation of the transmitter is shown in Figure 8.2.1.

H. The description of the proposed operation of the receiver section of the TV/Terminal interface assumes that the terminal has not been operated with the interface prior to this operation.

1. The parameters of the terminal should be set into the interface as in steps G1, G2, and G3 above.

2.   Upon detection of modulation on the TV channel, the raw digital data will be passed through the receiver and bit and character synchronization will be obtained.

3.   The header will be inspected to see for which terminal the message is destined. If the message is not for this terminal, it will be ignored.

4.   If the message is destined for this terminal, the checksum will be checked to insure that the message is error free. If there are errors, the message will be ignored.

5.   If the checksum is valid, the header will be inspected to see if the message indicates an acknowledgement or other control function. If so, appropriate action will be taken. For example, if the message is an acknowledgement, the transmitter will be notified to drop the copy of the last message sent and will be able to accept a new message.

6.   The message text will be received and stored in a buffer in the interface, and if an acknowledgement is required, the interface will transmit one to the message

TV/TERMINAL INTERFACE

TRANSMITTER FLOW CHART

```
                           ┌──────────────┐      ┌──────────────┐
                           │  INITIALIZE  │──────│   POWER ON   │
                           └──────────────┘      └──────────────┘
        ┌──────────────────────────┐ NO
        │                          ┌──────────────────┐
        │                          │      INPUT       │
        │                          │ MESSAGE READY ?  │
        │                          └──────────────────┘
        │                                 │ yes
        │                          ┌──────────────────┐
        │                          │ TRANSMIT PACKET  │
        │                          └──────────────────┘
  ┌───────────┐                    ┌──────────────────┐
  │  UNLOCK   │                    │ START ACK TIMER  │
  │ KEYBOARD  │                    └──────────────────┘
  └───────────┘         NO
              ┌───────────┐ NO  ┌──────────────────┐
      YES     │   ACK     │◄────│    ACK TIMER     │
              │   RCVD    │     │    RUN OUT?      │
              └───────────┘     └──────────────────┘
                                       │ YES
                                ┌──────────────────┐
                                │ TEST NUMBER OF   │ NO MATCH
                                │ RETRANS.AGAINST  │
                                │        N         │
                                └──────────────────┘
                                       │ MATCH
  ┌───────────────┐            ┌──────────────────┐
  │ RESET PRESSED │            │   SET ALARM      │
  │               │            │   INDICATOR      │
  └───────────────┘            └──────────────────┘
```

FIGURE 8.2.1

13.38
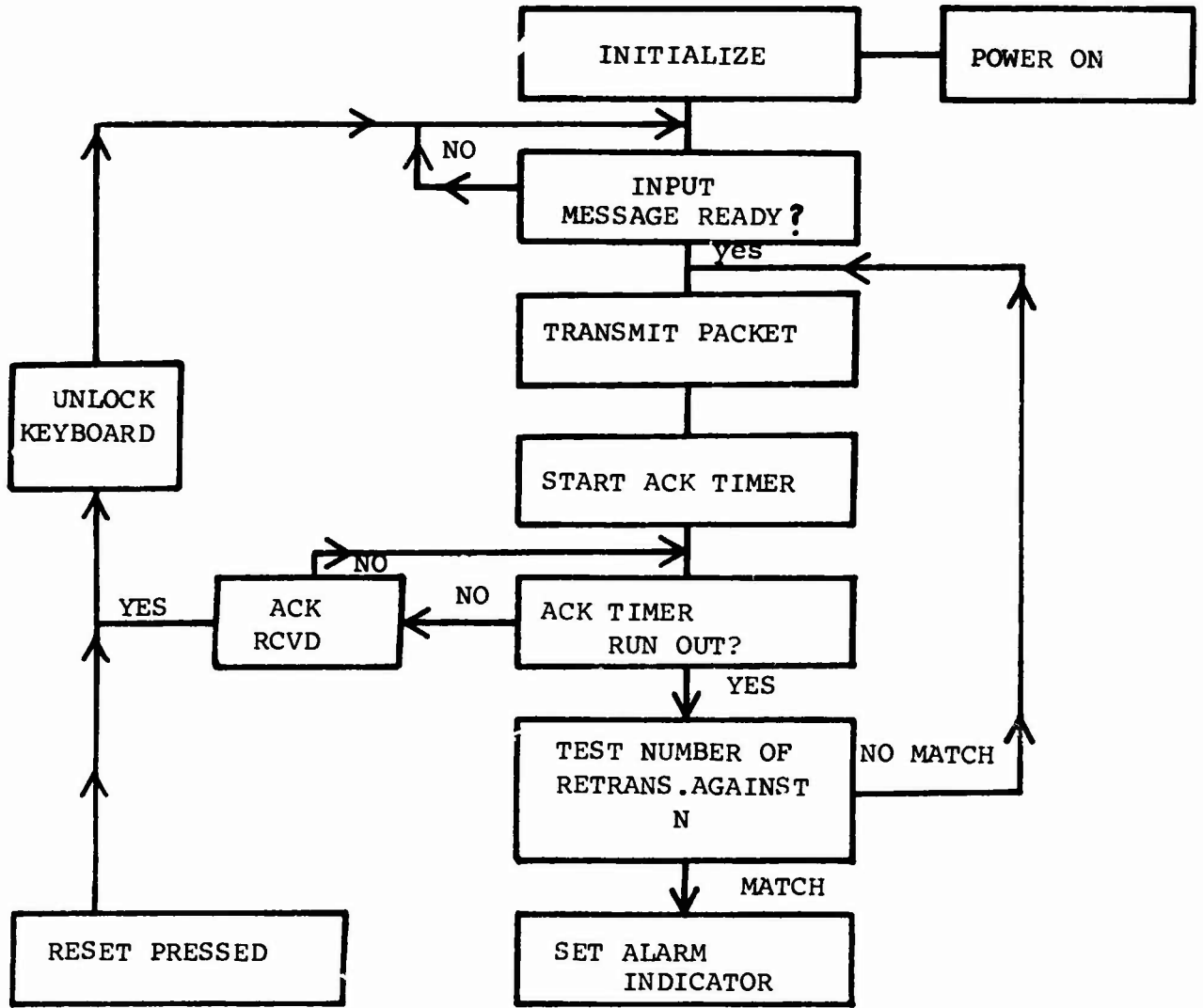
source. The interface will sense when the terminal is
turned off, using the Data Terminal Ready (DTR) signal
and will reject all packets received for that terminal
during terminal power off conditions. The head end will
not send messages to the same terminal interface without
waiting a fixed "safety" period. The duration of this
safety period will be determined at the head end by the
specific terminal receiver data rate information which
is a part of the header of all packets which originate
from that terminal interface. The acknowledgement from
the terminal interface will be sent to the head end
simultaneously with the sending of the message from the
interface to the terminal.

7. The number of the most recently received message
will be stored in the interface logic to allow detection
of duplicate messages.

8. A preliminary flow chart of the operation of the
receiver is shown in Figure 8.2.2.

I. A Cyclic Redundant Code (CRC) checksum will be employed on
the data for error control in the TV/Terminal system. The check-
sum for the message text will be chosen to provide adequate undetected
error probability under channel conditions which are expected to be
encountered in the TV system. The analysis and experience of the
Packet Radio and ALOHA systems will be used to pick the best checksum.
For hardware estimates, assume that the ARPANET type of 24 bit CRC
will be used in the TV/Terminal interface.

TV/TERMINAL INTERFACE

RECEIVER FLOW CHART
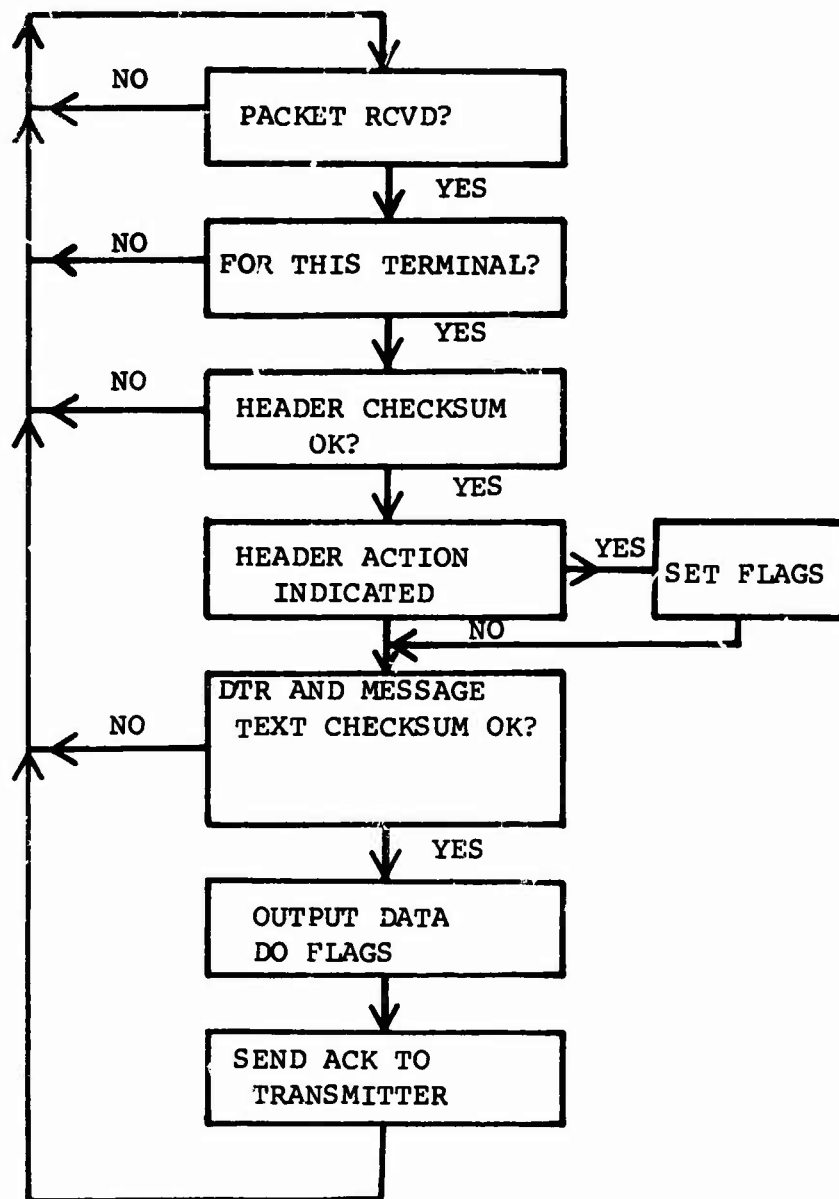
FIGURE 8.2.2

Electrical Specifications

A.   The interface shall be capable of being configured to
match a wide variety of terminal requirements. This flexibility
shall be provided either by having a set of the various possible
circuits required in each interface and selecting the proper form
by jumpers of plugs, by having a set of sub-modules which can be
interchanged in a given interface to adopt to a particular termin-
al's requirements, or, ideally (but possibly quite difficult), by
having a single "universal" type of interface circuit which can
be easily adapted to each terminal requirement. The types of
requirements which can be expected are:

1.   TTY (Current loop @ 20ma or 60ma)

2.   EIA-RS-232 (Either full or partial depending
     on the terminal)

3.   CCITT (Either full or partial depending on
     the terminal)

4.   MIL STD 188B

5.   Standard low level Signaling Interface

6.   Specials

In all cases, the interface circuits should prevent or minimize
damage to either the interface or to the terminal under conditions
of short circuit, open circuit, or voltage or current transients.
State-of-the-art isolation techniques will be used, including op-
tically-coupled interface circuits and protective diodes on sensi-
tive elements.

B.   The interface will be designed to match the choice of
modem. The same isolation and protective features used in the
connection of the interface to the terminal will be included in the
circuits which connect the interface to the modem.

C.   The power supply and interface circuits will be housed
together. Input power requirements are:

1.   110 volts A.C.

2.   60 Hz, single phase

3.   Estimated input power required less than
     100 watts

The power supplies will provide suitable voltages and currents for
the operation of all circuits and modules within the interface.
Standard modular off-the-shelf supplies will be used whenever possi-
ble.   The interface will have a master power switch, power-on indi-
cator, adequate protective fusing, transmission and reception status
indicators, and a reset button.

    D.   The logical design of the interface will be realized using
standard integrated circuits and components.   The possible use of
Large Scale Integration (LSI) logic modules, such as a microcomputers
and Read Only Memory (ROM), will be investigated.   The characteristics
of available microcomputers will be carefully studied and a choice will
be made based on the TV/Terminal interface requirements.   At a later
date, for reasons of improved microcomputer components or desired
compatibility with other packet systems, a different microcomputer can
be selected and programmed to perform the same functions without major
hardware changes.   The project will result in a reliable, cost effec-
tive interface with sufficient flexibility to allow reasonable system
modifications during testing.

Mechanical Specifications

A. The interface will be packaged in an enclosure which
is capable of standing alone. The estimated size of the interface
is approximately 19" wide, 20" deep, and 5" high. The interface
will have a 6' power cord and suitable connectors or terminal
strips to match the connections to the terminals and modem with
which the interface will operate. Sound construction practice will
be used throughout with an emphasis on ease of assembly and main-
tenance along with low cost. The estimated weight of the interface
is less than 30 pounds.

B. The front panel of the TV/Terminal interface will have a
power switch and whatever other switches and visual indicators are
required for operator assurance, such as power-on and incomplete
transmission. All other switches, connectors, and seldom used
controls will be mounted in a conveniently accessible, yet protected,
fashion. Some connectors will be mounted on the rear panel of the
interface, while other controls and terminals will be accessible
only with a set of tools such as a screwdriver or key to open a
service access panel or to remove the protective cover(s).

BLOCK DIAGRAM OF TERMINAL INTERFACE (FIGURE 8.2.3)

The cable modem data and control signals pass through appropriate level-conversion logic on entering the terminal interface. Modem control signals and status signals are set or sensed by the micro computer program via paths in the micro computer I/O multiplexer and controller. All incoming data are passed through a data examination register where special high speed character detection logic looks for synchronization characters and enables the checksum verification logic upon synchronization. All synchronized data are stored automatically in a high speed received packet buffer (First In, First Out - FIFO). Upon completion of reception of a packet, the micro computer is interrupted and begins to process the packet. The checksum is verified, and, if false, the buffer is cleared, and the sync logic is reinitialized. If the checksum is verified as true, the packet header is examined for destination, and if no match is detected, the receiver section is reinitialized and the buffer is cleared.

If a destination match is detected, the appropriate acknowledge and control information is stored in The Random Access Memor , (RAM), and the data is enabled to pass on a byte basis to the Uni rsal Asynchronous Receiver - Transmitter, (UAR-T), which converts the parallel data bytes to serial asynchronous data at the terminal speed. The data passes through appropriate terminal level conversion circuits before leaving the interface. The terminal control and status sense circuits are also connected to the micro computer and the I/O multiplexer. The micro computer can insert or delete characters in the actual data stream between the received packet buffer and the UAR-T, as for example in the case of an acknowledge for a previously transmitted message where no data would be passed to the terminal.

Data from the terminal is automatically stripped of start-stop bits by another UAR-T and stored in byte format in the transmit packet

buffer. Upon detection of the terminal special end-of-line character
or on filling up a packet, the micro computer is interrupted and
attaches the header information to the data in the transmit packet
buffer. At the appropriate randomized time, the data is gated to
the output register and the checksum generation logic is enabled.
The information about the message for retransmission purposes is
stored in RAM and the message is recirculated in the transmit buffer
at the same time it is sent to the modem. The micro computer controls
the number and timing of retransmissions based on acknowledge infor-
mation from received packets. The crystal clock provides timing
pulses to all circuits as required.
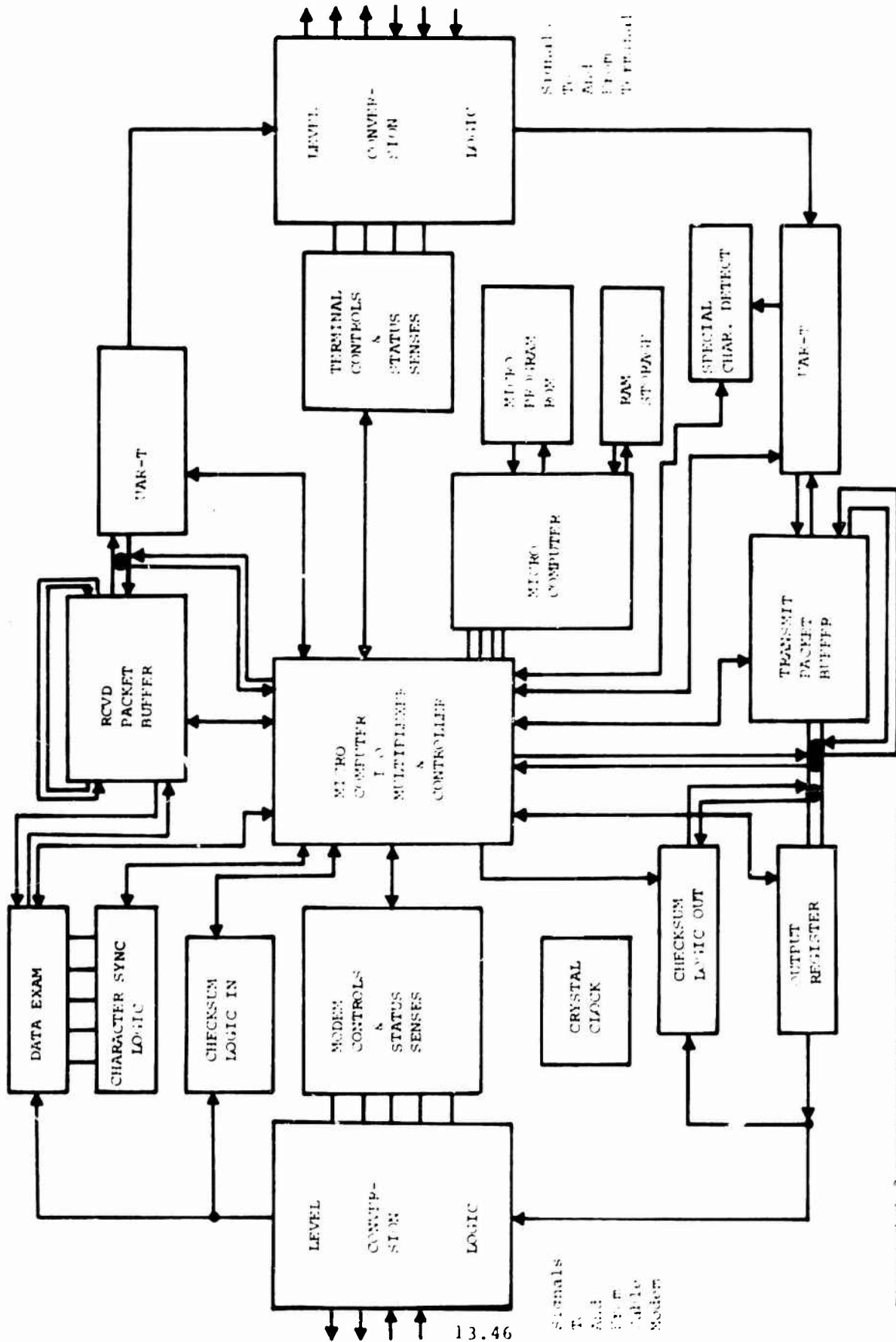
Figure 8.2.3 BLOCK DIAGRAM OF TERMINAL INTERFACE

8.3   MODEMS

For initial experiments a 2-phase differential phase shift keyed (DPSK), coherently detected signal with a 100 kilobit data rate is adequate. This selection of parameters enables the use of much off-the-shelf equipment while still giving the properties of a system meeting practical data requirements. For example, modems are commercially available to produce a baseband PSK binary data stream which can then be modulated onto a video carrier. These modems can produce a 0 dbmv signal on a 75 ohm line with good capture properties and excellent phase stability. The off-the-shelf modem will probably have poor acquisition time, on order of 100 milliseconds, which will limit the system to less than two hundred terminals in initial experiments.

Later experiments can be performed with 4 level DPSK and at a Megabit/sec data rate. However, the short acquisition times required for 1 Megabit/sec data rate would require special development of a Surface Acoustic Wave Device type detector [Matthaei,1973]. The final selection of parameters for a practical data transmission system depends upon several variables whose values are uncertain and subject to change with improvements in technology:

1.   The bandwidth of the head end minicomputer
2.   The amount of core at the head end minicomputer
3.   The bandwidth of the links to external test facilities.
4.   The throughput at the terminals
5.   The subscriber saturation level of CATV system

The specifications to be met by the final modems are given in Table 8.3.1.

type of modulation	DPSK
data throughput	100 KB/sec. or 1MB/sec.
carrier frequency	5-240 MHZ
nominal signal level	input 10 dbmv,output 32 dbmv
impedance	75Ω
reflection coefficient	-23 db
noise figure	7.5 db
power hum	60 db below signal level

Table 8.3.1

Modems meeting these specifications are available with perform-
ance parameters and resulting system degradation indicated in Table
8.3.2, [Cuccia, 1973].   The signal degradation is within
acceptable limits for error rates better than 10^{-20}.

Carrier Instability in Carrier Source	1/2° RMS in a PLL with Bandwidth = 0.03% Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Modulator	2°	0.10 db
Rise Time in Each Phase Change	1/4 Bit Period	0.40 db
Amplitude Unbalance in OPSK Modulator	0.2 db, 10°/db	0.10 db
Data Asymmetry from Data Source	+2%	0.05 db
Group Delay Distortion (Modulator)	25°	0.20 db
Clock Instability	1° RMS in the Bit Synchronizer PLL	0.10 db
	SOURCE TOTAL =	1.00 db

DEMODULATOR

Incidental FM from all Oscillators in RF Channel and Carrier Reconstruction	1/2° RMS in PLL with Bandwidth = 0.03% of Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Demodulator	2°	0.10 db
Reference Phase Noise in Reference PLO	1°	0.10 db
Total Group Depay Distortion (Total Channel from Modulator)	20°	0.35 db
Timing Jitter in Matched Filter Sampler	4%	0.25 db
Timing Bias Error in Matched Filter Sampler	1.5%	0.15 db
DC Offsets-Total Receive	4%	0.10 db
Data Waveform/Matched Filter Detector Mismatch	--	0.60 db
Table 8.3.2	SOURCE TOTAL =	1.70 db

8.4   TEST MINICOMPUTER

A.   General

The minicomputer used as the test head end controller in the Terminal/MATV-CATV system vill have the following character-istics:

   A. 16 bit word length

   B. ∿1 μ sec cycle time core memory

   C. Memory size expandable to ∿t least 32k words

In order to adequately test the Terminal/TV System, a variety of peripherals and interfaces provided by the mini computer vendor may be required. In addition, several custom interfaces may have to be developed for cases where standard interfaces are not available.

