E-1

The repeaters at d=5 do not have messages arriving from other stations. They only receive their own traffic at Poisson rate. Repeaters at $d < 5$ which are on the "axes" (denoted by circles) have messages arriving from the three neighbors at d+1, as well as their own Poisson traffic.

Repeaters off the axes at distance $d < 5$ have input at each time point from the two neighbors at d+1 as well as their own Poisson traffic.

The network is activated at t=0 by having random Poisson arrivals with mean λ at each of the 61 repeaters. This input traffic at each repeater is converted to received messages in each of the two possible modes for different values of m by use of the "transfer functions".

$$(1) \quad P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k-j,m-j)} (-1)^{\nu} \cdot \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \; ;$$

$$(2) \quad P^{*}_{kj} = \binom{m}{j} \sum_{\nu=0}^{j} (-1)^{\nu} \binom{j}{\nu} \left(\frac{j-\nu}{m}\right)^{k} . \qquad j=0, 1, 2, \ldots, \min(k,m)$$

These calculations give us $P^R_{(d,j)}(0)$ for all repeaters with coordinates (d,j), d=1, 2, 3, 4, 5, j=1,...,4d, an (0,0) the station at the origin.

We can now determine message traffic at each repeater by using equations which describe message transmission in the direction of of the "origin".

<u>For Time t=1</u>

<u>When d=5; j=1, 2,...,20</u>, the repeaters at d=5 receive only their generated Poisson traffic. Thus, for time 1 we generate 61 Poisson traffic numbers which describe direct (i.e., at the source) message input. When $d \le 4$, the repeater at coordinates (d,j) also receive traffic from its neighbors at further distance

by one unit. The following equations describe messages arriving at each repeater for arbitrary time $t > 1$.

On the Axis:

$$P_{(0,0)}(t) = P^R_{(1,1)}(t-1)+P^R_{(1,2)}(t-1)+P^R_{(1,3)}(t-1)+P^R_{(1,4)}(t-1)+\text{Poisson}$$

At d=1 $(P_{1,1}, P_{1,2}, P_{1,3}, P_{1,4})$

$$P_{(1,1)}(t) = P^R_{(2,1)}(t-1)+P^R_{(2,2)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

$$P_{(1,2)}(t) = P^R_{(2,2)}(t-1)+P^R_{(2,3)}(t-1)+P^R_{(2,4)}(t-1)+\text{Poisson}$$

$$P_{(1,3)}(t) = P^R_{(2,4)}(t-1)+P^R_{(2,5)}(t-1)+P^R_{(2,6)}(t-1)+\text{Poisson}$$

$$P_{(1,4)}(t) = P^R_{(2,6)}(t-1)+P^R_{(2,7)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

At d=2 $P_{2,1}, P_{2,3}, P_{2,5}, P_{2,7}$

$$P_{(2,1)}(t) = P^R_{(3,1)}(t-1)+P^R_{(3,2)}(t-1)+P^R_{(3,12)}(t-1)+\text{Poisson}$$

$$P_{(2,3)}(t) = P_{(3,3)}(t-1)+P^R_{(3,4)}(t-1)+P^R_{(3,5)}(t-1)+\text{Poisson}$$

$$P_{(2,5)}(t) = P^R_{(3,6)}(t-1)+P^R_{(3,7)}(t-1+P^R_{(3,8)}(t-1)+\text{Poisson}$$

$$P_{(2,7)}(t) = P^R_{(3,9)}(t-1)+P^R_{(3,10)}(t-1)+P^R_{(3,11)}(t-1)+\text{Poisson}$$

At d=3 $P_{3,1}, P_{3,4}, P_{3,7}, P_{3,10}$

$$P_{(3,1)}(t) = P^R_{(4,1)}(t-1)+P^R_{(4,2)}(t-1)+P^R_{(4,16)}(t-1)+\text{Poisson}$$

$$P_{(3,4)}(t) = P^R_{(4,4)}(t-1)+P^R_{(4,5)}(t-1)+P^R_{(4,6)}(t-1)+\text{Poisson}$$

$$P_{(3,7)}(t) = P^R_{(4,8)}(t-1)+P^R_{(4,9)}(t-1)+P^R_{(4,10)}(t-1)+\text{Poisson}$$

$$P_{(3,10)}(t) = P^R_{(4,12)}(t-1)+P^R_{(4,13)}(t-1)+P^R_{(4,14)}(t-1)+\text{Poisson}$$