B.   Data Rate Constraints

The data rates expected in the head end computer system are shown in Figure 8.4. The data to and from the cable modem will flow at the rates of either 10^5 or 10^6 bits per second, although the flow will not be continuou· in general. At the rate of 10^6 bits/second, a 16 bi⁺ computer word will be assembled in the inter-face every 16 μ seconds. The transfers of these words into and out of the computer will require the availability of a high speed data channel, or alternately a Direct Memory Access (DMA) channel as a required option on the computer. Transfers to and from memory will be accomplished automatically without requiring program inter-vention.
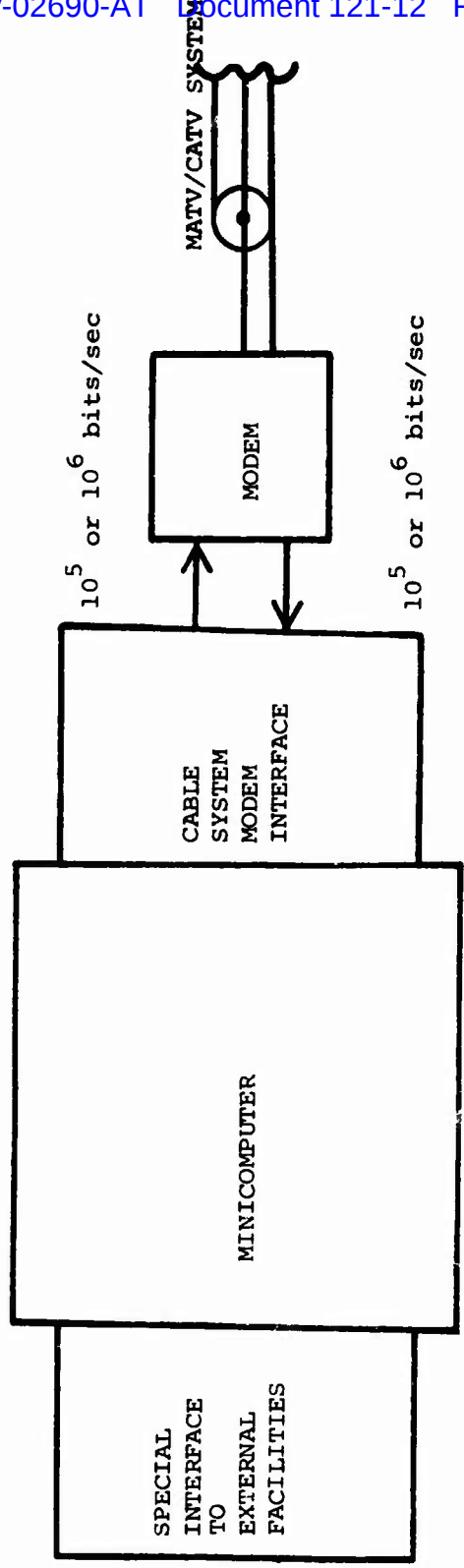
Figure 8.4.1 Head End Computer System Data Rates

C.   Functional Description

The computer at the head end must implement the programs to test the following functions for the system:

- Buffering of packets to and from terminals.

- Flow control of cable system

- Implementation of network protocols.

The headend computer will provide sufficient buffer storage to match the difference between the peak to average data rates seen on the cable system and any external sources of data. Initially, storage for a total of the order of 100 packets will be provided.

The head end computer will provide the required cable system flow control by means of acknowledges for received packets. When traffic from the cable terminals begins to congest the system, the head end will effect the flow control by refusing to acknowledge packets which cannot be accepted. This will cause the terminal to retransmit messages, producing the same effect as errors on the cable system. The head end computer will provide the implementation of all protocols required to communicate on the system.

The following is a list of some of the major program pieces which must be implemented:

1.   Cable modem interface handler
2.   Special interface to external facilities handler
3.   Monitor or supervisor for system
4.   Cable flow control routine
5.   Buffer allocation and management
6.   Miscellaneous background tasks such as garbage collect, timeouts, accounting, etc.

8.5 MINICOMPUTER TO CATV INTERFACE

This specification describes the interface between the head end computer and the MATV and/or CATV system. A diagram of the system is shown in Figure 8.5.1.

FUNCTIONAL SPECIFICATIONS

A.   The MATV-CATV computer interface will use a non-slotted ALOHA random access mode. The first transmission as well as subsequent retransmission of messages will be randomized over time.

B.   The interface will be able to simultaneously and independently transmit to and receive data from a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem, and receive modem clock signals along with modem data.

C.   The interface will be capable of receiving or transmitting 8 bit characters. The computer program shall be able to control the actual number of valid character bits in each received or transmitted character on a message by message basis.

D.   The interface will be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

E.   The following formats will be used for the TV/Terminal system:

(1)   Data Packets

HEADER	TEXT	CHECKSUM

(2)   Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D.	SOURCE I.D.	CONTROL CODES	MESSAGE NUMBER	TERMINAL RCVR RATE
40 bits	40 bits	8 bits	4 bits	4 bits

COMPUTER TO TV/TERMINAL CABLE SYSTEM INTERFACE
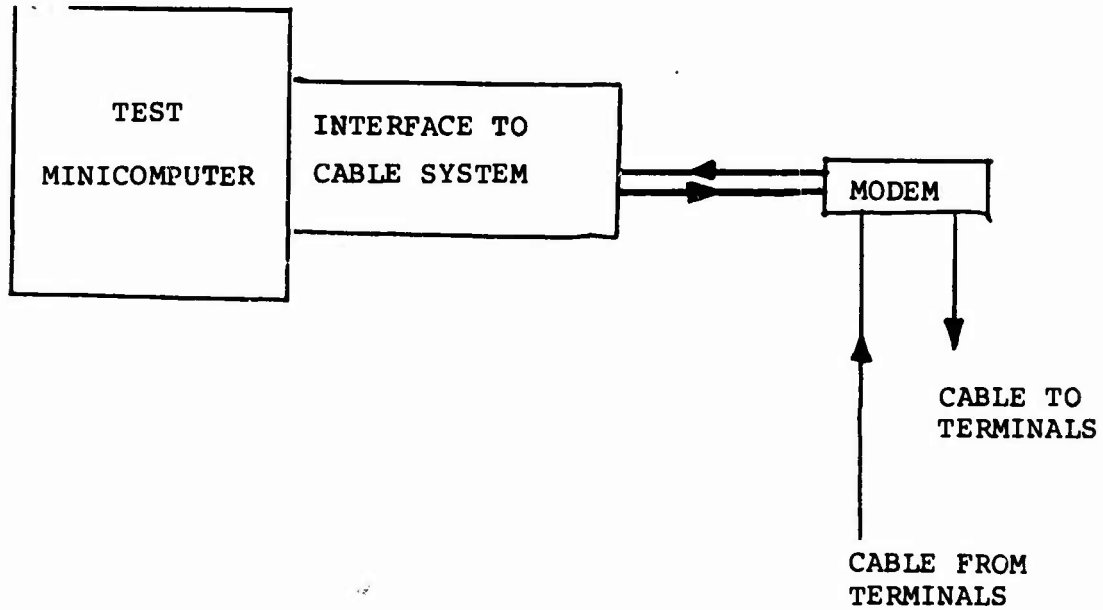
```
┌─────────────────┐
│                 │
│      TEST       │──────┐
│                 │ ┌────┴────────────┐
│   MINICOMPUTER  │ │  INTERFACE TO   │        ┌─────────┐
│                 │ │  CABLE SYSTEM   │◄──────  │  MODEM  │
│                 │ │                 │  ──────►│         │
└─────────────────┘ └─────────────────┘        └─────────┘
                                                    ▲  │
                                                    │  ▼

                                               CABLE TO
                                               TERMINALS


                                               CABLE FROM
                                               TERMINALS
```

FIGURE 8.5.1

13.54

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation. The destination and source I.D. fields will each be 40 bits long. This length is the lowest 16 bit word boundary which will contain the temporary 36 bit source and destination identifier.

F.   The transmitter section will operate in the following manner:

1.   Messages within the computer which are to be transmitted will be placed in one of two queues, a high priority acknowledgement queue or a regular message queue.

2.   All messages in the high priority queue will be transmitted before any regular messages are sent. The high priority queue will be serviced before each regular message transmission.

3.   Copies of transmitted acknowledgements will not be kept, since acknowledgements are never retransmitted.

4.   Copies of all transmitted regular messages will be kept in the computer until either receipt of an acknowledgement from the destination or the occurrence of a program-determined number of unsuccessful retransmissions.

5.   The transmitter program will set up a parameter table for each terminal active on the cable system. This table will contain terminal-specific information, such as terminal output rate and character set; and will be used to format each transmitted message to match the characteristics of the destination terminal and to allow the spacing of messages for each specific terminal in time to match terminal output speed.

6.   The transmitter program will operate in the computer in conjunction with other programs which will deal with the various functions required of the head end test mini computer system. There will be a monitor or executive program which will control the activities of

the cable system and external interface programs. This monitor
will also control or provide various required console related
activities and accounting functions.

7. Transfer of messages from the computer to the interface
logic will be by Direct Memory Access (DMA) of the head end computer.

8. The transmitter interface hardware will calculate the message
checksum for each message and append it to the message. In addition, the
hardware will automatically perform such routine tasks as synchronization
character generation, special escape character generation (for trans-
parency), and others as required. When the interface hardware completes
the transmission of each message, a program interrupt will be generated.

G. The receiver section will operate in the following manner:

1. Message transfers between the cable system interface and
the computer memory will be by means of DMA.

2. The interface logic will compute the checksum of the
received message and notify the computer by interrupt if an error has
occurred. In this case, the computer program will discard the data
for that message.

3. The interface logic will automatically acquire character
synchronization and will strip synchronization and control characters
from the input data stream.

4. The interface will allow the transparent reception of
full binary text.

5. As each message is received, its checksum will be verified
and a computer interrupt will be generated.

6. The receiver section of the computer program will, upon
notification by a receiver interrupt, examine the new message and
dispatch the message to the appropriate queue for further action.
Sample actions are given below:

> a. New message - determine destination queue and
> forward message, send acknowledgement to
> terminal, and check for retransmission.

b. Acknowledge - remove copy of acknowledged message from transmitter retransmission storage.

c. Duplicate - discard

7. The receiver section of the program will operate under the control of the computer monitor program and will interface to the transmitter and protocol programs.

ELECTRICAL SPECIFICATIONS

A. Modem Connection

The interface will be designed to match the choice of modem. Interface circuits should prevent or minimize damage to either the computer or the modem under conditions of short circuit, open circuit, or voltage or current transients. State-of-the-art isolation techniques will be used, including optically-coupled circuits and protective diodes on sensitive elements.

B. Computer Connection

The interface will be designed to connect to the Input/Output bus of the head end computer. The manufacturer's recommendations and specifications will be met in all cases to insure proper computer and interface operation.

C. Power

Power for the interface logic will be obtained from the computer power supply. Power for circuits beyond the isolation devices will be obtained by small modular power supplies which will be incorporated into the interface assembly.

D. Logical Design

The logical design of the interface will be realized using standard integrated circuits and components either of the types recommended by the minicomputer manufacturer or their equivalent. The project will result in a reliable cost effective interface with sufficient flexibility to allow reasonable system modifications during testing.

MECHANICAL SPECIFICATION

A.   The interface will be packaged on circuit boards of the type used by the minicomputer for interface circuits. The circuit boards will plug into the I/O bus of the minicomputer in the enclosure provided by the minicomputer and will connect to the modem with a cable and connector. Estimated weight of the interface is less than 10 lbs. The prototype version of the interface will most likely use wire-wrapped sockets for the integrated circuits to facilitate design changes during testing.

B.   There will be no operational controls associated with the interface other than program control. Test modes and switches for their control will be provided.

BLOCK DIAGRAM
OF MINICOMPUTER TO CATV INTERFACE

Figure 8.5.2

Assume that any frequency or phase acquisition is accomplished by the modem. The received bit stream is examined by the interface hardware for sync characters and synchronization is automatically accomplished without computer intervention. The interface hardware also handles DLE doubling and other special signaling conventions such as STX detection and ETX detection on the cable system automatically. As each character is received, it is packed into a minicomputer 16 bit word and when the 16 bits are ready, the word is transferred to core memory on a DMA cycle stealing basis. A computer interrupt is generated at the end of a packet or upon an error condition.

The transmit section of the interface is also automatic in that after initialization of the interface for a new packet, all communication discipline is accomplished automatically by the interface hardware, and words of data are simply requested by the interface via DMA from core, unpacked and serialized. The hardware also calculates and appends the checksum.
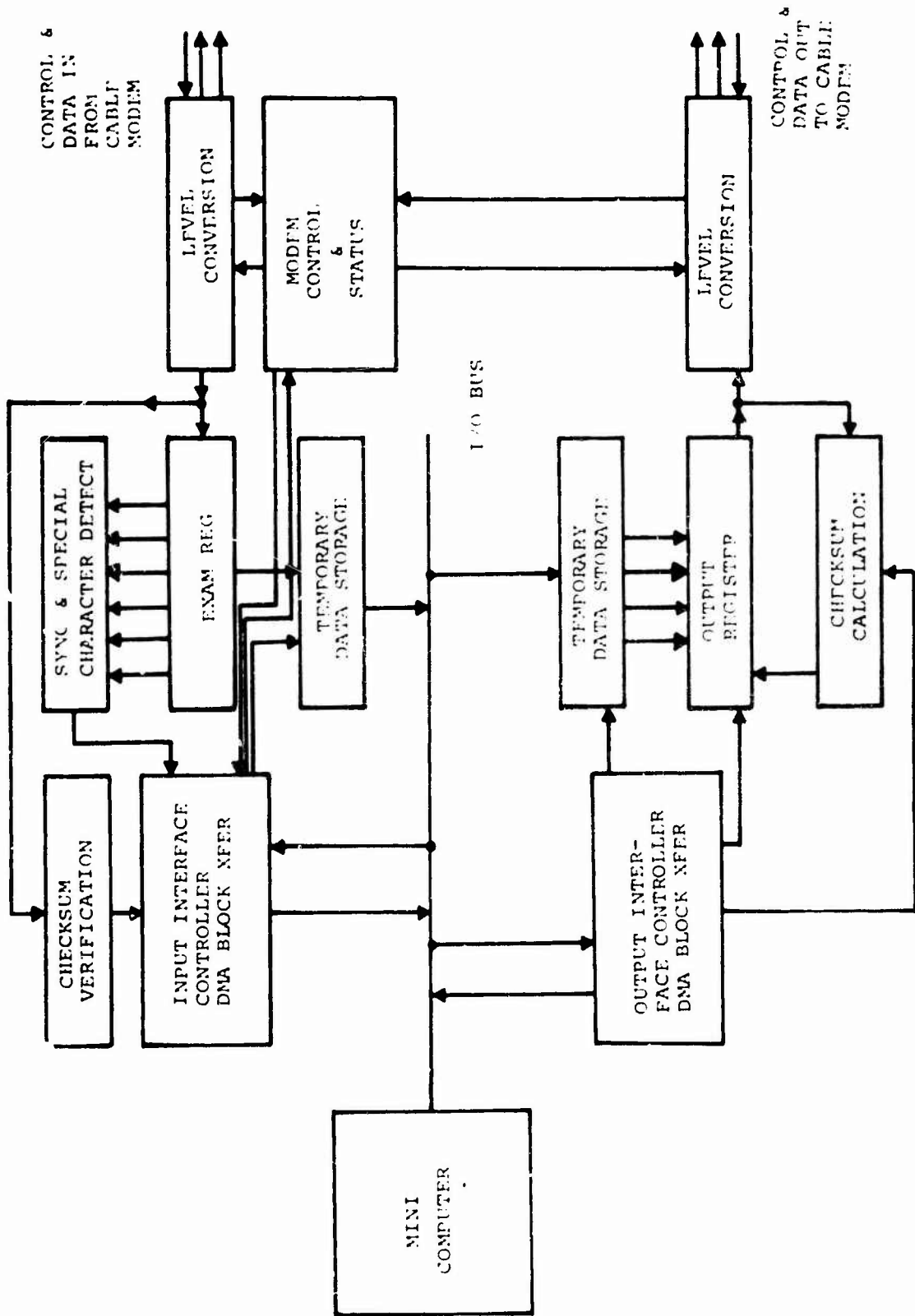
Figure 8.52 BLOCK DIAGRAM OF MINICOMPUTER TO CABLE SYSTEM INTERFACE

8.6   MINICOMPUTER TO EXTERNAL TEST FACILITIES INTERFACE

This specification describes the interface between the head
end test minicomputer and a communications link to external test
facilities.

Functional Specifications

A.   The interface will communicate with the external test
facilities over leased lines using commercially available modems,
and will provide all of the control signals required to operate
these modems.

B.   The interface will connect to the I/O system of the
selected minicomputer and will comply with all specifications
set forth for such connections.

C.   The interface will operate in full duplex mode at a rate
of up to at least 4800 bits per second.

Electrical Specifications

A.   The interface will be able to connect to a suitable data
modem operating at 4800 bps full duplex.   The connections will con-
form to EIA RS-232-C.   The interface will incorporate adequate
isolation features to prevent damage to itself or to the modem under
conditions of short circuit or open circuit signal leads.   In addition,
sensitive interface elements will be protected from damage by vol-
tage or current transients from external sources.

B.   Power Supplies

The interface will be powered from the minicomputer logic
power supplies, except for circuits which are to be isolated from
the computer system.   Any isolated sections of interface logic cir-
cuits will be powered by isolated power supplies with characteristics
to match the degree of isolation desired.   It is estimated that less
than 50 watts of DC power will be required for the interface circuits.

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;  <NAC>NSCENARIO.;1    MON 20-MAY-74 9:35AM              PAGE 1:1


     ONE OF THE FOLLOWING:
TAN <TAL>    TYPE ALL NODES(LINKS)
TN  <TL >    TYPE SPECIFIC NODES(LINKS)
SAN <SAL>    SET SAME VALUES TO ALL NODES(LINKS)
SN  <SL >    SET VALUES TO SPECIFIC NODES(LINKS)
FSN <FSL>    FORMATTED SET OF PROPERTY VALUES
ENTER COMMAND OR <CR> <TAN;WE LOOK AT NODE PROPERTIES OF ARPANET
     NODE        FP        LONG      LAT
    1 ETAC    1.0000000    77.000    38.930
    2 MBS     1.0000000    77.150    39.130
    3 ABER    1.0000000    77.000    39.000
    4 RUTG    1.0004000    74.450    40.480
    5 HARV    1.0000000    71.250    42.500
    6 NCC     1.0000000    71.250    42.500
    7 BBN     1.0000000    71.250    42.500
    8 CCA     1.0000000    71.330    42.500
    9 MIT2    1.0000000    71.200    42.500
   10 MITI    1.0000000    71.200    42.500
   11 LINC    1.0000000    71.330    42.580
   12 RADC    1.0000000    75.420    43.250
   13 CASE    1.0000000    81.750    41.500
   14 CMU     1.0000000    79.930    40.500
   15 BELV    1.0000000 >  77.000    39.080
   16 SDAC    1.0000000    77.150    38.920
   17 MITRE   1.0000000    77.000    39.000
   18 ARPA    1.0000000    77.000    39.000
   19 RML     1.0000000    80.570    28.250
   20 ISI     1.0000000   118.580    34.000
   21 RAND    1.0000000   118.580    33.920
   22 UCSD    1.0000000   117.160    32.660
   23 UCLA    1.0000000   118.520    34.070
   24 SDC     1.0000000   118.550    34.020
   25 USC     1.0000000   118.350    34.000
   26 DOCB    1.0000000   105.000    39.500
   27 GWC     1.0000000    96.000    41.000
   28 WPAFB   1.0000000    84.200    39.750
   29 ILLI    1.0000000    88.500    40.080
   30 UTAH    1.0000000   111.830    40.660
   31 LBL     1.0000000   122.280    37.830
   32 LLL     1.0000000   121.750    37.630
   33 SRI     1.0000000   122.160    37.360
   34 XEROX   1.0000000   122.170    37.300
   35 TYMSH   1.0000000   121.900    37.330
   36 FNWC    1.0000000   121.920    36.500
   37 UCSB    1.0000000   119.750    34.500
   38 STAN    1.0000000   122.170    37.300
   39 AMES    1.0000000   122.030    37.280
   40 MOFF    1.0000000   122.030    37.280
   41 BBNT    1.0000000    71.250    42.500
   42 AMET    1.0000000   122.030    37.280
```

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```
:  <NAC>NSCENARIO.11    MON 20-MAY-74 9:35AM            PAGE 1:2


ENTER PROPERTY COMMAND OR ?  <;WE HIT CARRIAGE RETURN TO MOVE UP THE COMM
                                    **AND HIERARCHY

ENTER SYSTEM COMMAND OR ? <E
ENTER EDITING COMMAND OR ? <AL 15 20;WE ADD A NEW CROSS COUNTRY LINK
LINK ADDED:       15  BELV       20  ISI
ENTER EDITING COMMAND OR ? <
ENTER SYSTEM COMMAND OR ? <P
ENTER PROPERTY COMMAND OR ? <SAN FP 0;WE ARE GOING TO DO A RELIABILITY A
                                    **NALYSISWHERE ONLY LINKS FAIL SO WE
                                    ** SET ALL NODE FAILURE PROBABILITIE
                                    **S TO ZERO

ENTER PROPERTY COMMAND OR ? <TAN ;WE CHECK TO SEE IF IT WORKEED
     NODE        FP         LONG      LAT
      1 ETAC    0.0000000    77.000    39.930
      2 NBS     0.0000000    77.160    39.130
      3 APEP    0.0000000    77.000    39.000
      4 RUTS    0.0000000    74.450    40.490
      5 HARV    0.0000000    71.250    42.500
      6 NCC     0.0000000    71.250    42.500
      7 BBN     0.0000000    71.250    42.500
      8 CCA     0.0000000    71.330    42.500
      9 MIT2    0.0000000    71.200    42.500
     10 MIT1    0.0000000    71.200    42.500
     11 LINC    0.0000000    71.330    42.580
     12 RADC    0.0000000    75.420    43.250
     13 CASE    0.0000000    91.750    41.500
     14 CMU     0.0000000    79.930    40.500
     15 BELV    0.0000000    77.000    39.030
     16 SDAC    0.0000000    77.160    38.920
     17 MITRE   0.0000000    77.000    39.000
     18 APPA    0.0000000    77.000    39.000
     19 RML     0.0000000    80.570    28.250
     20 ISI     0.0000000   118.580    34.000
     21 RAND    0.0000000   118.580    33.920
     22 UCSD    0.0000000   117.160    32.660
     23 UCLA    0.0000000   118.520    34.070
     24 SDC     0.0000000   118.550    34.020
     25 USC     0.0000000   118.350    34.000
     26 DOCB    0.0000000   105.000    39.500
     27 GWC     0.0000000    96.000    41.000
     28 WPAFB   0.0000000    84.200    39.750
     29 ILLI    0.0000000    88.500    40.090
     30 UTAH    0.0000000   111.930    40.660
     31 LBL     0.0000000   122.230    37.830
     32 LLL     0.0000000   121.750    37.630
     33 SRI     0.0000000   122.160    37.360
     34 XEROX   0.0000000   122.170    37.300
     35 TYMSH   0.0000000   121.900    37.330
     36 FNWC    0.0000000   121.920    36.500
     37 UCSB    0.0000000   119.750    34.500
```

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:  <NAC>NSCENARIO.;1    MON 20-MAY-74 9:35AM              PAGE 1:3


   38 ITAN     0.0000000   122.170   37.300
   39 AMES     0.0000000   122.030   37.290
   40 MOFF     0.0000000   122.030   37.290
   41 BBNT     0.0000000    71.250   42.500
   42 AMET     0.0000000   122.030   37.290
ENTER PROPERTY COMMAND OR ? <TL 15 20:WE MUST INPUT PROPERTIES OF NEW LI
                                      **NK
      NODE PAIR          FP       CAP     FLOW    COST
   15 BELV   20 ISI   0.0000000     0.00    0.00     0.00
ENTER PROPERTY COMMAND OR ? <

                         SL 15 20 FP=1. CAP.50:WE TRY ALTERNATE DELI
                                      **MITERS
ENTER PROPERTY COMMAND OR ? <TC

ENTER SYSTEM COMMAND OR ? <P
ENTER PROPERTY COMMAND OR ? <TL 15 20 00

      NODE PAIR          FP       CAP     FLOW    COST
   15 BELV   20 ISI   1.0000000    50.00    0.00     0.00
ENTER PROPERTY COMMAND OR ? <
ENTER SYSTEM COMMAND OR ? <REL:NOW WE GO TO THE RELIABILITY PROGRAM
ENTER RELIABILITY COMMAND OR ? <?
ONE OF THE FOLLOWING:
   R        RUN RELIABILITY:   LP    LIST PARAMETERS
   CP       CHANGE PARAMETER:  DP    DEFAULT PARAMETERS
   T        TABULAR OUTPUT:    G     GRAPHIC OUTPUT
   RJS      WRITE RJS FILE FOR CCN/91
ENTER COMMAND OR <CR> <
                      LP
RELIABILITY PARAMETERS ARE:
  NSAMP              100   NUMBER OF SAMPLES
  SEED                0   RANDOM NUMBER SEED
  MAXP          1.0000000  PROBABILITY FACTOR
ENTER RELIABILITY COMMAND OR ? <CP
ENTER   KEYWORD,VALUE   OR ? <NSAMP 10:WE REDUCE NUMBER OF SAMPLES IN SI
                                      **MULATION TO SAVE COMPUTER TIME
VALUE OF NSAMP CHANGED FROM
ENTER   KEYWORD,VALUE   OR ? <         100 TO             10
ENTER KEYWORD OR <CR>       <
ENTER RELIABILITY COMMAND OR ? <R:WE NOW RUN THE RELIABILITY ANALYSIS

COMPUTATIONS PROCEEDING
ENTER RELIABILITY COMMAND OR ? <T:ASK FOR TABULAR OUTPUT
ENTER OUTPUT COMMAND OR ? <?
ONE OF THE FOLLOWING:
  TTY:      TO TELETYPE:   F       TO DISK FILE
  <CR><CR> TO CANCEL
ENTER OUTPUT COMMAND OR ? <TTY:
TL
```

4.11

```
;   <MAC>NSCENARIO.11     MON 20-MAY-74 8:35AM
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PAGE 2

```
MAC RELIABILITY RUN:
NUMBER OF NODES=  42    NUMBER OF BRANCHES=  48
SEED=         0         NUMBER OF SAMPLES=         10
MAX.PROB.=  1.00000     PROB.INCR.=   0.05000
HIT <CR> TO CONTINUE; <N> TO STOP <
```

EXPECTED FRACTION OF NODE PAIRS NOT COMMUNICATING

FACTOR	EXPECTATION	VARIANCE	STANDARD DEV
0.00	0.0000000E+00	0.0000000E+00	0.0000000E+00
0.05	0.2984901E-01	0.4846934E+00	0.6961992E+00
0.10	0.1429733E+00	0.2440847E+01	0.1562321E+01
0.15	0.2430894E+00	0.4249271E+01	0.2061133E+01
0.20	0.4104530E+00	0.4663419E+01	0.2159495E+01
0.25	0.5305459E+00	0.1115992E+01	0.1056405E+01
0.30	0.7034843E+00	0.8507560E+00	0.9223644E+00
0.35	0.7773519E+00	0.6032070E+00	0.7766640E+00
0.40	0.8554007E+00	0.2932462E+00	0.5415221E+00
0.45	0.9049942E+00	0.1156519E+00	0.3400763E+00
0.50	0.9272938E+00	0.3703107E-01	0.1924346E+00
0.55	0.9345109E+00	0.3886824E-01	0.1971376E+00
0.60	0.9493612E+00	0.3178136E-01	0.1782733E+00
0.65	0.9610918E+00	0.2711382E-01	0.1646528E+00
0.70	0.9699197E+00	0.1457462E-01	0.1207254E+00
0.75	0.9764229E+00	0.1061702E-01	0.1030389E+00
0.80	0.9830430E+00	0.7367886E-02	0.8583639E-01
0.85	0.9892695E+00	0.3424797E-02	0.5852176E-01
0.90	0.9919861E+00	0.2309814E-02	0.4806053E-01
0.95	0.9962834E+00	0.4602207E-03	0.2145275E-01

```
HIT <CR> TO CONTINUE; <N> TO STOP <
```

PROBABILITY OF NET DISCONNECTED

FACTOR	EXPECTATION	VARIANCE	STANDARD DEV
0.00	0.0000000E+00	0.0000000E+00	0.0000000E+00
0.05	0.2000000E+00	0.1600000E-01	0.1264911E+00
0.10	0.7000000E+00	0.2100000E-01	0.1449138E+00
0.15	0.7000000E+00	0.2100000E-01	0.1449138E+00
0.20	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.25	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.30	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.35	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.40	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.45	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.50	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.55	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.60	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.65	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.70	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.75	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.80	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.85	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.90	0.1000000E+01	0.0000000E+00	0.0000000E+00
0.95	0.1000000E+01	0.0000000E+00	0.0000000E+00

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```
;  <NAC>NSCENARIO.;1    MON 20-MAY-74 9:35AM              PAGE 2:1


DONE.
ENTER RELIABILITY COMMAND OR ? <
ENTER SYSTEM COMMAND OR ? <ROUT;NOW WE DO A ROUTING ANALYSIS FOR THE SAM
                           **E NET

ENTER ROUTING COMMAND OR ? <?
ONE OF THE FOLLOWING:                   DF - DEFAULT ROUTING PARAMETERS
R  - RUN ROUTING ANALYSIS                Q - QUIT ROUTING SECTION
LP - LIST ROUTING PARAMETERS            TR - CHANGE TRAFF. REQUIREMENTS
CP - CHANGE ROUTING PARAMETERS           O - SEE OUTPUT     D - DRAW NETWORK

ENTER ROUTING COMMAND OR ? <LP

ROUTING PARAMETERS ARE:
KEYWORD              CURRENT VALUE        DESCRIPTION

 ITMAX                   5            DESIRED NUMBER OF ITERATIONS
 PKLEN                0.500000        AVG. PACKET LENGTH (KBITS)
 DELAY                0.200000        AVG. PACKET DELAY (SEC)
 PROVR                0.350000        PROTOCOL OVERHEAD (FRACTION)
 RUOVR                0.070000        ROUTING OVERHEAD (FRACTION)
 NDPRS                0.001000        NODAL PROCESSING TIME (SEC)
 UNIRO                1.000000        UNIFORM TRAFFIC REQUIREMENT (KB/S)
 PRPGA                0.000008        LINE PROPOGATION DELAY (SEC/MILE)
 THACC                0.000100        THROUGHPUT ACCURACY   (%)
 TIACC                0.000100        TIME DELAY ACCURACY   (SEC)
 NCAP                    2            NUMBER OF DIFFERENT CAPACITIY OPTIONS
 CAP1                 50.000          CAPACITY(KB/S) OF CAP OPTION # 1
 FIX1                950.000          SUM OF ALL FIXED COSTS FOR CAP OPT. # 1
 RAT1                  5.000          RATE/MILE FOR CAPACITY OPTION # 1
 CAP2                230.000          CAPACITY(KB/S) OF CAP OPTION # 2
 FIX2               1300.000          SUM OF ALL FIXED COSTS FOR CAP OPT. # 2
 RAT2                 30.000          RATE/MILE FOR CAPACITY OPTION # 2

FOR HELP ON THESE PARAMETERS, ENTER THE ROUTING COMMAND <CP>
ENTER ROUTING COMMAND OR ? <CP
ENTER   KEYWORD,VALUE   OR ? <HELP;WE WANT MORE INFORMATION
ONE OF THE FOLLOWING:
<KEYWORD>,<VALUE>       TO CHANGE VALUES
<CR><CR>               TO TERMINATE
LIST                   LISTS KEYWORDS AND VALUES
HELP <KEYWORD>         DESCRIBES <KEYWORD>
HELP ALL               DESCRIBES ALL KEYWORDS
ENTER   KEYWORD,VALUE   OR ? <
                           HELP ITMAX
ITMAX: MAX NUMBER OF ROUTING ITERATIONS DESIRED;
ERATIONS TO SAVE RECOMMENDED MAX IS 10, GOOD APPROXIMATION AFTER 5
ENTER   KEYWORD,VALUE   OR ? <
                           ITMAX=2;WE REDUCE ITMAX TO SAVE TIME
VALUE OF ITMAX CHANGED FROM           5 TO              2
ENTER   KEYWORD,VALUE   OR ? <
ENTER KEYWORD OR <CR>
```

4.13

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```
;  <NAC>NSCENARIO.;1    MON 20-MAY-74 8:35AM            PAGE 2:2


ENTER ROUTING COMMAND OR ? <R
 COMPUTATIONS PROCEEDING
AT ITERATION  1, THRUPUT=    280.993 KB/S      DELAY=  0.19997 SEC.
AT ITERATION  2, THRUPUT=    329.897 KB/S      DELAY=  0.19992 SEC.

  DESIRED NO. OF ITERATIONS  REACHED

THRUP=  329.897 KB/S     WHICH IS   19.158 % OF THE BASE REQUIREMENT
 DELAY=  0.19992 SEC    TOTAL COST=  103319. $/MO
ELAPSED TIME :  0 MIN., 21 SEC.         CPU TIME :  0 MIN., 10 SEC.
CONTINUE ROUTING WITH MORE ITERATIONS ?  ENTER Y OR N
 <N

ENTER ROUTING COMMAND OR ? <O:WE LOOK AT OUTPUT
ENTER OUTPUT COMMAND OR ? <?

ONE OF THE FOLLOWING:
    LF   - LOOK AT LINK FLOWS AND COSTS OF MOST RECENT RUN
    T    - LOOK AT THROUGHPUT-DELAY TABLE OF MOST RECENT RUN
    S    - DRAW THROUGHPUT-DELAY CURVES
    Q    - QUIT OUTPUT
ENTER OUTPUT COMMAND OR ? <T
SPECIFY OUTPUT DEVICE OR ?<?
ONE OF THE FOLLOWING :
TTY  - OUTPUT TO TELETYPE       F - OUTPUT TO DISK FILE
?    - PRODUCES THIS LIST       Q - QUIT THIS SPECIFIC OUTPUT
HC - OUTPUT TO HARDCOPY DEVICE (IF SLAVE IS AVAILABLE)
SPECIFY OUTPUT DEVICE OR ?<F

ENTER 5 CHARACTER FILE NAME<ROUTO

FILE  ROUTO.DAT  OPENED ON DISK UNIT # 21
SPECIFY OUTPUT DEVICE OR ?<
SPECIFY OUTPUT DEVICE OR ?     <

ENTER OUTPUT COMMAND OR ? <

ENTER OUTPUT CONMMAND OR ?     <

ENTER ROUTING COMMAND OR ? <
ENTER ROUTING COMMAND OR ?     <
ENTER SYSTEM COMMAND OR ? <
ARE YOU ALL DONE? Y OR N? <Y


CPU TIME: 45.10 ELAPSED TIME: 35:14.15
NO EXECUTION ERRORS DETECTED

EXIT.
```

4.14

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IMPACT OF INTERACTIVE GRAPHICS

ON NETWORK DESIGN

I.   INTRODUCTION

Recently NAC has developed an interactive program that analyzes packet switching networks for cost, throughput delay, and reliability performance. Input and output is handled via a graphic terminal, which displays network topology, allows easy topological reconfiguration and shown curves of network performance (delay, reliability, etc.) versus a variety of network parameters (throughput, failure rates, etc.).

The details of the algorithms used in the analysis are presented in [1], [2], and [3]. This chapter is a description of the interactive program and it's use, the program's performance, a short comparison with an equivalent bath program and implications of interactive graphics on network design.

II.   FUNCTIONAL SPECIFICATIONS OF THE INTERACTIVE NETWORK ANALYSIS PROGRAM

NAC's interactive network analysis program called RELROUT is located on the (NAC) directories at USC-ISI and BBN for use by any network user. It runs on a PDP-10 using the TENEX operations system. The program can be run from any type of terminal, but will only support graphics on an IMLAC PDS-10 graphics display unit.

The program analyzes a packet switching network for routing and reliability performance.

The user must specify:

- A network topology.

- Properties of nodes and links.

- General parameters such as packet size, etc.

The topology can be entered in two ways:

1. Create a new network configuration through the use of network editing commands; or

2. Read an existing configuration from a previously generated data file.

The second method of inputting a network is very useful when repeated evaluation of the same basic topology are required. On an IMLAC terminal, the user may move the position of the nodes on the CRT in order to obtain a clearer and more intelligible graphic representation of the network.

5.2

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The user defines the following node and link properties:

A.   Node Properties

  1.   Name (Optional)

  2.   Location (longitude and latitude, for cost calculation in the routing analysis.

  3.   Failure probability (for reliability analysis).

  4.   Symbol type - □,△,◊,O (for display purposes).

B.   Link Properties

  1.   Capacity or line speed (for routing and cost analysis)

  2.   Failure probability (for reliability analysis).

  3.   Link type - solid or dotted (for display purposes).

After the user has defined the network configuration and properties, he can request either a routing or a reliability analysis to be performed. Associated with each analysis are various general parameters.
For a routing analysis these are:

  1.   Average packet delay.

  2.   Average packet length.

  3.   Overhead

  4.   Tariff Structure, etc.

5.3

For the reliability analysis the parameters are:

1.   Number of samples.

2.   Random number seed.

3.   Range of variation for the probabilities.

Initially all of these parameters are defaulted to specified values. However, the user can change any or all of them. The program does error and validity checking on the parameters that are changed. Also, upon user request, a short description of each parameter can be provided.

When the user is satisfied with the values of the various parameters, he can ask for the analysis to begin. In most cases, the processing is done locally in the PDP-10; however, the reliability analysis can be performed remotely, as discussed later in this chapter.

When the analysis is completed, the user can examine various outputs. The routing analysis output consists of a list of link flows, lengths, and costs, and values for global throughput, delay and cost.

There is also a table of network throughput and delay as a function of relative traffic. Reliability output consists of tables of P_{nc} (probability of network disconnected) and F_{nc} (fraction of node pairs not able to communicate) as a function of component failure rates. If the user's terminal can support graphics (IMLAC), the program will plot curves for throughput vs delay and for P_{nc} and F_{nc} of the most recent run along with at most two previous runs. (See Figures 1,2, and 3).

After examining the output of the specific analysis, the user typically will do one of the following:

1.   Perform a sensitivity analysis on the present network configuration by varying the input parameters.

2.   Switch to the other analysis (routing reliability or vice versa).

3.   Edit the topology (add nodes and/or links) and/or change node or link property values and proceed with another routing or reliability analysis.

4.   Exit the program.

At any time during the interactive session, the user may save his network topology on a data file for use at a later time. After leaving the program, the user is returned to the TENEX operating system.

NAC's interactive program assists the network analyst by:

1.   Offering a schematic representation of the current network configuration on a graphic display device;

2.   Allowing interactive error checking of topology and input parameters so that the user can correct the input data immediately;

3.   Removing the necessity of explicitly entering all the data for each run and allowing flexibility in the order and format of input;

4.   Using a flexible command structure, with numerous prompts to tutor the novice user if he requests help; and with short, quick commands for the experienced user.

5.   Providing network performance results with small enough response time, so that an effective man-machine interactive design can be carried out.
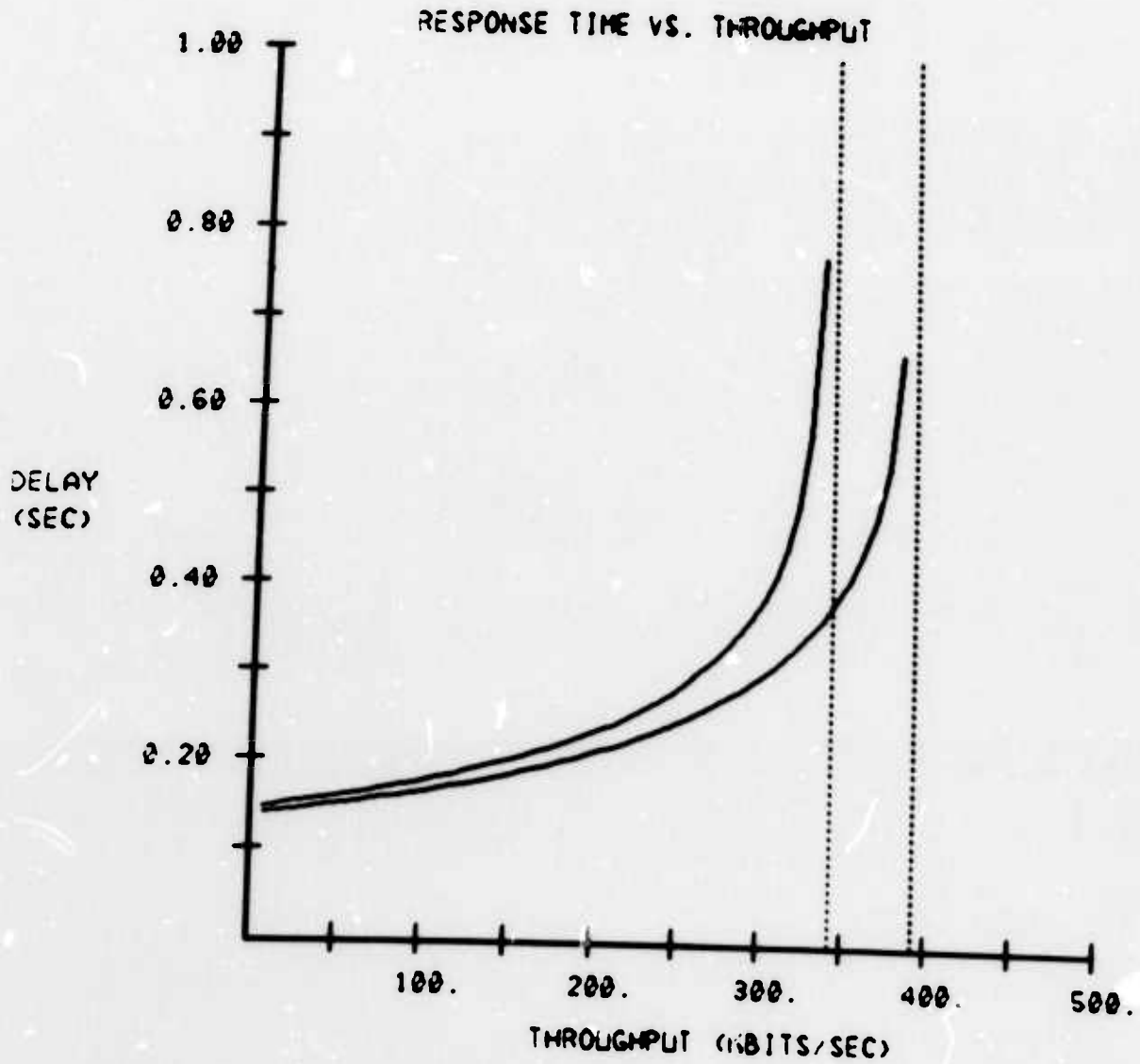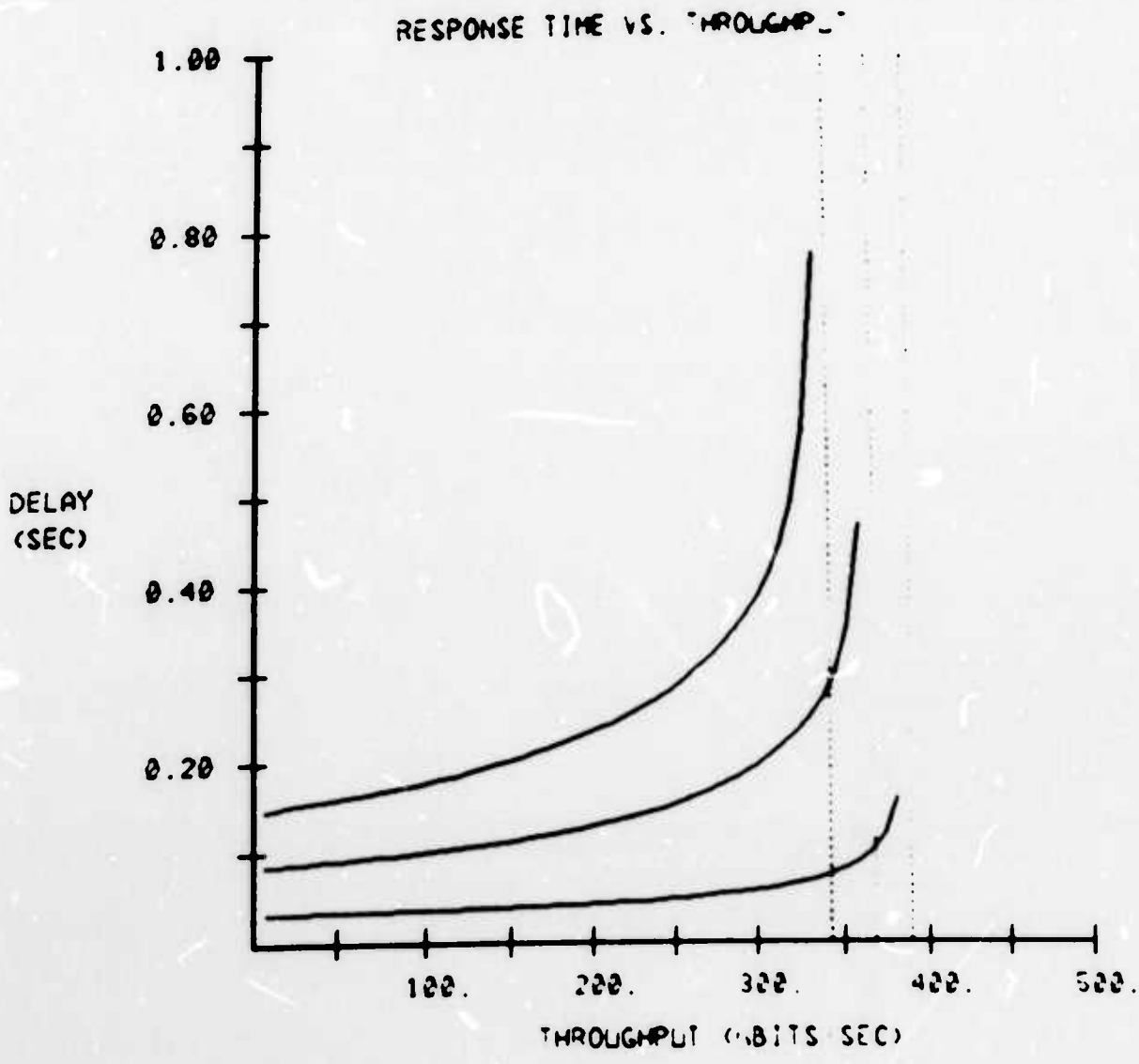
FIGURE 1:

42 NODE ARPANET CONFIGURATION, ALL 50 KB/S LINES VS. INCREASE
OF 1 CROSS COUNTRY CHAIN TO 230 KB/S

ENTER OUTPUT COMMAND OR ? < _

FIGURE 2:

42 NODE ARPANET CONFIGURATION 100, 500, 1000 BIT PACKETS
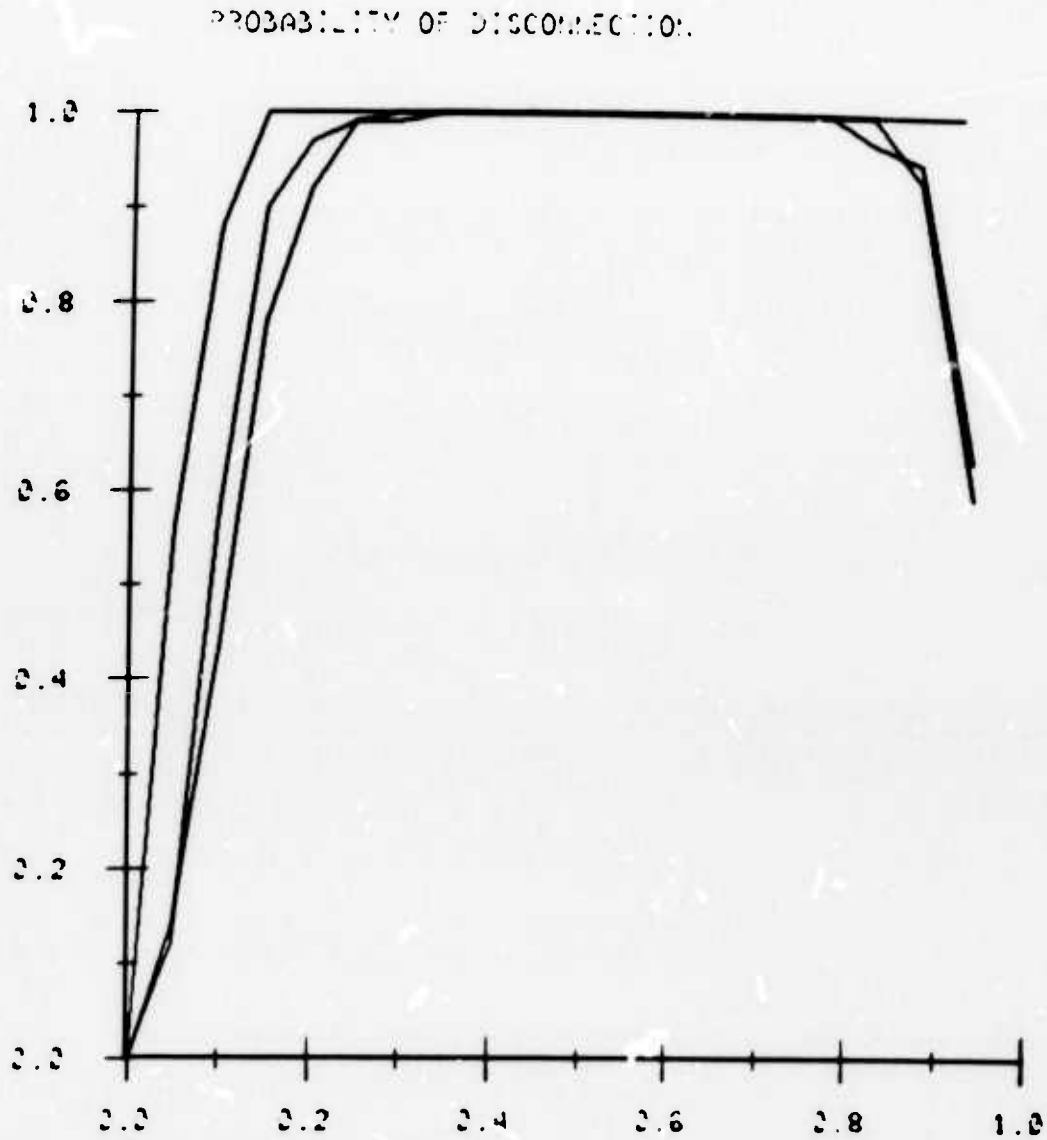
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PROBABILITY OF DISCONNECTION

FIGURE 3:

100 SAMPLE RELIABILITY RUNS, ONLY NODES FAIL, ONLY LINKS FAIL, BOTH NODES AND LINKS FAIL

5.9

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III. TURNAROUND DELAY PERFORMANCE OF INTERACTIVE PROGRAMS

A. General

Turnaround delay is probably the most important performance criterion for the evaluation of the effectiveness of an interactive analysis and design program. In fact, the purpose of an interactive program implementation is to provide the systems analyst with faster answers and, therefore, better man-machine interaction than he could obtain with the batch version of the same program. In this section, the turnaround time performance of our routing and reliability interactive programs is evaluated in a variety of load conditions.

Turnaround time is here defined as the delay between the time the RUN command (which starts execution) is entered and the time the output comes back on the screen. In a time sharing environment, such a delay is approximately proportional to the average number of tasks (system and user) simultaneously requesting the CPU; in a Tenex system, such a number is referred to as the Load Average.

B. Routing Program Turnaround Delay

NAC's routing program is based on an iterative algorithm which attempts to raise network throughput while maintaining the specified delay constraint. Therefore, the CPU time is proportional to the number of iterations performed. In the following experiments, five iterations were allowed for each run, since at that point, a sufficient accuracy was generally obtained.

Several routing runs were performed on three network examples with 10, 26, and 42 nodes, respectively, using NAC's interactive program at USC-ISI. The same network was analyzed at different times during the day, and with different load averages. The curves

in Figure 4, 5, and 6 show the elapsed time versus load average for each network application.

For networks on the order of 10 nodes, the elapsed time is quite tolerable for any reasonable value of load average (Note: the typical range of load averages is between 2 and 8). Furthermore, intermediate throughput and delay results are printed at the terminal after each routing iteration, to keep the user's attention while computation is proceeding.

For larger networks (20 nodes or more), load average has a more critical impact on elapsed time. In the case of high load average, the user can avoid long delays by reducing the number of iterations and thus sacrificing accuracy; alternatively, the user can transfer his files to another site with lower load average and run his programs there. This is one of the attractive features offered by a resource sharing network such as ARPANET.

C.   Performance of Interactive Reliability Analysis

In general, the reliability of a network with a size such as ARPANET's does not vary drastically with the insertion or deletion of a few links. Furthermore, the reliability criteria, P_{nc} and F_{nc}, will not change at all if only line speeds are varied. The routing performance, on the other hand, can be substantially affected by the above modifications. Therefore, network reliability is usually not evaluated after each topological modification and routing performance evaluation. The designer might, however, want to examine the network reliability after several topological changes. Therefore, an inter-active network design "package" should include a reliability program.

The reliability program currently included in NAC's inter-active network analysis and design program is essentially a simulation program [2]. The accuracy of the results is, therefore, related to

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the number of samples generated in the simulation. Table 1 shows
the results for several reliability analyses using 1000 samples for
networks of 10, 26, and 42 nodes. The runs were made at ISI at
approximately 8:30 EDT (5:30 PDT).

TABLE 1

RELIABILITY PROGRAM PERFORMANCE

	10 Nodes	26 Nodes	42 Nodes
Load Average	1.6	1.5	2.5
Elapsed Time	1 min. 49 sec.	3 min. 58 sec.	9 min. 35 sec.
CPU Time	59 sec.	2 min. 37 sec.	4 min. 18 sec.

As can be seen, the elapsed time for the reliability analysis, even
at very low load averages, tends to reach levels which are intolerable
for interactive analysis.

Because of the extensive amount of computation required for
a reliability analysis, it would be desirable to have the analysis
be done in batch on a big "number crunching" machine, and yet allow
the user the flexibility of interactive editing and validation of data
and graphic display of output. This feature can actually be implemented
on ARPANET. In fact, with the remote job service (RJS) in association
with the IBM 360/91 at UCLA, the user does have the option of local
or remote processing of the reliability analysis. He can direct the
interactive program to perform the analysis locally by entering the
command R(UN). Or, he could request the interactive program to create

an RJS data file complete with the job control language (JCL) needed for execution. The user can then leave the interactive program, enter the RJS subsystem, and submit the reliability job file. He can wait for the output or he could return to the interactive program and, perhaps, continue with a routing analysis; periodically checking to see if the IBM 360/91 has completed the execution of the reliability analysis.

Once the output is ready, the data file is read into the interactive program and various reliability curves can be displayed. Efforts are currently being made to make this RJS feature invisible to the user. However, even now this feature of parallel processing, made available by ARPANET resource-sharing capability, is very useful for any interactive computation.
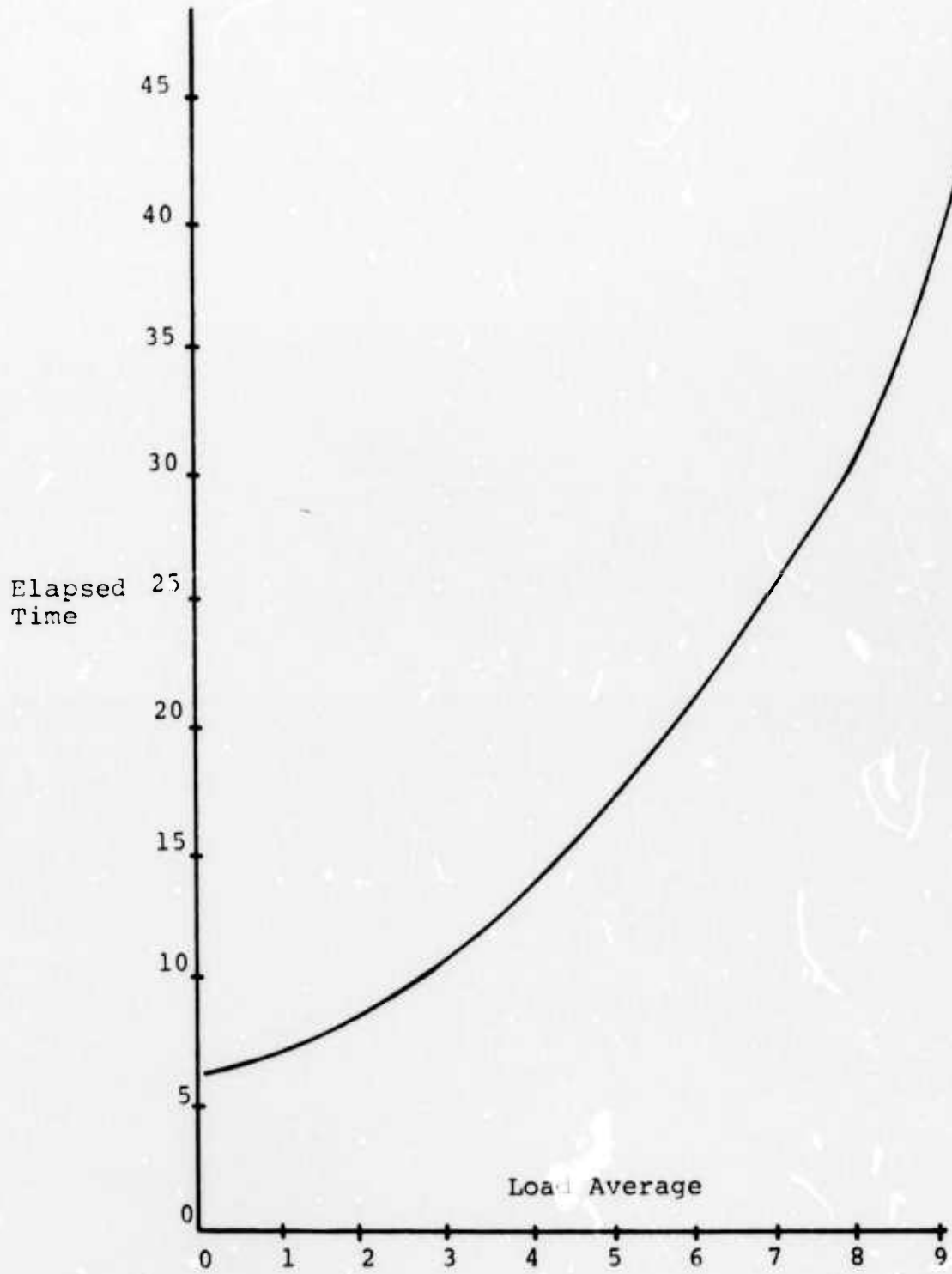
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FIGURE 4:
10-NODE NETWORK CPU TIME 3 SEC.

5.14
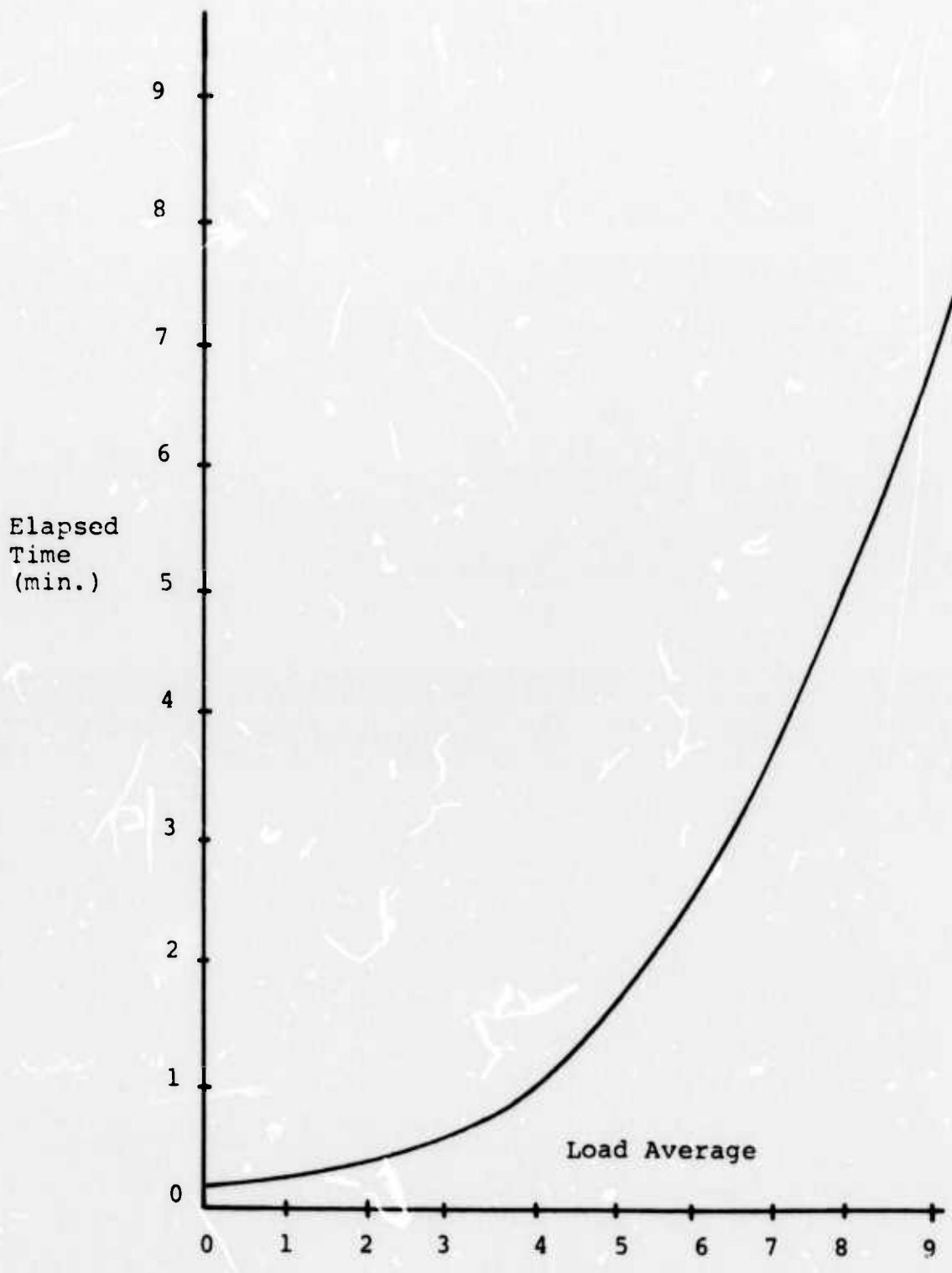
FIGURE 5:

26-NODE NETWORK CPU TIME - 11 SEC.
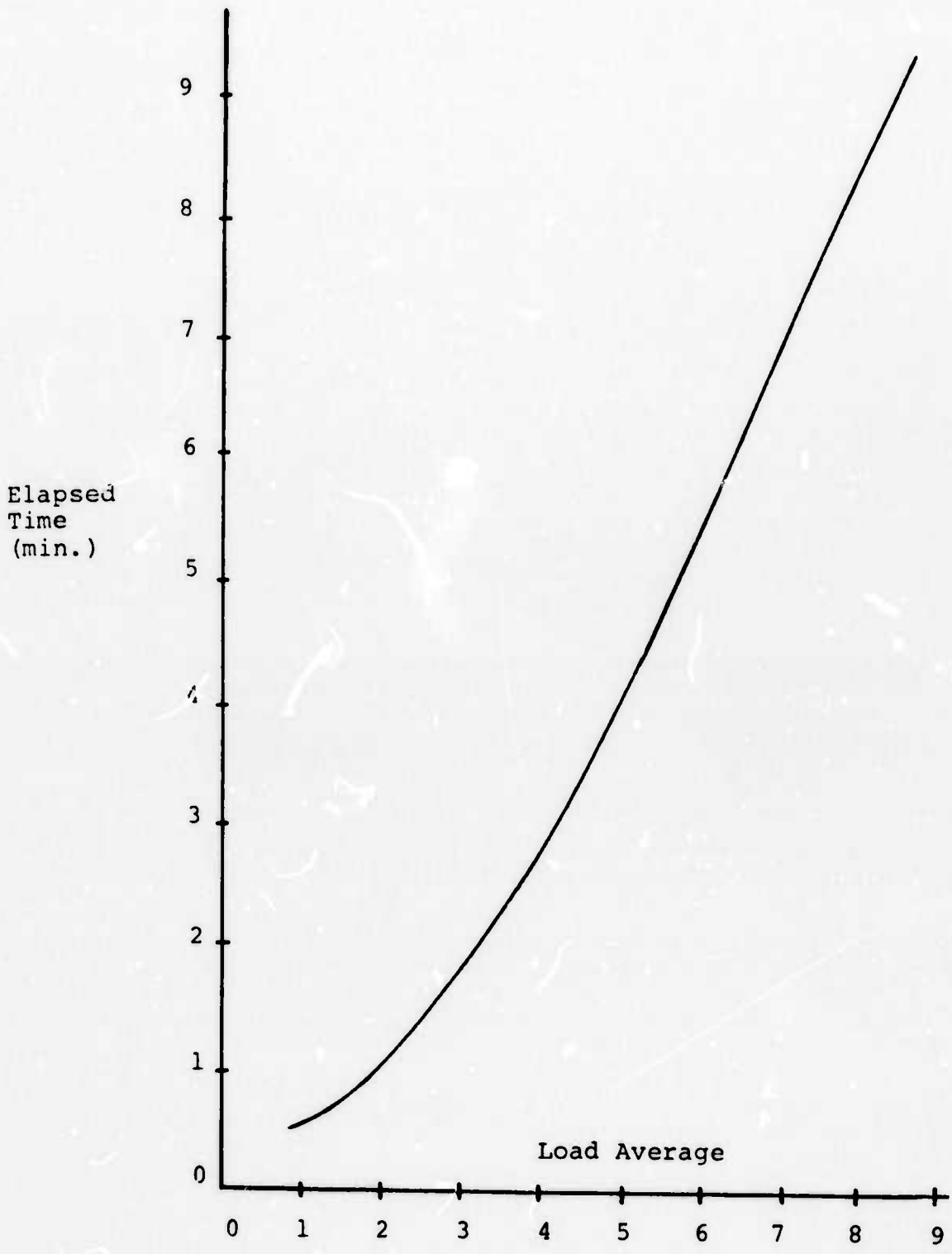
5.15

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FIGURE 6:

42-NODE NETWORK CPU TIME 24 SEC.

5.16

IV.   INTERACTIVE VERSUS BATCH NETWORK DESIGN

It is possible that a one-time network evaluation could be performed faster and more economically in a batch mode than in an interactive mode. However, what we are concerned with here is a complete design session in which a network is analyzed and modified repeatedly using man-computer interaction. In this section, we compare the overall length of time involved in a design session using either a batch program or an interactive program. A design session consists of: analysis of a given network configuration; use of results to modify the topology in order to improve network performance (reduce cost, improve throughput and reliability); re-evaluation of the new network configuration, etc.

Table 2 shows the steps a designer would take in a "design session", evaluating routing performance, using an interactive or a batch approach.

In Step I1, the user sets up interactively the network configuration and the data base internal to the program. In the batch case, the designer must first draw the configuration and then keypunch the appropriate cards. The setting up of the routing parameters (I2 and B2) requires basically the same amount of time in both interactive and batch mode; however, the interactive program displays on the screen a description for each input parameter, and performs validity and error-checking on the input data, thereby avoiding error and confusion.

In Step I3, the user enters the R(UN) command and execution of the routing analysis begins. In the batch case, (B3.1 → B3.5), the user initiates the job and receives the output. Even though the actual batch computation is generally faster than the interactive computation, (5 sec. for a 42 node network on a CDC 6600 as opposed to 24 sec. on a PDP 10 Tenex), there is quite a fixed delay before and after job execution which is independent of CPU time requirements (read-in deck, job waiting on input and output queue, wait for printer).

5.17

This delay can be on the order of 15 - 20 minutes, sometimes even longer.

In Step I4 or B4, the designer examines the results of the analysis. The time required to examine the data on the screen is comparable to the time required to read the same data on the print out; however, in the interactive mode, the designer can instantly compare the present results with those obtained from previous runs (most notably plots of throughput versus delay). After the topological modifications are made on the configuration displayed on the CRT (Step I5), the designer is ready to perform another analysis and returns to Step I3. In the batch case, on the other hand, the designer must redraw his new configuration by hand and then keypunch the appropriate cards. He then must return to the card-reader to initiate a new analysis.

Thus, for the first iteration of the analysis, the effort on the designer is comparable for both the interactive and batch approaches. However, for the subsequent iterations, the effort is considerably less with the interactive procedure.

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INTERACTIVE

| Create Topolog. On Graphics Terminal | I1 |

↓

| Set   Up Routing Parameters | I2 |

↓

| Enter R (un) Command, Job Executes | I3 |

↓

| Designer Looks At Results On Terminal | I4 |

↓

| Makes Modifi- cations of Topology on Display Unit | I5 |

BATCH

| B1.1 | Draw Map Manually |

↓

| B1.2 | Keypunch topology Cards |

↓

| B2 | Keypunch Routing Parameter Cards |

↓

| B3.1 | Read in Deck |

↓

| B3.2 | Job Waits On Input Queue |

↓

| B3.3 | Job   Executes |

↓

| B3.4 | Job Waits On Output Queue |

↓

| B3.5 | Printer Prints Out Results |

↓

| B4 | Designer Looks At Results |

↓

| B5.1 | Make Modifications On Map |

↓

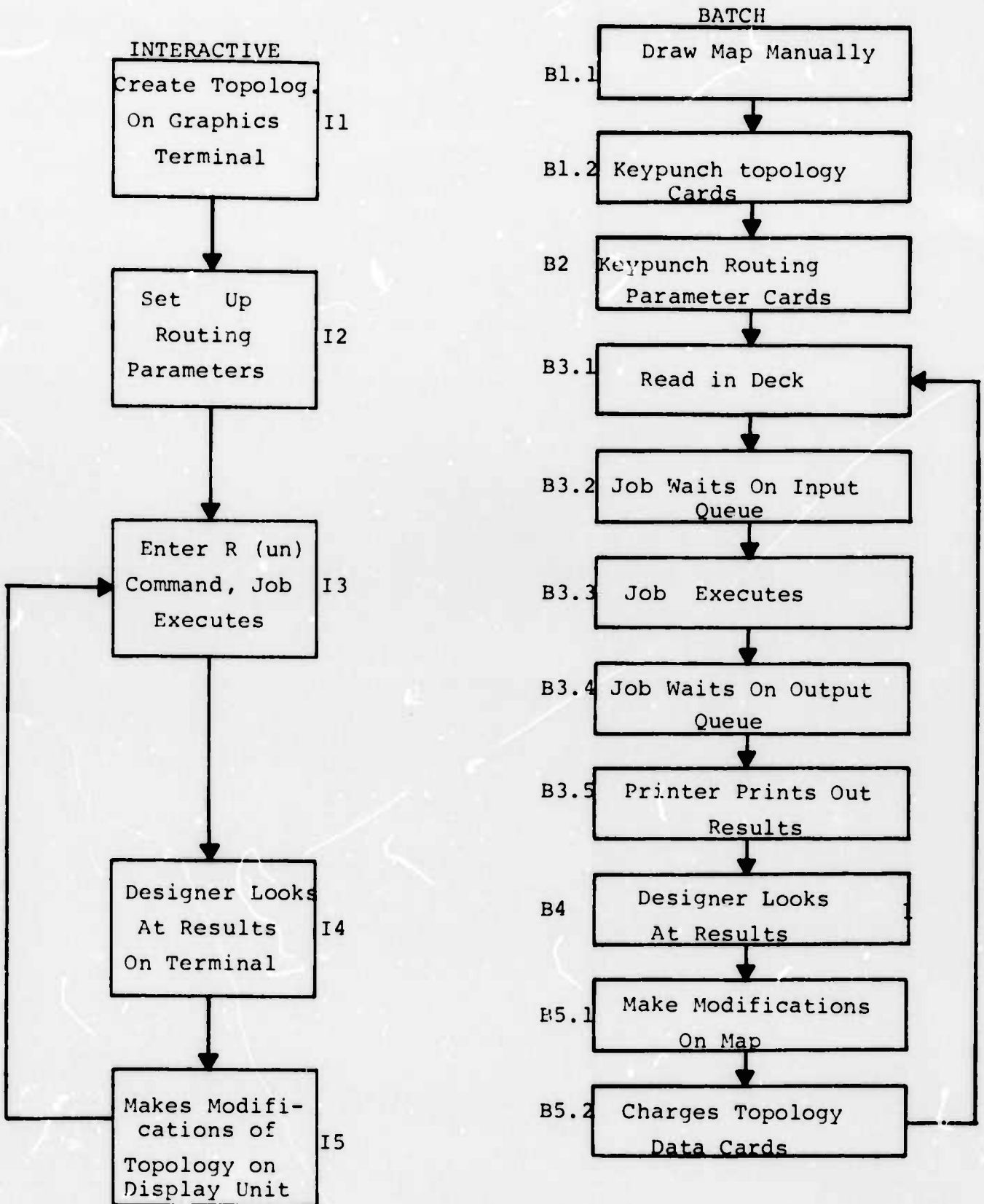| B5.2 | Charges Topology Data Cards |

TABLE 2

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V.   IMPACT OF INTERACTIVE GRAPHICS ON NETWORK DESIGN

The most conspicuous effect of interactive graphics on network design is that of speeding up the overall design process in a very substantial manner. The amount of time saved and the increase in productivity clearly depend on many factors (efficiency of the interactive graphic system, programmers experience, load on the computer, etc.); our experience indicates that the design process is typically speeded up by 5 to 10 times.

Another important effect of interactive graphics on network design is the interactive use of analysis programs to develop better design algorithms. In fact, most network design algorithms are based on heuristics, and good heuristics are obtained by combining physical intuition, careful observation of several network properties, and evaluation of several examples. Therefore, an interactive program is an extremely valuable tool in the hands of a network designer who is trying to establish experimentally some general relationship between network configuration, topological transformations and network performance.

Besides assisting the network designer in the development of better heuristics, interactive graphics can also provide a way to monitor design algorithms which are completely automatized, and in principle would not require human intervention. Since most network design algorithms are iterative and typically perform a topological transformation at each step, it is possible to implement them in an interactive mode so that they display the current solution at each iteration. The designer, therefore, can evaluate the cost effectiveness of each transformation, and can stop, correct and restart the algorithm, whenever he identifies some inadequacy in the current solution.

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The monitoring and verification function of interactive graphics is very valuable for any heuristic design. In fact, it is extremely rare to find heuristics that perform well on all possible problems. In general, there are always cases in which the heuristic solution is very bad. Sometimes the solution is so bad that the designer can detect and correct the inconsistencies by simple visual inspection. Thus, the importance of visually monitoring the solutions.

VI. EXAMPLE

A practical application of the interactive program arose when
NAC was asked to evaluate the possibility of significantly reducing
the communication costs of ARPANET and the impact of potential cost
reductions on the network's performance. Below is the report sum-
marizing the results.

Requirements used in the study are as follows:

1.   Average packet delays under 0.2 seconds
throughout the net.

2.   Capacity for expansion to 64 IMPs without
major hardware or software redesign.

3.   Average total throughput capability of
200 - 300 kilobits/second for all Hosts.

4.   Peak throughput capability of 40 - 80
kilobits/second per pair of IMPs in an otherwise
unloaded network.

5.   High communication subnet reliability
subject to economic constraints.

The time delay and throughput requirements imply that 50 kilobit/
second communication lines are needed within the network. Factors
impacting network designs have been:

1.   Nine month lead times for obtaining lines
from AT&T.

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2.   Rapid expansion of the number of IMPs and TIPs in the net (averaging one new node per month).

3.   Rapid increase in traffic in the network.

The traffic growth in the net prompted us to study, within the present contract year, the problem of increasing the network's traffic capacity and the associated costs of such increases. Now that the network traffic has become relatively stable because of the saturation of serving Hosts and the reduction of the rate of addition of new nodes to the network, the object of the present study was to reevaluate network costs as a function of the various parameters in the network.

Several options are available to reduce cost in the network. These are:

1.   Rearrangement of lines.

2.   Reduction of line capacities in the network either on a limited basis or throughout the network.

3.   Introduction of new technology to reduce overall line costs.

A constraint imposed on all alternatives is that communication subnet reliability should not significantly decrease from that of the present network. This constraint dictates that the network remain two connected and imposes certain other technical conditions upon network topology.

Major modifications to the present network cause reduction to network performance with respect to one or more of the network's performance parameters. A simple rearrangement of lines to reduce cost also reduces the network's expansion capability for at least nine months (since this is the time required to obtain new 50 kilobit/second circuits from AT&T). Throughput is also somewhat reduced.

Decrease of line capacities substantially reduces total network throughput, increases time delay, and most significantly, theoretically increases file transfer time by at least a factor of 2.6 (based on available AT&T circuit options). The cases where some or all of the lines are reduced in speed were considered.

If lower speed lines, (in particular, 19.2 kilobit/second lines) are used, the use of a new device called a biplexer to further reduce the cost of some lines is possible. Although many have not yet been installed and thoroughly tested, the technical concept is sound and should a decision to reduce line speeds to 19.2 be made, several biplexers should be obtained and tested, and, if successful, used wherever appropriate. The analyses consider networks both with and without biplexers.

Finally, the case where line speeds are decreased to 9.6 kilobits/second was considered. This results in a network which can only marginally handle existing network traffic, has no expansion capability, very high average time delays (over 1 second) and file transfer times more than five times as great as those achievable at present. While the communication costs of such a network would only be 1/3 of the present networks cost, its performance would be so poor that this option was not extensively examined.

Figure 7 summarizes the available alternatives and communication costs. Options that we prefer given that a decision is made to degrade ARPANET performance are also indicated as are the disadvantages of each alternative. An important point to be understood is that communication costs do not reflect all of the issues that must be

5.24

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made in changing the present ARPANET approach. For example, if all of
the network's lines were reduced from 50 kilobits/second to 19.2
kilobits per second (and the network were reconfigured), communication
line costs would decrease from about $1.1 million per year to either
$736,000 (without biplexers) or $694,000 (with biplexers). However,
file transfer time would increase by a factor of 2.6 and the computer
connect time costs, to transfer these files, could increase by as
much as $72,000 per year. (These calculations are based on available
network measurement data).

Using the interactive program, this study took approximately
four hours. Using programs that run in batch, this study could have
taken several days.
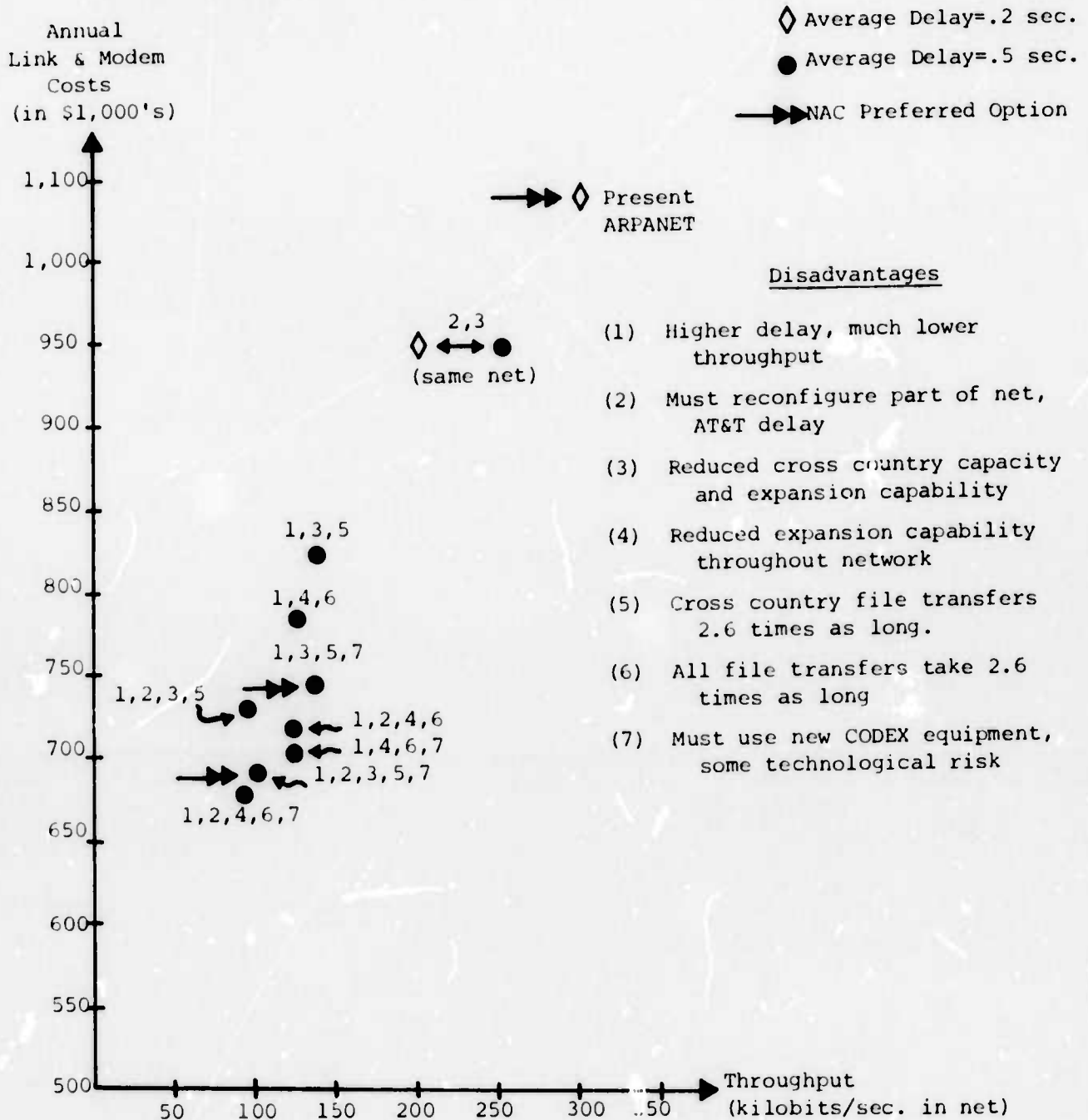
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Graph of options, costs and throughputs. All nets have approximately the same reliability. Numbers next to points indicate applicable disadvantages when compared to present ARPANET. NAC preferred options, in addition to present network, incorporate 50 and 19.2 kilobit/second lines and biplexers where appropriate.

FIGURE 7

5.26

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VII. <u>FUTURE RESEARCH</u>

Future research plans in the area of interactive network design
tools include the following items:

       1.   Implementation of the Cut-Saturation Network
Design Algorithm [4] as an interactive program.

       2.   Development of more efficient and less time
consuming reliability analysis algorithms, which
would allow more frequent reliability evaluations
during the design process.

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COMPUTATIONAL COMPLEXITY OF NETWORK
CONNECTIVITY ALGORITHMS

I. INTRODUCTION

A basic property of a graph is its connectivity structure; that is, how many connected pieces it divides into. The determination of this simple property is fundamental to many more complex calculations. It is equivalent to determining equivalence classes. One important application is in network reliability calculations. Suppose we are given n nodes for a network. There are $n(n-1)/2$ possible distinct undirected links (not counting loops) among them. Suppose we add these links sequentially and in random order. What is the expected number of links we must add to connect all n nodes? This value allows one to estimate a bound on the average running time for a simulation technique for reliability analysis [23]. In this chapter we describe closed form solutions to this and related problems for finite n. The method used to consider the process as Markov processes on the lattice of partitions.

The problem we discuss arises in the evaluation of algorithms for determining spanning trees and/or the connected component structure of graphs and networks. Some of these algorithms are given in [9], [13], [14], and [22]. Mathematically, the problem has its own history and seems to have appeared (in the form studied here) for the first time in [6], although forms of it were studied earlier in [10] and [11].

The problem can be succinctly described as follows: At each point in "time" (sequentially) we select a subset of size two (edge) from a set of n-labelled objects (nodes, vertices) "at random". At random means that each edge not previously selected is equally likely to be picked. A graph generated "at random" is called a "random graph".

What is the probability that the "random graph" is connected after
j-edges have been selected? What is the "mean time" to connectedness?
More generally, given a connected component structure π, what is the
mean time to obtaining a graph with component structure π. We
obtain "closed form" solutions and relatively simple computing
equations for the problems just posed as well as other combinatorial
functions related to the process of generating random graphs. A
number of these results have been obtained by other methods by the
authors mentioned and others. Our methods are based on the work
in [20], [21].

The problems described above have a long and interesting history
going back to 1956 and to questions about trees in the late nine-
teenth century. The problem of determining the probability that
a random graph has a given connected component structure was
apparently first posed in [6]. The question of determining the
number of connected graphs on n-nodes with j edges was posed and
solved in terms of generating functions in [19], [11], [1] and
[2] and perhaps elsewhere between 1959 and 1971.

In analyzing the computational complexity of algorithms for
determining the connected component structure of a graph, it is
useful to know the mean-time to obtaining that structure as well
as the sojourn time in various structures sequentially obtained.
To our knowledge, these problems have not been discussed in the
literature. In the special case of a connected graph with (n-1)
edges we ask for the number of trees on n-vertices. This question
has its own history starting with [4], [5], [3], see especially [15].
Some results of these authors are obtained herein as special cases
of our results.

In [21] the suggestion was offered that a number of probability
questions are more "naturally" posed and solved by mapping sample
spaces into semilattices (semilattice variables) rather than to the
real line (random variables). In some examples, the distribution

function of the mapping (called a generating function) can be de-
termined and inverted using the Möbius-Rota inversion therorem
(see e.g. [20]) to obtain the density function (distribution
function in [21]) of the mapping. Indeed this is a useful
method for obtaining the probability that a "random graph" with
j-edges has a given connected component structure. The approach
yields a complicated but closed form formula from which the
generating function is easily obtained. Furthermore, we show
that the process of adding edges at random is a Markov process
and hence, that theory can be used to study the generation of
random graphs. It is easy to obtain the probability transition
function for the Markov process, which turns out to be non-stationary.
None of the formulae obtained are useful for computing so we close
the paper with a pair of coupled equations which can be used for
computing the combi atoria' functions of interest. This is carried
out in Sections III-IV. Notations and basic definitions are
introduced in Section II.

The question how "connected" is a graph has been subjected to
numerous definitions. The commonly accepted definition is the size
of a minimal cut-set. There are many reasons why this is not a
particularly good definition. Two obvious reasons are:

1)   It can not be applied to disconnected graphs, although
obviously some disconnected graphs are more connected than
others.

2)   Many connected graphs have the same size minimal cut-set
although the complexity of their internal structure is quite
different.

A definition of "connectivity" which would differentiate between connected graphs by measuring their internal structure and also differentiates between disconnected graphs would seem to have many obvious merits and applications. We propose and discuss such a measure in Sections VI-XI.

II. THE LATTICE OF PARTITIONS

A lattice is a partially ordered set where each pair of elements has a greatest lower bound and least upper bound (g.l.b. and l.u.b.) We denote the statement that a is less than or equal to b in the partial ordering by $a \leq b$. In a lattice as on the real line we have three types of intervals, or segments. $[x,y], [x,y), (x,y]$ which are respectively the sets of a's for which $x \leq a \leq y$; $x \leq a < y$; $x < a \leq y$. It is easy to see that $[x,y]$ is a sublattice of the original lattice when endowed with the same ordering. An element b is said to cover an element a when, $a<x<b$ is satisfied by no x, or equivalently when the interval $[a,b]$ contains exactly two elements. A Hasse diagram is a pictorial depiction of a lattice where the elements are represented as points and a line is drawn from b to a when b covers a, e.g.
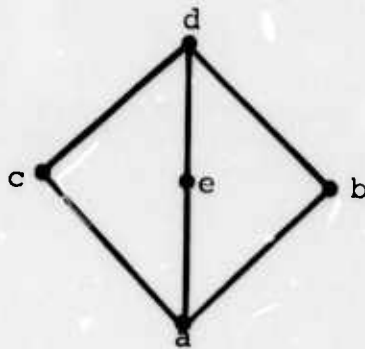
An element of a lattice is minimal (maximal) if it covers no element (is covered by no elements). A unique minimal (maximal) element is called a zero or least (unit or greatest). When the zero or unit exist they are denoted by 0 an I respectively. An atom is an element which covers a minimal element while a dual atom is an element covered by a maximal element.

6.5

Let V_n be a finite set with n-labelled objects. A <u>partition</u> of V_n is a family of disjoint subsets of V_n, say $P_1, P_2, \ldots, P_k$, called <u>parts</u> or <u>blocks</u> of π, whose union is V_n. A natural ordering on the set π_n of partitions of V_n is given by $\sigma \leq \pi$ if every <u>block</u> or <u>part</u> of σ is a <u>subset</u> of some part or block of π. It is not difficult to verify that the set Π_n of partitions of V_n is partially ordered with the ordering just defined. In fact, Π_n is a lattice with that ordering. Furthermore, the lattice Π_n has a zero 0, or least element given by the partition with n-singleton parts. The lattice Π_n also has a unit I greatest element given by the partition containing one block V_n itself. The following result about covers in Π_n is obvious.

<u>Lemma 2A</u> : An element $\sigma \epsilon \Pi_n$ <u>covers</u> $\pi \epsilon \Pi_n$ if and only if σ is obtained by combining any two parts or blocks of π.

The lattice Π_n is endowed with a very useful rank or numerical ordering as will be evident from an examination of the Hasse diagram for small n, or from Lemma 2A.   A partially ordered set is said to satisfy the Jordan-Dedekind chain condition if:

      1. It has a zero and unit.

      2. All totally ordered subsets having a maximal number of elements have the same number of elements.

A totally ordered subset of a partially ordered set is called a chain, $a > b > \ldots > w$, e.g.. A chain is maximal if it cannot be enlarged. When a lattice satisfies the Jordan-Dedekind chain condition one can introduce a <u>rank</u> <u>function</u> <u>R (p)</u> on the lattice. The functions $r(p)$ is defined as the length of a maximal chain in the segment $[o, p]$ minus one. The rank of zero is zero, the rank of an atom is one, etc. If the rank of I is n-1 then the rank of a dual atom is n-2. It is easy to verify that Π_n satisfies the Jordan-Dedekind chain condition and indeed the rank of a partition $\pi \epsilon \Pi_n$ is n minus the number of blocks or parts of π.

Another fundamental descriptive combinatorial notation for partitions is the <u>type</u> or <u>class</u> of π. A partition π is of type or class $(k_1, k_2, \ldots, k_n)$ when k_i is the number of blocks or parts with i elements. Obviously, if $\pi \epsilon \Pi_n$ then $\sum_{i=1}^{n} k_i = n - r(\pi)$ where $r(')$ is the rank function, and $\sum_{i=1}^{n} i k_i = n$. The following structure lemma is fundamental for forming recurrences on Π_n.

<u>Lemma 2B</u>:   Let Π_n be the lattice of partition of a set with n-elements. If $\pi \epsilon \Pi_n$ is of rank k then the segment or interval $[\pi, I]$ is isomorphic to Π_{n-k}. If π is of class of type $(k_1, k_2, \ldots, k_n)$ then the interval $[0, \pi]$ is isomorphic to the direct product of k_i lattices isomorphic to Π_1, k_2 lattices isomorphic to $\Pi_2, \ldots, k_n$ lattices isomorphic to Π_n.

Among other applications of lemma 2B it was shown in [20] that it can be used to compute the Mobius function of Π_n. The Mobius function was shown by Rota to be an important invariant of lattices and hence a distinguished member of the incidence algebra over a partially ordered set.

If P is a partially ordered set, the <u>incidence</u> <u>algebra</u> of P denoted by I(P) is the algebra of real valued functions $f: P \times P \to R; \exists x \not\leq y \implies f(x,y) = 0$; addition and scalar multiplication are defined as usual and the product (convolution) is given by $e = f * g$, $e, g, f \epsilon I(p)$

$$e(x,y) = \sum_{x \leq z \leq y} f(x,z) g(z,y).$$

It is easy to verify that I(p) is an algebra. The convolution becomes essentially matrix multiplication when P is finite. Among some distinguished elements of the incidence algebra are the functions:

zeta function : $\zeta(x,y) = 1$ if $x \leq y$ and 0 otherwise
delta function: $\delta(x,y) = 1$ if $x = y$ and 0 otherwise
incidence function: $n(x,y) = \zeta(x,y) - \delta(x,y)$.

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Since the delta function is easily verified to be an identify in I(P)
certain elements of I(P) will be invertible in I(P) with respect to δ.

The next and last set of definitions can be found in [21]
Let S be an arbitrary finite set and W ($\cdot$) be a weight function on
S.   (S can be thought of as a sample space and W ($\cdot$) a probability
measure).   Let Π be a lattice.   A mapping $X:S\to\Pi$ is called a semilattice
variable analogous to probability theory and random variables.
Let A (Π) be the algebra of singlevalued functions $f:\Pi\to R$ with addition
and scalar multiplication as usual, and $e=f*g$, $f,g,e\epsilon A(\Pi)$ given
by

$$e(x) = \sum_{a\wedge b=x} f(a)g(b),$$

where $a\wedge b$ is the g.l.b. of the pair $\{a,b\}$. With this notation,
if the weight function is a probability density on S then there
is a function $f\epsilon$ A(Π) which represents the density function of a
semilattice variable X given by

$$f(\pi) = \sum_{t:\, x(t)=\pi} W(t)$$

The generating function F (probability distribution function) of
f is given by

$$F(\sigma) = \sum_{\pi\leq\sigma} f(\pi) = \sum_{\pi\epsilon[o,\sigma]} f(\pi);$$

and plays the role of a probability distribution function. The
density function and distribution functions can be computed from
each other by the Möbius Rota inversion formula. This approach is
useful in "random graph" theory. For other examples see [21].

III.   STRUCTURE OF RANDOM GRAPHS

Let the set $V_n = \{v_1, v_2, \ldots, v_n\}$ denote a set of n-nodes or vertices of a graph. Let $\varepsilon_n = \{(e_1, e_2, \ldots, e_{\binom{n}{2}}) \mid e_i$ is a subset of size two from $V_n\}$ be the set of sequences or all permutations of the edges of the complete graph on V_n. We define the lattice stochastic process $X_1, X_2, \ldots, X(\binom{n}{2})$; $X_j : \varepsilon_n \to \Pi_n$ by $X_j(e) = X_j(e_1, e_2, \ldots, e_{\binom{n}{2}}) =$ the partition of V_n determined by the connected components of the graph $(V_n, \{e_1, e_2, \ldots, e_j\})$; $j=1, 2, \ldots, \binom{n}{2}$. Thus for example $X_1(e)$ is always a partition of rank 1 and type $(n-2, 1, 0, 0, \ldots, 0)$; $X_2(e)$ is always a partition of rank 2 and $(n-2)$ parts but can be of two possible types $(n-3, 0, 1, 0, \ldots, 0)$ or $(n-4, 2, 0, 0, \ldots, 0)$. In fact for each $e \in \varepsilon_n$, $X_1(e) \leq X_2(e) \leq \ldots \leq X_{\binom{n}{2}}(e) = I$ so that the process is monotonic nondecreasing in the natural ordering on Π_n. Furthermore, each pair $(X_j(\cdot), X_{j+1}(\cdot))$ has $X_{j+1}(e) = X_j(e)$ or is a cover of $X_j(e)$ so that no wild jumps take place.

We assume that each $e \in \varepsilon_n$ has the same probability $\dfrac{1}{\binom{n}{2}!}$ so that we can define the density function;

$$f_j(\pi) = P\{X_j = \pi\} = \frac{\text{no. of } e \in \varepsilon_n \text{ for which } X_j(e) = \pi}{\binom{n}{2}!}$$

and the one step transition function.

$$P_j(\sigma, \pi) = P\{X_j = \pi \mid X_{j-1} = \sigma\}.$$

Thus if $X_j(e) = \pi$ then $X_{j+1}(e) = \pi$ if the $(j+1)^{th}$ edge of e is a subset of some part of π, otherwise, $X_{j+1}(e)$ is a cover of π obtained by combining the two parts of π which contain the points of e_{j+1}. The next theorem should be fairly obvious.

6.9

<u>Theorem 3A</u>:   The semilattice process $X_1, X_2, \ldots$ is Markovian.
The state I is absorbing and the process is absorbed with probability
one.

<u>Proof</u>:   The state I is absorbing since eventually the graph becomes
connected. To see that the process is Markovian we observe that
the probability of moving to any state depends only on the number
of edges in the graph and the type of the partition. The number
of ways of staying in the same state is computed by selecting edges not
already selected from within the blocks of π, this is independent
of the history. The number of ways of moving to any given cover
depends only on the sizes of the parts to be combined which is also
independent of the history of the process.

It is easy to determine the transition functions which are
incidently, members of the incidence algebra I (Π_n), and invertible.

<u>Theorem 3B</u>:   The transition function is given by,

$$
P_j(\sigma, \pi) =
\begin{cases}
\dfrac{h(\sigma, \pi)}{\binom{n}{2} - (j-1)} & \text{when } \pi \text{ covers } \sigma. \\[2ex]
1 - \displaystyle\sum_{\sigma \lessdot \pi} \dfrac{h(\sigma, n)}{\left(\binom{n}{2} - (j-1)\right)} & \text{when } \sigma = \pi, f_j - 1(\pi) \neq 0 \\[2ex]
1 & \text{When } \pi = \sigma, f_{j-1}(\pi) = 0 \\[1ex]
0 & \text{Otherwise; } j = 1, 2, \ldots, \binom{n}{2};
\end{cases}
$$

where $h(\sigma, \pi)$ is the number of edges connecting the pair of blocks
of σ which is one part of π when π covers σ and zero otherwise.

Proof: The formulae for $p_j(\sigma,\pi)$ are obvious since at each stage each unselected edge is equally likely to be chosen.

Theoretically theorem 3A and 3B can be used to answer all questions about the process since we know the initial conditions and the transition functions. We will use these observations in the next section to compute various combinatorial functions. We can use the Mobius-Rota Inversion formula to obtain a closed form solution for the probability of being in any given state at any given time since the distribution function is easy to compute. First, however, it is interesting to obtain a new formula in the form of the transition functions for the number of trees on n-vertices.

Theorem 3C: (Chapman-Kolmogoroff Equations).   The following formulae hold;

$$f_j(\pi) = \Sigma \; p_1(0,\pi_1) p_2(\pi_1,\pi_2) \ldots p_j(\pi_{j-1},\pi)$$
$$0 \leq \pi_1 < \pi_2 \leq \cdots \leq \pi_{j-1} \leq \pi .$$

$$\pi \varepsilon \Pi_n; \; j=1,2,\ldots,\binom{n}{2}.$$

Corollary: If T_n is the number of trees on n vertices then

$$T_n = \binom{\binom{n}{2}}{n-1} \Sigma \; p_1(o,\pi_1) \; p_2(\pi_1,\pi_2) \ldots p_{n-1}(\pi_{n-2},I).$$
$$0 < \pi_1 < \pi_2 \leq \cdots \leq \pi_n \leq I$$

Let $F_j(\pi)$ be the distribution function of X_j; $j=1,2\ldots\binom{n}{2}$
i.e. $F_j(\pi) = \sum\limits_{\sigma \leq \pi} f_j(\sigma)$ or the probability that the partition of

connected components is a refinement of the partition π after
j-edges are introduced.

<u>Theorem 3D</u>:   If π is of type $(k_1, k_2, \ldots, k_n)$ then

$$F_j(\pi) \quad = \quad \frac{\binom{\sum\limits_{i=1}^{n} k_i \binom{i}{2}}{j}}{\binom{\binom{n}{2}}{j}} \quad ; \quad j=1,2,\ldots,\binom{n}{2}.$$

<u>Proof</u>:   The j-edges are to be chosen from within the parts of π.

Since $F_j(\cdot)$ is known $f_j(\cdot)$ can be computed by the Mobius-Rota inversion formula.

<u>Corollary 1</u>:   The probability that the graph is connected after j
edges have been added is;

$$f_j(I) \quad = \frac{n!}{\binom{\binom{n}{2}}{j}} \sum_{k=1}^{n} (-1)^{k-1} (k-1)! \sum_{\substack{(k_1,\ldots,k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \frac{\binom{\Sigma k_i \binom{i}{2}}{j}}{1^{k_1} (2!)^{k_2} \ldots (n!)^{k_n} k_1! k_2! \ldots k_n!}$$

Proof: By Theorem 2A

$$f_j(I) = \sum_{k=1}^{n} \sum_{r(\sigma)=n-k} F_j(\sigma)\mu(\sigma,I)$$

$$= \sum_{k=1}^{n} \sum_{\substack{(k_1,\ldots k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} B(n;k_1,\ldots,k_n)F_j(\sigma)\mu(\sigma,I)$$

where σ is of type $(k_1,\ldots,k_n)$ and $B(n;k_1,k_2,\ldots k_n)$ is the number of partitions of type $(k_1,\ldots k_n)$ with k parts. It is easy to see that,

$$B(n;k_1,\ldots,k_n) = \frac{1}{k_1!k_2!\ldots k_n!} \frac{n!}{1^{k_1}(2!)^{k_2}\ldots(n!)^{k_n}}$$

So that

$$f_j(I) = \sum_{k=1}^{n} \sum_{\substack{(k_1,\ldots,k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \frac{n!\binom{\sum_{i=1}^{n} k_i \binom{i}{2}}{j}\mu(\sigma,I)}{k_1!k_2!\ldots k_n!1^{k_1}(2!)^{k_2}\ldots(n!)^{k_n}\binom{\binom{n}{2}}{j}}$$

Since $\mu(\sigma,I)=(-1)^{k-1}(k-1)!$ when $[\sigma,I]$ is isomorphic to Π_k the result is proved.

6.13

IV.   DESCRIPTIVE COMBINATORIAL QUANTITIES

We will now derive formulae and generating functions for the interrelated combinatorial quantities of interest in the evaluation of algorithms. We will count sequences rather than deal with additional symbols for the associated probabilities. Obviously, each of the formulae are easily converted to probabilities and moments can be computed. We will treat n, the number of vertices as a parameter since it will be helpful in the next section where we give a relatively simple way of computing mean-time to a given component structure. The basic functions are:

A. $h_n(\pi;j)$ = the number of edges that can be added to a graph with j-edges on n-vertices with component structure π so that the component structure stays at π.

B. $p_j(\sigma,\pi)$ = the number of ways of adding an edge to a graph G with (j-1) edges and component structure σ to change the component structure to π.

C. $f_n(\pi,j)$ = the number of sequences of j-edges on n-vertices which determine the connected component structure π.

D. $C_n(\pi,j)$ = the number of sequences of j-edges on n-vertices which "enter" π for the <u>first</u> time with the addition of the jth edge.

E. $S_n(\pi,j,k)$ = the number of sequences of k-edges which enter π for the first time on the jth edge and leaves π on the kth edge. This quantity is called the conditional sojourn time in π.

If a partition σ covers a partition π then the pair of numbers (i,j), where i and j are the sizes of the parts of π which are combined to form σ, is called the <u>cover type pair</u> of (π,σ).

<u>Lemma 4A:</u>   The quantity $p_j(\sigma,\pi)$ is given by

F.

$$
p_j(\sigma,\pi) =
\begin{cases}
0 \text{ if } \sigma \neq \pi \text{ and } f_n(\sigma,j-1) = 0, \\[2ex]
1 \text{ if } \sigma = \pi \text{ and } f_n(\sigma,j-1) = 0, \\[2ex]
h_n(\pi,j-1) \text{ if } \sigma = \pi \text{ and } f_n(\pi,j-1) > 0, \\[2ex]
i \cdot k \text{ if } (i,k) \text{ is the pair cover type} \\
\quad \text{of } (\sigma,\pi), \\[2ex]
0 \text{ otherwise.}
\end{cases}
$$

<u>Proof</u>:   The proof is immediate, actually, this is a repeat of Theorem 3B, restricted for completness.

<u>Lemma 4B:</u>   The quantity $h_n(\pi,j)$ is given by

G.
$$
\binom{n}{\sum_{i=2} k_i \binom{i}{2}} - j
$$

when π is of type $(k_1,k_2,\ldots,k_n)$.

Proof: Obvious.

Lemma 4C: The quantity $C_n(\pi,j)$ is given by,

H. $C_n(\pi,j) = \sum\limits_{\sigma \leq \pi} f_n(\sigma,j-1) p_j(\sigma,\pi)$.

Proof: Immediate.

Lemma 4D: The quantity $S_n(\pi;j,k)$ is given by.

I. $S_n(\pi;j,k) = \sum\limits_{\sigma > \pi} C_n(\pi,j) \left[\prod\limits_{\eta=1}^{k-j-1} n_n(\pi,j+\eta) \right] P_k(\pi,\sigma)$.

Corollary: The number of sequences which spend k-units of time in π denoted by $s_n(\pi,k)$ is given by,

J. $s_n(\pi,k) = \sum\limits_{j=1}^{\binom{n}{2}-k} S_n(\pi;j,j+k)$; $k=0,1,2,\ldots$,

The proofs of all of the above lemmas follow from the Markovian property of the process $X_1,X_2,\ldots,X_{\binom{n}{2}}$. All of the quantities are computable from the above equations and the "closed form" equation for $f_n(\pi,j)$ as given in the previous section. However, the calculations are cumbersome and can be done more directly, as we will do in the next section. We close this section by obtaining the generating function for $f_n(\Gamma,j)$ and hence, for all the quantities given above.

Theorem 4A: If $F_n(y) = \sum\limits_{j=1}^{} \dfrac{f_n(\Gamma,j)}{j!} y^j$,   then,

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K. $\quad F_n(y) \; = \; - \, n! \displaystyle\sum_{k=1}^{n} \sum_{\substack{(k_1,\ldots,k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \frac{(k-1)!}{k_1! k_2! \ldots k_n!} \prod_{i=1}^{n} \left(\frac{-B_i(y)}{i} \right)^{k_i}$

Where $B_i(y) = (1+y)^{\binom{i}{2}}$, (This formula is related to Faa Di Bruno's formula for derivatives and the classic Bell polynomials.)

<u>Proof</u>:   From the result of Section 3,

$\dfrac{f_n(I,j)}{j!} = n! \displaystyle\sum_{k=1}^{n} \sum_{\substack{(k_1\ldots,k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \frac{(-1)^{k-1}(k-1)!}{k_1! k_2! \ldots, k_n!} \frac{\binom{\Sigma k_i \binom{i}{2}}{j}}{1^{k_1}(2!)^{k_2}\ldots(n!)^{k_n}}$

Thus

$F_n(y) = n! \displaystyle\sum_{k=1}^{n} (k-1)! \; (-1)^{k-1} \sum_{\substack{(k_1;\ldots,k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \sum_{j \geq 1} \frac{\binom{\Sigma k_i \binom{i}{2}}{j} y^j}{1^{k_1}(2!)^{k_2}\ldots(n!)^{k_n}}$

$= n! \displaystyle\sum_{\substack{k=1 \\ }}^{n} \sum_{\substack{(k_1,\ldots k_n) \\ \Sigma k_i = k \\ \Sigma i k_i = n}} \left[\frac{(1+y)^{\Sigma k_i \binom{i}{2}}}{1^{k_1}(2!)^{k_2}\ldots(n!)^{k_n}} \right] (k-1)! \; (-1)^{k-1}$

and the result follows by expanding

$$(1+y)^{k_1 \binom{1}{2} + k_2 \binom{2}{2} + \ldots + k_n \binom{n}{2}} = (1+y)^{\binom{1}{2} k_1} (1+y)^{\binom{2}{2} k_2} \ldots (1+y)^{\binom{n}{2} k_n}$$

V.   CALCULATIONS:

We can determine pairs of equations from which we can re-cursively compute $f_n(I,j)$ from $c_n(I,j)$ and conversely. These equations can be derived from the Markovian property or from the fundamental isomorphic decomposition of Π_n.

Theorem 5A: (Basic Recurrence)

If π is of type $(k_1, k_2, \ldots, k_r)$ then

$$f_n(\pi,j) = \sum M(w_{2,1}, w_{2,2}, \ldots, w_{2,k_2}; w_{3,1}, w_{3,2}, \ldots, w_{3,k_3};$$

$$\ldots; w_{n,1}, w_{n,2}, \ldots, w_{n,k_n})$$

$$\prod_{j=1}^{k_2} f_2(I, w_{2,j}) \prod_{\tau=1}^{k_3} f_3(I; w_{3,\tau}) \ldots \prod_{\nu=1}^{k_n} f_n(I, w_{n,\nu}).$$

where $M(j, w_{2,1}, w_{2,2}, \ldots, w_{2,k_2}; w_{3,1} w_{3,2}, \ldots, 3, k_3; \ldots;$

$w_{n,1}, w_{n,2}, \ldots w_{n,k_n})$ is the multinomial coefficient and the sum is over all partitions of the integer j into $\sum_{\tau=2}^{n} k_\tau$ parts.

Proof: If π is a partition of type $(k_1, k_2, \ldots, k_n)$ then if a sequence of j edges produces π the sequence must contain places for the $k_{\nu, \tau}$ edges which come from the τth part of those parts which have ν-vertices. The set of subscripts $\{1, 2, \ldots, j\}$ can be partitioned in $M(j; \ldots)$ such parts. The edges in the part with $w_{\nu, \tau}$ edges can then be arranged in $f_\nu (I, w_{\nu, \tau})$ ways. The product formulation follows from the basic isomorphism theorem.

Corollary 1:   If π is a dual atom of type $(r, n-r)$ (i.e. one part with r-elements the other with (n-r) elements) then

$$f_n (\pi, j) = \sum_{w=1}^{j} \binom{j}{w} f_r (I, w) \cdot f_{n-r} (I, j-w)$$

when $n > 2$, $r > 1$, $(n-r) > 1$;

$$f_n (\pi, j) = f_{n-1} (I; j) \quad \text{when} \quad r = 1 \text{ or } n-1 = r;$$

$$f_2 (I, 1) = 1, \quad f_2 (I, j) = 0 \quad j > 1$$

Corollary 2:   [5], [3], [16].

If $T(n)$ is the number of trees on n-vertices then,

$$T(n) = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} \binom{n}{i} T(i) \cdot (n-i) \cdot i \cdot (n-i)$$

Proof: If we set $j = n-2$ in Corollary 1 then,

$$f_n (\pi, n-2) = \sum_{w=1}^{n-2} \binom{n-2}{w} f_r (I, w) f_{n-r} (I, n-2-w).$$

This expression has one non-zero term so,

$$f_n (\pi,n-2) = \binom{n-2}{r-1} f_r(I,r-1) \cdot f_{n-r}(I, n-2 - (r-1)).$$

Since obviously $f_r(I,r-1)= T(r) \cdot (r-1)!$ we obtain

$$f_n(\pi,n-2) = (n-2)!T(r)\cdot T(n-r).$$

Since π has two parts we get a tree on n-vertices by a single edge connecting those parts. Summing over all such partitions then yields all trees on n-vertices.

<u>Theorem 5B:</u>   The quantity $C_n (I,j)$ is given by

$$C_n (I,j) = \sum_{\substack{\pi \text{ is} \\ \text{a dual atom}}} f_n (\pi,j-1)\ h (\pi,I)\ n>2$$

$$C_1 (I,j) =0\ \forall j;$$
$$C_2 (I,1) =1\quad C_2 (I,j) =0\ j>1.$$

<u>Proof:</u>   The proof follows by the Markovian property. If we are to enter I for the first time on the jth then after (j-1) we must be in a dual atom.

<u>Corollary</u>:

$$C_n(I,j) = \frac{1}{2} \sum_{r=2}^{n-2} \binom{n}{r} f_n (\pi_{r,n-r};j-1)\cdot r\cdot (n-r)$$

$$+ n\ (n-1)\cdot f_n (\pi_{1,n-1};j-1)$$

(The notation $\pi_{i,j}$ means π is a dual atom of type (i,j).)

Proof: Substitution of the formula of Corollary 1 to Theorem 5A into Theorem 5B.

Theorem 5C: The quantity $C_n(I,j)$ can be obtained from f_{n-1}, f_{n-2}, $\cdots$ using

$$C_n(I,j) = \frac{1}{2} \sum_{r=2}^{n-2} \binom{n}{r} \sum_{w=1}^{j-1} \binom{j-1}{w} \cdot f_r(I,w) \cdot f_{n-r}(I,j-1-w) \cdot r \cdot (n-r)$$

$$+ n(n-1) \cdot f_{n-1}(I;j-1).$$

Proof: Combine Theorem 5A and 5B

Theorem 5D: The quantity $f_n(I,j)$ can be computed from $C_n(I,j)$ by the equation

$$f_n(I,j) = \sum_{k=n-1}^{j} C_n(I,k) \left(\binom{n}{2}-k\right) \left(\binom{n}{2}-(k+1)\right) \cdots \left(\binom{n}{2}-j\right).$$

Proof: This follows from the Markovian property. If we are to be in state I after j steps then we arrived for the first time on the kth step and stayed in I.

Theorems 5C and 5D provide a coupled pair of equations to compute C_n from f_{n-1} and then f_n from $C_n, \ldots$ and so forth. Some sample calculations are, $f_2(I,1) = 1$, and $f_2(I,j) = 0$ $j>1$

We compute C_3 from f_2;

$C_3(I,1) = 0$

$C_3(I,2) = \frac{1}{2}$ (empty sum) $+ 3 \cdot 2 \cdot f_2(I;1) = 6.$

$C_3(I,j) = 0$ $j>2$.

Now we compute f_3 from C_3

$$f_3(I,2) = \sum_{k=2}^{2} C_3(I,k) = C_3(I,2) = 6$$

$$f_3(I,3) = \sum_{k=2}^{3} C_3(I,k) \cdot (3-k) = C_3(I,2) \cdot 1 = 6$$

$$f_3(I,j) = 0, \quad j>3.$$

We can compute C_4 from f_3

$$C_4(I,3) = \frac{1}{2} \sum_{r=2}^{2} \binom{4}{2} \sum_{w=1}^{2} \binom{2}{w} f_r(I;w) \cdot f_{n-r}(I,j-1-w) \cdot r(4-r)$$

$$+ 4 \cdot 3 \cdot f_3(I;2)$$

$$= \frac{6}{2} \sum_{w=1}^{2} \binom{2}{w} f_2(I;w) \cdot f_2(I;2-w) \cdot 2 \cdot 2 + 4 \cdot 3 \cdot 6$$

$$= 12 [2 \cdot f_2(I;1) \cdot f_2(I,1) + 1 \cdot f_2(I;2) \cdot f_2(I,0)] + 72$$

$$= 12 [2 \cdot 1 + 0] + 72 = 96$$

$$C_4(I,4) = \frac{1}{2} \sum_{r=2}^{2} \binom{4}{2} \sum_{w=1}^{3} \binom{3}{w} f_r(I,w) \cdot f_{n-r}(I;3-w) \cdot r \cdot (4-r)$$

$$+4 \cdot 3 \cdot f_3(I,3)$$

$$= 12 [3 \cdot 0 + 3 \cdot 0] + 72 = 72$$

$$C_4(I,j) = 0; \quad j \geq 5.$$

Similarly, we can compute f_4 from C_4 and obtain $f_4(I,3) = 96$, $f_4(I,4) = 360$, $f_4(I,5) = 720$ and $f_4(I;j) = 0$, $j>5$. The coupled equations themselve can be used to obtain the exponential generating functions for the sequences f_n, and C_n. These generating functions turn out to be exponential convolutions and have been recently studied [17].

6.22

Network Analysis Corporation

It is possible to compute f_j (I) from the formula of corollary l but the calculation is lengthy. An example follows:

Example (n=4). For n=4 T_n=16, hence the probability of a tree should be $\frac{16}{20} = \frac{4}{5}$ when j=3.

Parts (k)	Class or Type	$\frac{\sum\limits_{i=2}^{4} k_i \binom{i}{2}}{6}$	$\left(\frac{\sum\limits_{j} k_i \binom{i}{2}}{20}\right)$
1	(0,0,0,1)	6	20
2	(1,0,1,0)	3	1
2	(0,2,0,0)	2	0
3	(2,1,0,0)	1	0
4	(4,0,0,0)	0	0

$$f_3(I) = \frac{24}{20}\left[\frac{20}{4!}\right] = \frac{24}{20}\left[\frac{96}{(24)\ (6)}\right] = \frac{16}{20} = \frac{4}{5}$$

VI.   A NEW MEASURE OF CONNECTIVITY

Let G be a connected graph with n nodes and m edges.   Let
$S(G)$ be the set of the m! permutations of edges, viewed as a set
of m!-tuples.   For each sequence s $S(G)$ define $C_c(s)$ as the index
of the first edge in s for which the graph with the first $C_c(s)$
edges is connected.   The number $m-C_c(s)$ then measure "how long"
the sequence s has been a connected graph.   Intuitively, if many
of the sequences in $S(G)$ have large $m-C_c(s)$ then the graph G is
more connected.   In particular we take the average or first moment
of numbers $\{m-C_c(s)\}$ as the definition of the connectivity of G.
We can also take higher moments which would lead to more precise
measures of connectivity.

Definition: The connectivity or "mean connectivity" of a
connected graph G, denoted by $C(G)$, is given by:

$$C(G) = \frac{1}{m!} \sum_{s \in S(G)} (m-C_c(s)) = m - \frac{1}{m!} \sum_{s \in S(G)} C_c(s)$$

Alternatively, if $C_G(k)$ is the number of sequences in $S(G)$
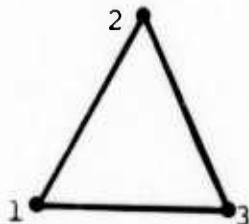for which $C_c(s) = k$ then,

$$C(G) = m - \frac{1}{m!} \sum_{k=n-1}^{m} k\, C_G(k)$$

Example: If G is a tree on n nodes then $m=n-1$ and $C_c(s) \equiv n-1$
for all $s \in S(G)$, so that

$$C(G) = (n-1) - \frac{1}{(n-1)!}(n-1)\,(n-1)! = (n-1) - (n-1) = 0.$$

Thus, all these have connectivity zero.

Example: Consider a triangle:

There are six edge sequences

$$S_1 = \{1,2\}, \{1,3\}, \{2,3\} \ , \quad C(s_1) = 2$$

$$S_2 = \{1,2\}, \{2,3\}, \{1,3\} \ , \quad C(s_2) = 2$$

$$S_3 = \{1,3\}, \{2,3\}, \{1,2\} \ , \quad C(s_3) = 2$$

$$S_4 = \{1,3\}, \{1,2\}, \{2,3\} \ , \quad C(s_4) = 2$$

$$S_5 = \{2,3\}, \{1,3\}, \{1,2\} \ , \quad C(s_5) = 2$$

$$S_6 = \{2,3\}, \{1,2\}, \{1,3\}, \quad C(s_6) = 2$$

$$C(G) = 3 - \frac{1}{6} \sum_{s \subset S(G)} 2 = 3 - \frac{1}{6} [12] = 3 - 2 = 1$$

So that the connectivity of the complete graph K_3 is $C(K_3) = 1$. The connectivity of K_2 is zero since K_2 is a tree.

Problem 1:   Determine the sequence $C(K_2)$, $C(K_3)$,...; or its generating function; i.e.,

$$C_k(z) = \sum_{i=2}^{\infty} \frac{C(K_i)}{i!} z^i$$

We now turn our attention to disconnected graphs.   Let G be a disconnected graph and let S(G) be the set of all permutations

of edges <u>not</u> in G. For each $s \varepsilon S(G)$ let $C_d(s)$ be the index of the first edge in the sequence s for which G would become connected if all the preceeding edges were added to G.

<u>Definition</u>: If G is a disconnected graph with m-edges we define its connectivity,

$$C(G) = - \sum_{s \varepsilon S(G)} \frac{C_d(s)}{(\binom{n}{2}-m))!} = - \frac{1}{(\binom{n}{2}-m!} \sum_{s \varepsilon S(G)} C_d(s)$$

<u>Example</u>: The empty graph Ω_2 on two nodes, has m=0 so C(s)=1. Therefore,

$$C(\Omega_2) = - \frac{1}{(\binom{2}{2}-0)!} = -1$$

<u>Example</u>: We compute the connectivity of Ω_3. There are six sequence in S(G) and $C(s) \equiv 2$.
Therefore,

$$C(\Omega_3) = - \frac{1}{3} \sum_{s \varepsilon S(G)} 2 = - \frac{12}{6} = -2$$

The computation of the sequence $\{C(\Omega_n)\}_{n=2}^{\infty}$ or its generating function is carried out in the next sections.

<u>Example</u>: We compute the connectivity of the graph G depicted below:

The graph G is disconnected. The edges not in G are $\{1,3\}$, $\{2,3\}$, there are two permutations, and $C(s) = 1$ for both so that,

$$C(C) = - \sum_{s \in S(G)} \frac{1}{2!} = - \frac{1}{2} (1+1) = - 1$$

Many questions, problems and conjectures are already apparent, some of these are:

Problem 2: There seems to be a duality in terms of connectivity between a graph and its complement. Can the connectivity of a graph be obtained in terms of its complement? The answer is yes and the appropriate result is given in Section VII.

Problem 3: Is the connectivity of a graph related to the reliability of its network? The answer is yes and this is discussed at an elementary level in Section VIII.

Problem 4: If we are given a graph G, then all sequences $s \in S(G)$ have "connectivity weights" $C(s)$. What are the minimal and maximal values of $C(s)$ over all $s \in S(G)$. In statistical terms we are asking for the range of $C(s)$ over $S(G)$; i.e. $\max_{s \in S(G)} C(s) - \min_{s \in S(G)} C(s)$,

Along these same lines given a class G of graphs what are the minimal and maximal values of the connectivities of the graphs in G? Characterize the classes of graphs which achieve these values. This problem is discussed in Section IX.

Problem 5: Can the connectivity of a graph be determined by the connectivity of various bipartite graphs. What is the connectivity of a bipartite graph consisting of two connected components? This question is discussed in Section X.

Problem 6:   There are many descriptive combinatorial functions associated with a graph, minimal cutset size, girth, diameter, number of spanning trees, cliques, etc. How are these related to connectivity? The definitions of these quantities are given in Section XI.

Problem 7:   There are many combinatorial questions associated with the sequence of graphs determined by a sequence of edges in S(G). Many of these questions concern the number and type of the connected component structure associated with the sequence of graphs associated with each $s \in S(G)$. Some of the questions are formulated in Section XI.

Problem 8:   There should obviously be relationships between connectivity of a graph and its chromatic polynomial or other such coloring properties. In Section XI we give Whitney's derivation of the chromatic polynomial of a graph. It seems that the Whitney approach gives a tie in with connectivity since it deals with colorings of subgraphs and that seems to be a way of approaching connectivity.

Problem 9:   When is the connectivity of a graph greater than the connectivity of every subgraph?

Problem 10:   If two graphs have the same number of edges but different size cutsets, does it follow that their connectivities have the same properties?

Problem 11:   How many "moments" are required to uniquely determine a graph or isomorphic classes of graphs?

Problem 12:   How do we construct "highly" connected graphs, or build on existing graphs to increase connectivity at low cost?

VII. A GRAPH AND ITS COMPLEMENT

We investigate Problem 2 and derive a relationship between a graph and its complement.

Definition: It G is a graph then its complement G' is the graph on the nodes of G whose edge set is the set of all edges not in G.

Theorem 7A: If G is a disconnected graph with m edges on n nodes then

$$C(G') - C(G) = \binom{n}{2} - m + \frac{1}{\binom{n}{2} - m} \sum_{s \in S(G)} (C_d(s) - C_c(s))$$

Proof: If G is disconnected then G' is connected and G' has $(\binom{n}{2} - m)$ edges.   Since $S(G)$ and $S(G')$ are identical as sets of $(\binom{n}{2} - m)$-tuples of edges we have by direct calculation;

$$C(G') - C(G) = (\binom{n}{2} - m) - \frac{1}{(\binom{n}{2} - m)!} \sum_{s \in S(G')} C_c(s)$$
$$+ \sum_{s \in S(G)} \frac{C_d(s)}{(\binom{n}{2} - m)!}$$

$$= (\binom{n}{2} - m) + \frac{1}{(\binom{n}{2} - m)!} [\sum_{s \in S(G)} C_d(s) - \sum_{s \in S(G')} C_c(s).]$$

$$= (\binom{n}{2} - m) + \frac{1}{(\binom{n}{2} - m)!} \sum_{s \in S(G)} [C_d(s) - C_c(s)].$$

Corollary 1:   For the complete and empty graphs;

$$C(K_n) + C(\Omega_n) = \binom{n}{2}.$$

6.29

<u>Proof</u>: Set $G = \Omega_n$ in Theorem 2.1 and note that $C_d(s)$

$$= C_c(s) \; \forall \; s \varepsilon S(\Omega_n).$$

<u>Corollary 2</u>: For the generating functions $C_k(z)$ and $C_\Omega(z)$ of $\{C(K_m)\}$ and $\{C(\Omega_n)\}$, respectively we have

$$C_k(z) + C_\Omega(z) = \frac{z^2 e^z}{2}$$

<u>Proof</u>:   By direct calculation!

VIII. <u>APPLICATION TO RELIABILITY AND OTHER AREAS</u>

The analysis of sequence sets such as S(G) arises in many areas other than in a "pure" study of graph connectivity. In set-merging algorithms we have a partition of a set of n points which we may think of as nodes of a graph. Unordered pairs of points are presented sequentially which we may think of as edges of the graph. If the two points of an edges are in different parts of the partition then the two parts are merged (their set theoretic union is formed). Otherwise, the same analysis is carried out with the next edge. The obvious questions concern the statistical moments of the number of edges need to connect the graph (merge all parts into one part) or to reach some other "state". Many of these questions can be restated in terms of the statistics of S(G).

Another area in which the set S(G) arises is in the use of "random edges" to generate a spanning tree for a graph or network with n-nodes. Edges are randomly generated and added to a graph (beginning perhaps with the empty graph). The question arises in the analysis of such a process as to how long it will take before the resulting "random graph" is connected. The answer to this question if we start with an empty graph on n nodes is $C(\Omega_n)$. It should also be clear that this question is the same as the set merging question.

A large area of potential application of the notion of connectivity is the study of the reliability of networks. Let G be a graph and $R_p(G)$ be the probability that the graph G is connected if p is the probability that is branch (independently) is "on" or "working" or "up". Clearly if G has m edges.

$$R_p(G) = \sum_{k=n-1}^{m} A_k p^k (1-p)^{m-k}$$

where A_k is the number of connected subgraphs of G with n nodes and

6.31

k edges.   Alternatively

$$R_p(G) = 1 - \sum_{i=w}^{m} B_i q^i p^{m-i},$$

where b_i is the number of disconnected subgraph of G with n nodes and (m-i) edges.   W is the size of a minimal cutset of G.

A simple revision for $R_p(G)$ relates the reliability of graph or networks to subgraphs.   This might be helpful in relating reliability to connectivity and to coloring.   Let e be a given edge in G. so that:

$$R_p(G) = P\{G \text{ is connected } | \text{ e is open}\} \, P\{e \text{ is open}\}$$
$$+ P\{G \text{ is connected } | \text{ e is closed}\} \, P\{e \text{ is closed}\}$$
$$= P\{G \text{ is connected } | \text{ e is open}\} + q \, R_p(g')$$

where G' is the subgraph of G obtained by removal of e.   Let G'' be the graph obtained by identifying the nodes incident to e.   Thus

$$R_p(G) = pR_p(G'') + aR_p(G')$$

where

$$R_p(G'') > R_p(G) > R_p(G')$$

The same type of formula can be obtained by taking a subset of edges of G which is not a cutset.   This would help tie in with coloring problems.

The Moore-Shannon definition of reliability is similar although it asks for the probability p_{ij} that a given pair of nodes {i,j} is connected by some path in G.   Clearly:

$$P_{ij} = \sum_{k=1}^{m} A_k p^k (1-p)^{m-k}$$

6.32

Where A_k is the number of distinct k edge subgraphs of G which contain a path between nodes i and j, and m is the number of edges. Alternatively, if B_k is the number of k edge subset of edges which when removed leaves nodes i and j disconnected then,

$$1-P_{ij} = \sum_{k=1}^{m} B_k (1-p)^k p^{m-k}$$

It is easy to relate $R_p(G)$ to $C(G)$ and it should be easy to relate P_{ij} to $C(G)$. Let G be a graph on n nodes with m edges. Let $f_G(k)$ be the number of sequences in $S(G)$ for which the first k edges on the n nodes form a connected graph.

Theorem 8A: Let $E_w(p)$ be the expected value of X! when X is a binomial random variable $b(w;p)$. Let $C_k(p) = \dfrac{C_G(k) p^k}{k!}$, then

$$R_p(G) = \sum_{k=0}^{m} E_{m-k}(p) \, C_k(p)$$

Proof: By definition:

$$R_p(G) = \sum_{k=n-1}^{m} A_k p^k (1-p)^{m-k}$$

Now $A_k = \dfrac{f_G(k)}{k!} = \dfrac{1}{k} \sum_{w=n-1}^{k} C_G(w) \, (m-w)_{(k-w)}$

where $(t)_{(n)} = t\,(t-1)\ldots(t-k+1)$ is the falling factorial.

It follows by substitution that

$$R_p(G) = \sum_{k=n-1}^{m} \frac{1}{k!} \sum_{w=n-1}^{k} C_G(w) \, (m-w)_{(k-w)} p^k (1-p)^{m-k}$$

Rearranging terms and collecting by $C_G(w)$ yields.

$$R_p(G) = \sum_{k=n-1}^{m} \frac{C_G(k) p^k}{k!} \sum_{j=0}^{m-k} (m-k)_{(j)} \, p^j (1-p)^{m-j-k}$$

which proves the theorem!

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Problem 13: Devise an algorithm for computing $C_G(k)$. Perhaps a Markovian type algorithm as used in [22] for the complete graph might work.

Problem 14: Study the behavior of the sequence $C_k(p)$ in terms of the connectivity of connected graphs. In fact if $C'_k(p)$ is the derivative of $C_k(p)$ with respect to p, then obviously the connectivity of G is,

$$C(G) = [m - \frac{1}{m} \sum_{k=n-1}^{m} k! \ C'_k(p)]_{p=1}.$$

Problem 15: Suppose two graphs or networks with the same number of nodes and edges have different reliability functions. To what extent can the differences be explained by the "moments" of connectivity?

Problem 16: Relate statistical connectivity to the Moore-Shannon definition of reliability.

IX.   GRAPHS WITH MINIMAL AND MAXIMAL CONNECTIVITY

In Problem 4, we asked about graphs with minimal and maximal connectivity, we shall examine the case of connected graphs with n nodes and n edges. The connected graphs with (n-1) edges all have connectivity zero. What is the minimal connectivity of a graph with n edges? What is the maximal value? Obviously if G has n nodes and n edges and is connected $0 < C(G) \leq 1$. More generally if G has m edges $0 < C(G) \leq m-(n-1)$. There are no connected graphs with connectivity zero which have more than (n-1) edges.
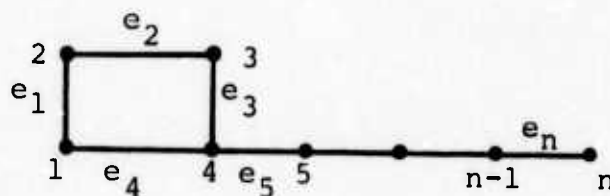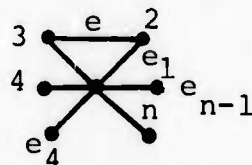
The following classes of graphs with n nodes and n edges have connectivity $\frac{3}{n+1}$.
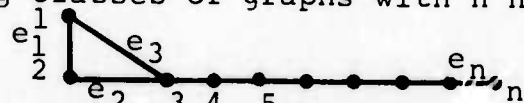
There are 3(n-1)! sequences of the n edges whose associated graph becomes connected for the first time on the $(n-1)^{st}$ edge, and (n-3) (n-1)! sequences which become connected graphs for the first time with the nth edge.   Therefore

$$C(G) = m - \frac{1}{m} \sum_{k=n-1}^{n} kC_G(k) = n - \frac{1}{n!} \sum_{k=n-1}^{n} k\ C_G(k)$$

$$= n - \frac{1}{n!} \ [(n-1)(n-1)! \cdot 3 + n(n-3)(n-1)!]$$

$$= n - 3\frac{(n-1)}{n} - (n-3) = 3 - 3\frac{(n-1)}{n} = \frac{3}{n}$$

Another graph with the same connectivity is,

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It should be easy to prove the following conjectures.

<u>Conjecture 1</u>:   Among the class of connected graphs with n nodes and n edges the minimum value of the connectivity is 3/n and is achieved by those graphs which contain a triangle.

More generally.

<u>Conjecture 2</u>:   If a connected graph on n nodes with n edges contains a k-gon then its connectivity is k/n.

Conjecture two applies to the maximum value of the connectivity. Indeed the cycle graph C_n.
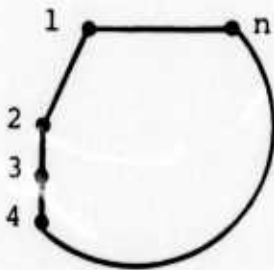
has connectivity one since the ommission of any edges does not destroy connectivity.   Many other questions are now apparent!

<u>Problem 17</u>: Generalize conjectures 1 and 2 for graphs with more than n edges.

<u>Problem 18</u>:   Are there parallel results for the connectivity of disconnected graphs.

<u>Problem 19</u>:   In what sense are the reliabilities of the above networks minimal and maximal.

X.   CONNECTIVITY OF BIPARTITE GRAPHS

Let G be a graph on n nodes with two connected components C_1 and C_2 with m_1, m_2 edges and V_1, V_2 nodes, respectively.

There are $\binom{n}{2}$ possible edges of which $(m_1 + m_2)$ are already in the graph G so that we form sequences from the $\binom{n}{2} - (m_1 + m_2)$ edges not in the graph G. Let X be the random variable "time to connectivity" on S(G). There are $(V_1 \cdot V_2)$ good edges and $\binom{n}{2} - (m_1 + m_2) - (V_1 \cdot V_2)$ "bad" edges from which to select at random (form sequences) and add to G one at a time. Let $P_k = P\{X=k\}$, so that

$$P_1 = P\{X=1\} = \frac{V_1 \cdot V_2}{\binom{n}{2} - (m_1+m_2)}$$

$$P_2 = P\{X=2\} = (1-P_1)\left(\frac{V_1 \cdot V_2}{\binom{n}{2} - (m_1+m_2)-1}\right)$$

$$\vdots$$

$$P_n = \left[\prod_{i=1}^{n-1}(1-P_i)\right]\left(\frac{V_1 \cdot V_2}{\binom{n}{2} - (m_1+m_2)-(n-1)}\right)$$

Let $G(z) = \sum_{k=1}^{\infty} P_k z^k$ be the generating function for the sequence $\{P_n\}$.

Theorem 10A: The function G(z) satisfies the differential equations;

$$(1-z)\ G'(z) + [(1-z)\ (W+1) = (wz)]\ G(z) + \frac{w}{w}\ (z-w-1) = 0,$$

where $w = V_1 V_2$ and $W = \binom{n}{2} - (m_1+m_2)$.

<u>Proof</u>: It is easy to see that P_n satisfies the recurrence relation

$$P_n = P_{n-1} \quad (\frac{W-w-(n-2)}{W-(n-1)}) \text{ for } n \geq 2 \text{ with}$$

$$P_1 = \frac{w}{W}.$$

The theorem follows by direct application of the generating function and a little algebra.

The connectivity of the bipartite graph is given by $G'(1)$. Perhaps a more detailed analysis of the differential equation will yield some more information.

It is possible to obtain an upper bound for the connectivity of a bipartite graph which we conjecture to be exact in the limit.

Let Y be the time to connectivity of the bipartite graph when the edges are chosen with replacement. Obviously $E[Y] \geq E[X]$. The time to connectivity for the replacement process is easy to compute since the distribution of y is geometric. In fact;

$$E[y] = \sum_{k=1}^{\infty} k \ (1-\frac{w}{w})^{k-1} (\frac{w}{w}) = \frac{W}{W}.$$

so that we have proved.

<u>Theorem 10B</u>: Let G be a bipartite graph on n nodes with connected components C_1 and C_2 with m_1, m_2 edges and V_1, V_2 nodes respectively, then,

$$C(G) \geq - \frac{\binom{n}{2} - (m_1 + m_2)}{V_1 \cdot V_2}.$$

<u>Problem 20</u>: Obtain an asymptotic estimate in Theorem 10B by suitably restricting the ranges of m_1, m_2, V_1, V_2.

Problem 21:   If we interpret connectivity of a graph in re-
liability terms as edges going out one at a time, selected at
random, what is a reasonable definition for the reliability of
a bipartite graph?   If terms of Moore-Shannon it might be the
average time for two nodes (one in each component) to communicate.
Interpret and calculate "reliability of a bipartite graph" and
relate it to connectivity.

XI. DESCRIPTIVE COMBINATORIAL QUANTITIES IN A GRAPH

In the literature of graph theory, there are many combinatorial type quantities which measure internal structure of a graph. Each of the quantities should be related to connectivity. We will not spell out the specific questions since they should be obvious!

Definition: If a graph G has p connected components, n nodes and m edges then the rank of G denoted by $\rho(G)$ = n-p and its cyclomatic number denoted by $\nu(G)$ = m-$\rho(G)$.

Theorem 11A: (Known). Let G be a graph and G' be a graph formed from G by adding a new edge between node i and j (arbitrary) then,

$\rho(G')$ = $\rho(G)$ and $\nu(G')$ = $\nu(G)$ if i and j are in the same component, while $\rho(G')$ = $\rho(G)$+1, and $\nu(G')$ = $\nu(G)$ if i and j are in different components.

Corollary: $\rho(G) \geq 0$ and $\nu(G) \geq 0$.

Definition: The chromatic number of a graph is the smallest number of colors needed to color the graph so that no adjacent vertices have the same color.

Theorem 11B: (König) A graph is bi-chromatic if and only if it contains no cycles of odd length.

It is not difficult to verify that the zeta function is invertible. The inverse of the zeta function is called the Mobius function and is denoted by $\mu(.,.)$. These observations of the Mobius Inversion Theorem.

Theorem 2A: (Mobius-Rota Inversion Theorem)

If $f:P \rightarrow R$ is a real valued function on a finite partially ordered set P and p so that $f(x) = 0$ when $x \cdot p$ and,

$$g(x) = \sum_{y \leq x} f(y) \qquad \text{then we have,}$$

$$f(x) = \sum_{y \leq x} g(y)\mu(y,x),$$

where $\mu(.,.)$ is the Mobius function of the partially ordered set P.

Some useful results for computing the Mobius function are given in [20]: In Particular the next formula for the Mobius function of Π_n.

Theorem 2B: The segment $[x,y]$ is said to be of class $(C_1, C_2, \ldots, C_n)$ when the lattice $[x,y]$ is isomorphic to the direct product of C_1 lattices isomorphic to Π_1, C_2 lattices isomorphic to $\Pi_2, \ldots, C_n$ lattices isomorphic to Π_n. If $[x,y]$ is of class $(c_1, c_2, \ldots, c_n)$ then

$$\mu(x,y) = (-1)^{c_1 + c_2 + \ldots c_n - n} (2!)^{c_3} (3!)^{c_4} \ldots ((n-1)!)^{c_n}$$

Corollary: $\mu(o,1) = (-1)^{n-1}((n-1)!$

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SIMULATION OF PACKET COMMUNICATION NETWORKS

I.   INTRODUCTION

In developing a large scale communication network, one
encounters many problems which cannot be formulated or solved
analytically. Consequently, one resorts to simulation. In this
chapter, we outline the structure of a simulation program for
packet communication networks; specify the problems which can be
resolved by a simulator; give the description of the simulator de-
veloped for the packet radio network; present results obtained by
the simulator; and discuss the future development of the simulator
for the packet radio network.

In general, there are various degrees of simulation depending
on the amount of knowledge (or assumed knowledge) about the system
operation, and the objectives of the simulation, i.e., the problems
to be resolved. We particularly distinguish in this chapter between
a simulation for design and a simulation for development.

A simulator for design (e.g., in a design loop) is developed
when the operation of the system is completely specified, and the
objective of the simulator is to simulate specific parts of the
system which cannot be analytically modeled or whose solution is
computationally infeasible. The efficiency of such a simulator is
of major importance; consequently, one attempts to avoid simulation
[8] wherever possible, in the simulation program.

The objectives of a simulation for development are much broader.
In this case, one has a set of theoretical (untested) hypotheses
which state one or several possible ways for system operation.

These include routing algorithms, protocols, etc. The objectives
are to test (verify) the hypotheses, to complete the specification
of portions of the system, to compare alternative modes of system
operation, to identify system bottlenecks, and finally, to deduce
measures of system efficiency by obtaining estimates of the major
parameters.

II.   OBJECTIVES OF SIMULATOR

We outline specific problems for which a simulation approach solution is most appropriate.

A.   Routing Algorithms

The object is to compare the efficiency of existing and newly developed routing algorithms in terms of throughput and delay on the one hand, and storage and processing requirements of the algorithms at the switching nodes on the other hand. The objective of this comparison is to suggest a small set (two or three) algorithms for implementation and further testing in an experimental system. We note that it is not mandatory that a single routing algorithm be used in a network. For example, in the broadcast network that we discuss later, a simple routing algorithm is used to load the switching nodes (repeaters) with a more sophisticated algorithm. Furtheremore, one may examine the possibility of using an alternative routing algorithm under overload conditions or in different hierarchies of the network, if a hierarchical network is investigated.

B.   Protocols

There may be many protocols in a communication network considered, depending on the type of communicating devices and on the type of application. The following protocols are examples: terminal-terminal, terminal-switching node, terminal-host computer, switching node-switching node, switching node-host computer, host computer-host computer. Some of the above may contain more than one protocol depending on the application, and others may be

needed in a hierarchical network. The objective of the simulation is to test the efficiency of these protocols in a dynamic environment, to change them when necessary, and to complete details which may be missing.

C.   Identify System Bottlenecks

One of the advantages of the simulator is that it enables one to observe and trace the detailed flow of packets in the network. For example, a specific packet may be traced, such as an information packet, an acknowledgement packet, or various priority packets, from the origination node to the destination node. This allows investigation of questions such as, whether the bottleneck is at the switching nodes or due to the limited capacity of the channel, given a proposed configuration, and communication protocols. It may also suggest improvements in communication protocols.

D.   Flow Control

The simulator is the only tool which allows testing and improvement of theoretically developed flow control algorithms.

E.   Software Transfer

A simulator can be coded so that subroutines or sections of code are identified with specific software programs of the switching nodes. This will reduce the effort of software development by transfering or coding according to the programs in the simulator. Furthermore, it will allow testing of sections of the software of communication devices by comparing these with the corresponding sections in a simulated device.

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F.   Trade-Offs

Among the trade-offs which can be investigated are:
the trade-off between the storage requirements at switching nodes
and the channel capacity of links, given the topology and the
offered traffic rates; trade-offs between average delay, maximum
delay, and delay as a function of priority and link and switching
node capacities.

G.   Stand-by Network

The simulator can be useful beyond the stage of develop-
ment. It can be used to study particular problems which may be
encountered in the network, once it becomes operational, and to
test suggestions for improvement.

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III. GENERAL STRUCTURE OF SIMULATOR

It is important to separate between data structure and management functions on the one hand, and the communication and device functions on the other hand. There are several advantages in doing so; firstly, one communication device does not have access to information available in other devices; and secondly, it is easier to identify and distinguish the communications part of the program for the purpose of modification or software transfer. We have developed efficient data structures which can be used for simulation programs of communication systems. The proposed data structures are described in Section V, and includes the fourteen (14) subroutines of Section VIII, subsection A.

It is useful to have one main subroutine for each type of communication device. For example, one subroutine for all switching nodes of the same type. The differences between the devices (e.g. switching node) can be recorded in a state vector associated with each device. The state vector will include information such as, the switching nodes to which this device has channels and the data rates of these channels, the state of occupancy of the storage buffers of the node, the routing algorithm that this node is currently using, and others. In addition there will be buffers associated with each device in which the content of specific packets (e.g. packet type, priority) will be stored.

There are distinguisable functions which may be used by more than one type of device (e.g. a modem), these functions can be coded in separate subroutines.

7.6

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A. Performance Measures

There should be an extensive measurement program to enable the resolution of the problems outlined in the objectives. The measurements are divided into network performance measurements, performance of communication devices, and trace information.

B. Network Measurements

· Throughputs

We distinguish between throughput of packets and throughput of information. Packets which successfully travel from origin to destination contribute to the packet throughput measure. Packets which are also acknowledged contribute to the information throughput measure. The distinction comes from the fact that when protocols are not efficient or when delays are very large, the origination node may reissue another copy of a previously transmitted packet. When the network delivers both packets, then the two packets contribute to the packet throughput measure but in terms of information transfered only one packet was delivered. Network throughputs are measured as a function of time. This enables one to determine whether the network can maintain a steady state throughput when offered a given traffic rate, to estimate the time needed to obtain steady state, and to observe the behavior of the network under either minor or major perturbations.

· The Average Number of Links That A Packet Traverses

This measure when compared with the average number of links of the input (offered) rate for a given topology, reflects on the amount of alternate routing. It may also enable one to detect looping in the network.

- <u>Delays</u>

   Many delays should be measured in the network, we outline some of these:

   1.   Delay to negotiate protocol between a terminal and a switching node.

   2.   Delay to negotiate protocol and reserve storage at destination switching node.

   3.   Delay of an information packet of a given priority from origination to destination node.

   4.   Delay for receiving an end-to-end acknowledgment back at the origination node.

   5.   Delay for the delivery of a maximum size message and receiving an acknowledgment.

   6.   Delays of special priority packets.

C.   <u>Performance of Communication Devices</u>

- <u>Utilization of Devices</u>

- Fraction of time that the device is transmitting or receiving.

- Number of Packets stored in device as a function of time.

- Number of Packets successfully switched and number of packets discarded due to buffer overflow or other reasons.

D.   <u>Trace Information</u>

   This information includes the listing of significant communication events. The listing contains a unique identifier of the packet, its type and priority, its origin and destination, and

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the time of the event. These are needed to allow a detailed ob-
servation of flow, to trace of specific packets, and to follow
specific interactions between source and destination.

Remark: Other measurements may be needed to evaluate the
efficiency of flow control algorithms.

IV.   PACKET RADIO NETWORK SIMULATOR

The main features which distinguish the broadcast network from a point-to-point (PTP) packet switching network such as the ARPANET are: (i) devices in the network transmit packets by using a random access scheme, and (ii) devices broadcast so that signals can be received by several devices simultaneously.

When using a random access scheme, there is a possibility that several packets are simultaneously received by a receiver due to independent transmissions of several devices; in the event, none of these packets are correctly received, and the corresponding devices must retransmit their packets. This implies that, unlike PTP network, the probability of error due to overlapping packets is much higher than the probability of error due to other causes such as Gausian or impulse noise. Furthermore, the probability of error varies widely depending on the amount of traffic on the channel. The broadcast nature of the network implies that there is correlation between the probabilities of successful transmission in different parts of the network.

The communication system simulated contains three types of devices, terminals, repeaters, and stations. The station is considered as an interface communication device of the broadcast network to a higher level network or to a computer installation. Terminals are considered as traffic sources and traffic sinks, they transmit packets to the station and receive response packets from the station; they may be mobile. The basic function of the repeater is to extend the effective range of the terminals and the station. The network simulated contains repeaters and stations which are placed at fixed locations chosen at random. Terminals originate at random times and are placed in random locations on the plane. A terminal is considered to depart from the system once it completes its communication. A

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more detailed description of the system simulated can be found in
[4], and [5].

The program was coded in FORTRAN. It includes a total of 33
subroutines, approximately 3,000 statements. The compilation time
on a CDC 6600 takes approximately 20 CP sec. , and the running time
for meaningful results takes 100-300 CP sec. The storage re-
quirement of the program is $245,000_8$ 60 bit words.

The simulator has already been used for improving communication
protocols, and to answer questions related to the trade-offs be-
tween device range and device interference, and the trade-offs between
a single and a dual data rate system.

V.   DATA STRUCTURES OF THE SIMULATOR

The global information for the Packet Radio Simulation Program is contained in five (5) data structures:

1)   Event Structure;
2)   Active Message Structure;
3)   Active Packet Structure;
4)   Repeater-Station Structure;
5)   Data Collection Tables.

The meaning, configuration, and elements of these structures will be explained in the next five sub-sections. In the last sub-section, the use of these structures in the context of the entire program will be indicated.

A.   Event Structure

The simulation program is event - driven. That is, periodically the Event Structure is consulted to determine the time of occurrence of the next event. The Event Structure also contains information telling the program what the event is. Examples of events are arrivals of messages to terminals, transmission of packets, arrival of packets at receivers, and arrival of messages to the stations. As events are executed, they are deleted from the structure; and, periodically, newly-generated events are added to the structure. Since there are a large number of events; many more than the number of exogenous message arrivals, for example, the process of efficiently determining the next event is of vital importance. To this end the event times are maintained in a "heap." Corresponding to each event "i" is its time "t_i," the device subroutine "d_i" to which it refers (e.g. station, terminal, repeater), the index "w_i" of the device in question (i.e. which repeater, which station, etc.), a number representing the point at which the routine is entered, and,

finally, the packet number of the packet in question. In addition, there is the heap index vector itself. "h_j" points to the event which occupies the jth position in the heap. A heap is a structure in which $t_{hj} \leq \text{Max } \{t_{h2j}, t_{h_{2j+1}}\}$. At all times t_{h_1} is the smallest t_i over all the events. This structure allows for quite rapid selection of the minimum t_i while using a minimal amount of storage, [2], [1], [3]. In order to efficiently eliminate old events and to reuse the space created by the elimination, a garbage stack of unused locations is maintained.

B.   Active Message Structure

The external or exogoneous traffic which flows into the Packet Radio Network consists of messages which arrive at terminals or stations and represent information that users wish to send to a location using the net. Messages are to be distinguished from packets which carry the message information internally in the net. In general, there may be several packets in the network carrying copies of the same message or of parts of that message. Messages are added to the Active Message Structure when the event corresponding to their generation occurs. It stays on the list until the last packet containing the message is dealt with. Associated with each message "i" is its arrival time "t_i;" the current number of packets representing the message "n_i" (When "n_i" is reduced to zero (0), the message is removed from the structure.); its length "l_i;" "x_i" and "y_i", the co-ordinates of the terminal or station originating the message; and other pieces of data, such as the repeater with which the terminal communicates, the total number of packets, and arrival time of message, which may be needed for output statistics. In order to eliminate old messages and add new ones efficiently, the messages are kept in a doubly-linked list structure. There is a

7.13

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list kept for messages and one for unused message spaces. This information is kept in two vectors, "f" and "b". Thus, "f_i" represents whatever follows the space in question ("i"), be it message or empty space; while "b_i" represents whatever precedes the space. The unit being followed or preceded may be an empty space or may contain a message.

C.   Active Packet Structure

The active packets are kept in a list. Associated with the ith packet is a pointer to its corresponding message and several words which state the variable part of the packet label for use by the routing algorithms. A garbage stack similar to the one used for events keeps track of vacant spaces in the Packet Structure.

D.   Repeater-Station Structure

The Repeater-Station Structure is basically a list of the repeaters and stations, their locations, their possible neighbors (repeaters and stations within range of their transmitters and receivers), and their state. Associated with the ith station or repeater are "x_i" and "y_i", the co-ordinates of the device; and a state vector which, among others, specifies items such as whether the repeater or station is busy, free, turned off, or failed; and whether the device is a station or a repeater. In addition, there is a list of repeaters and stations adjacent to the ith repeater or station.

E.   Data Collection Tables

Sufficient statistics for the evaluation of the system performance measures (see Section VI subsection D) are kept in the data collection tables.

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F.   An Outline of the Use of Data Structures

The global simulation structure is based on events of packets arriving at devices: stations, terminals, and repeaters. Figure 1 schematically illustrates the program flow. It is quite simplified, one simplification being especially important and needing emphasis. The events of retransmissions and acknowledgements resulting from a packet's arriving at a repeater, station, or terminal are not necessarily generated immediately, but may depend on the arrival of packets at the device subsequent to the packet which gives rise to the new events.
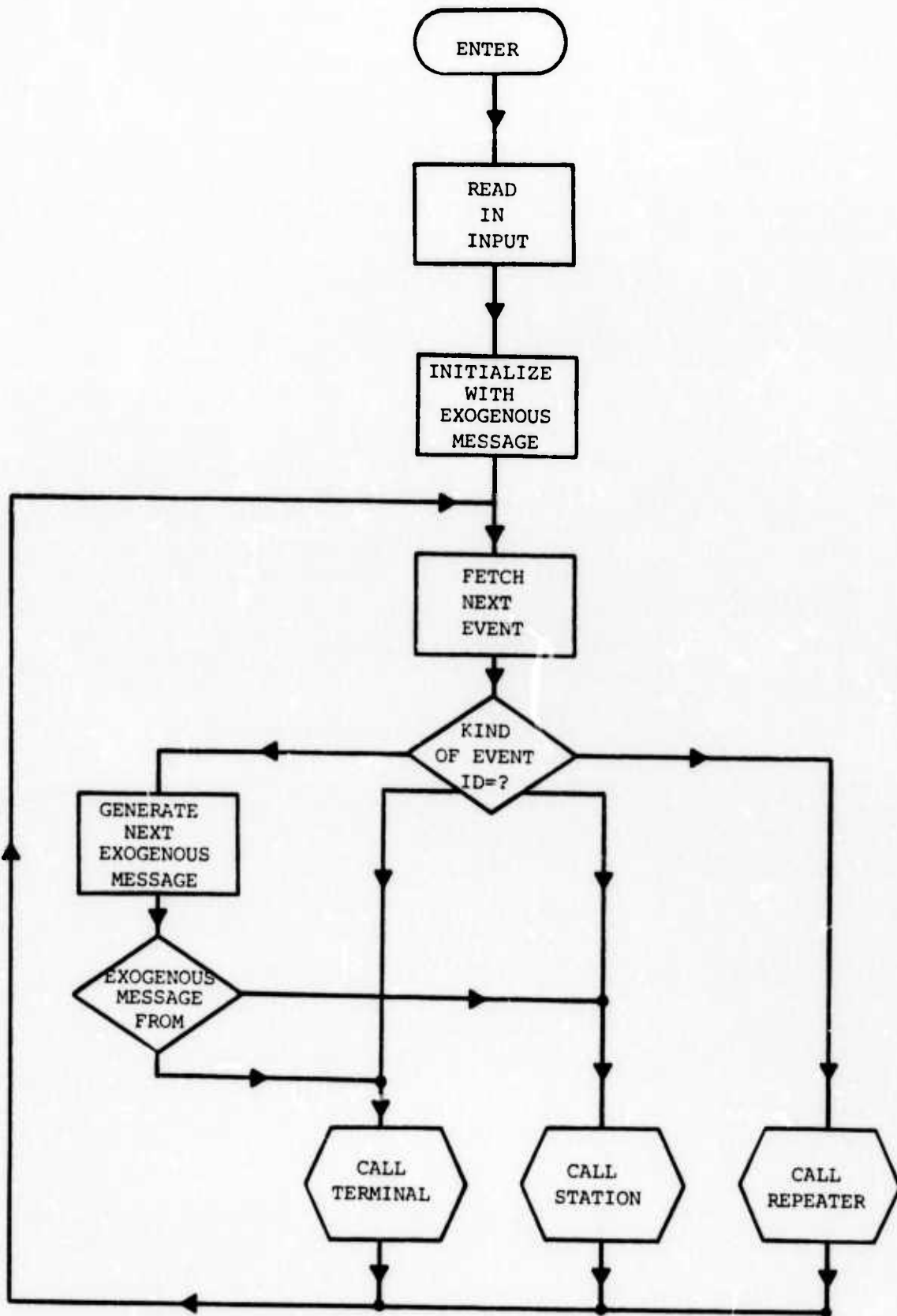
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FIGURE 1

GENERAL PROGRAM FLOW

7.16

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VI.   THE COMMUNICATION SYSTEM SIMULATED

A.   General System Description

•   Channel and Access Mode

The communication channel is shared for transmission in both directions, to the station or from the station to terminals. The channel access mode is the Non-Persistent Carrier Sense Multiple Access (CSMA) [7]. That is, when a packet is ready for transmission, the device senses the channel and transmits the entire packet if the channel is idle. If the channel is busy, the device reschedules the packet for some future random time at which it senses the channel again, and the procedure repeated.

•   Capture

A zero capture system is simulated. That is, whenever the reception of more than one packet overlap in time, none of the packets is correctly received.

•   Packet Types Simulated

IP   -   Information Packet

ETE   -   End-to-End Acknowledgement; Short Packet (Assumed to be 10% of the length of an information packet).

SP   -   Search Packet, transmitted by Terminal or Repeater to all devices and aimed to identify a specific receiver (short packet).

7.17

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RSP   -   Response to a Search Packet, transmitted by
Repeater or Station which received a SP and
is available for handling packets. This
packet contains the label of the transmitting
device and is addressed to all devices.

B.   Routing Algorithms

Three routing algorithms which are implemented in the
simulator are briefly described in the following paragraphs. A
detailed description of the routing schemes is given in [5].

•   Hierarchical Labeling:

The hierarchical labeling routing scheme enables point-
to-point routing between devices along an "efficient path".
It is obtained by assigning to every repeater a label, which
forms, functionally, a hierarchical structure. The label
assigned contains the following information:

1)   A specific address of the repeater for
routing purposes.

2)   The minimum number of hops to the nearest
station.

3)   The specific address of all repeaters on the
shortest path to the station, and the address of
the repeater to which a packet has to be transmitted
when destined to the station.

In the hierarchical labeling algorithm an information packet (IP)
is addressed to one device. If it is received by a device to
which it is not addressed, then the receiving device is closer
to the destination than the device to which the packet was
addressed. If the preassigned path is temporarily blocked,

7.18

the packet may depart from it. It then uses the most efficient path from its new location. The departure from the preassigned path is obtained by a search procedure.

The Response to Search Packet (RSP) is transmitted by repeaters after a random waiting time. This time randomization is essential in a no-capture system. Otherwise, if more than one receiver wish to respond to the SP, the transmissions will overlap at the searching device. The station, on the other hand, transmits the RSP immediately after receiving the SP (Note that if we assume zero processing time then the channel is idle at this time since otherwise the SP would not have been correctly received). The above enables searching devices within range of an idle station to communicate directly with the station.

A repeater makes one attempt to transmit a RSP and if the channel is busy it discards it, rather than store the RSP for future transmissions. This allows control of the level of terminal blocking by specifying the number of transmissions of the SP by a terminal. Thus, when the system is conjested in the geographical neighborhood of the terminal it will not be able to "enter" the system. This feature also makes repeaters more available for handling information packets.

- Directed Routing (One Level Labels)

Directed routing is a simplified version of the hierarchical labeling algorithm in which the only information preserved is the direction TO or FROM station. Repeaters are assigned labels that indicate the hierarchy level, or the number of hops to the nearest station. When a device transmits

a packet, the packet is addressed to all repeaters (stations)
that are closer to the destination than the transmitting device.
Many devices can receive the same packet and it may arrive at
more than one station. The acknowledgement schemes, the
station-station protocol, and the station-terminal protocol
must then resolve this problem.

- Flooding Algorithm (Plus Repeater Memory)

In this algorithm, there is no directionality of trans-
mission. A packet is addressed to all devices that can hear
it. To control the problems of cycling and looping, repeaters
are assigned storage for unique identifiers of packets that
they recently repeated. When a packet is received by a re-
peater, it compares its identifier with those stored and
discards the packet if a match occurs. A maximum handover
number (MHN) in the packet will prevent it from being propagated
for very long distances. This feature is also used in the
other routing algorithms.

C.   Acknowledgement Schemes

The acknowledgement scheme has particular significance
in this system because of the broadcast feature and the limited
capability of the repeater for processing and storage. The following
acknowledgements are used:

- End-to-end acknowledgement (ETE Ack) between
  station and terminal to ensure message integrity.
  The frequency and precise meaning of the Ack depends
  on the particular protocol used, and is part of the protocol.