At d=4 $P_{(4,1)}(t)$, $P_{(4,5)}(t)$, $P_{(4,9)}(t)$, $P_{(4,13)}(t)$;

$$P_{(4,1)}(t) = P^R_{(5,1)}(t-1)+P^R_{(5,2)}(t-1)+P^R_{(5,20)}(t-1)+\text{Poisson}$$

$$P_{(4,5)}(t) = P^R_{(5,6)}(t-1)+P^R_{(5,5)}(t-1)+P^R_{(5,7)}(t-1)+\text{Poisson}$$

$$P_{(4,9)}(t) = P^R_{(5,10)}(t-1)+P^R_{(5,11)}(t-1)+P^R_{(5,12)}(t-1)+\text{Poisson}$$

$$P_{(4,13)}(t) = P^R_{(5,15)}(t-1)+P^R_{(5,16)}(t-1)+P^R_{(5,17)}(t-1)+\text{Poisson}$$

Off the Axes:

$$P_{(d,j)}(t) = P^R_{(d+1,j)}(t-1+P^R_{(d+1,j+1)}(t-1)+\text{Poisson};\quad j=2,3,\ldots,d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+1)}(t-1)+P^R_{(d+1,j+2)}(t-1)+\text{Poisson};\quad j=d+2,d+3,..2d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+2)}(t-1)+P^R_{(d+1,j+3)}(t-1)+\text{Poisson};\quad j=2d+2,..3d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+3)}(t-1)+P^R_{(d+1,j+4)}(t-1)+\text{Poisson};\quad j=3d+2,..4d;\ d=2,3,4.$$

These equations relate arriving and received messages over neighboring time points and repeaters. Thus, the arriving number of messages can be computed in the grid at each point in time and each repeater.

In terms of a flow diagram, the procedure for analyzing this and all finite grids follows:

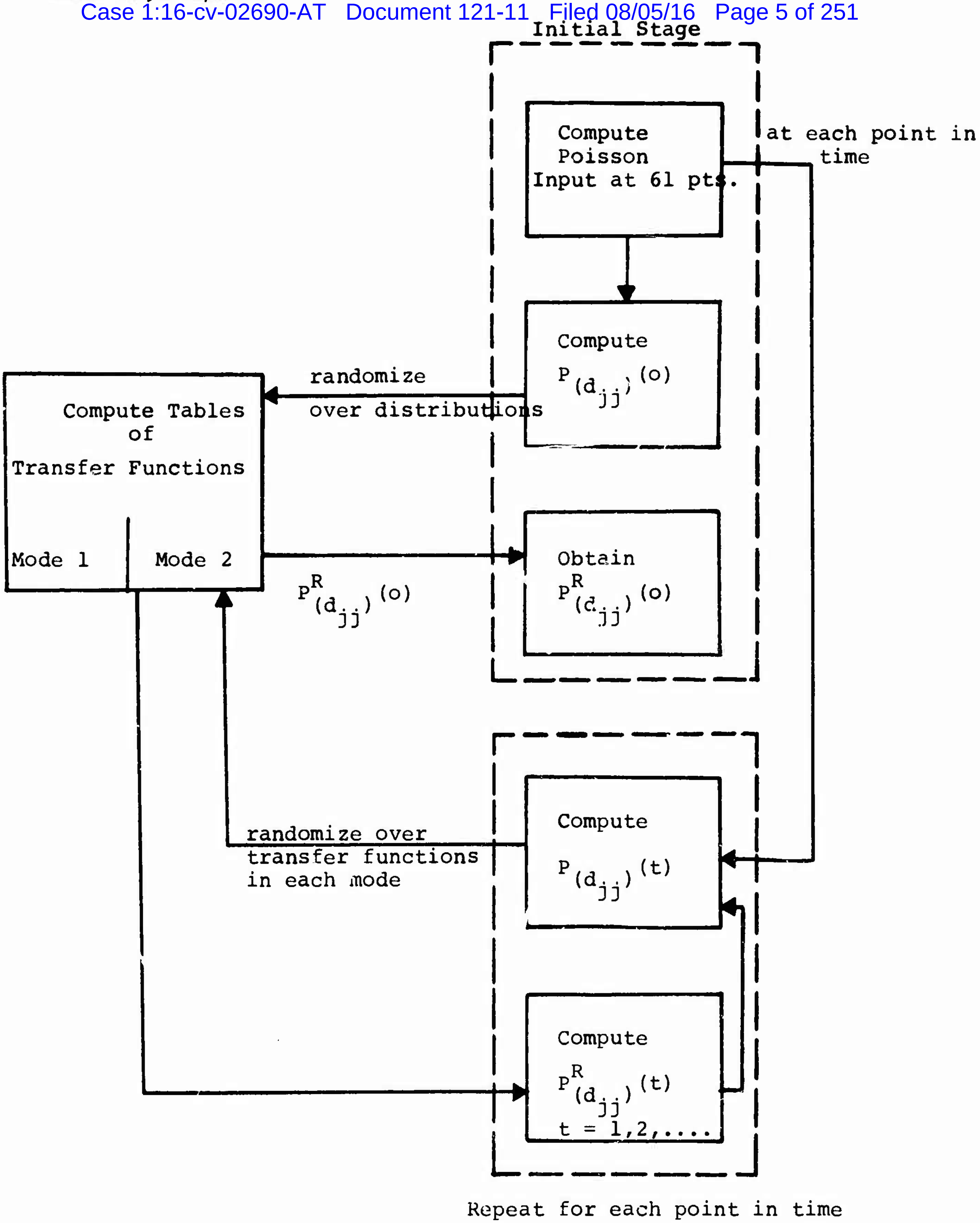

Initial Stage
Compute Poisson Input at 61 pts.
at each point in time
Compute $P_{(d_{jj})}(o)$
randomize over distributions
Compute Tables of Transfer Functions
Mode 1
Mode 2
$P^R_{(d_{jj})}(o)$
Obtain $P^R_{(d_{jj})}(o)$
randomize over transfer functions in each mode
Compute $P_{(d_{jj})}(t)$
Compute $P^R_{(d_{jj})}(t)$ t = 1,2,....
Repeat for each point in time

FIGURE 2

The parameters are, λ = mean Poisson arrival at each time point, at each repeater, m the number of slots in each mode one and two.

The output of the computer analysis is processed and presented in two forms, tabular and graphical. The tabular format is for each m and λ mode,

t	0	1	2	3	4	5	6	7	. . .
$P_{(0,0)}(t)$									. . .
$P^R_{(0,0)}(t)$									. . .
$P_{(1,1)}(t)$									. . .
$P^R_{(1,1)}(t)$									
$P_{(1,2)}(t)$									
$P^R_{(1,2)}(t)$									
⋮	⋮	⋮	⋮	⋮					

Various graphical analyses are also obtained.

A. A graph of arrived and received messages at the origin as a function of time for various values of m and λ.

B. A frequency histogram of arrivals off the axis. There are 24 points of the axis at distance 2, 3, 4,...

We take for each time t;

$$f(x) = \frac{\text{number of stations with x arrivals at time t}}{24} .$$

This is plotted for each time point.

C. The same histograms as in b except on the axis. There are 16 points on the axes at distances 1, 2, 3, 4.

D. The mean number of arrived and received messages $\hat{\lambda}(t)$ and $\hat{\lambda}^R(t)$ as a function of time on and off the axis. These are given by,

$$\hat{\lambda}(t) = \sum_{x=1} xf(x), \quad \hat{\lambda}^R(t) = \sum_{y=1} xf^R(x)$$

where f(x) if the frequency of arrivals and $f^R(x)$ is the frequency of received messages.

$$\hat{\lambda}_A(t) = \sum_{x=1} xf_A(x), \quad \hat{\lambda}_A^R(t) = \sum_{x=1} xf_A^R(x),$$

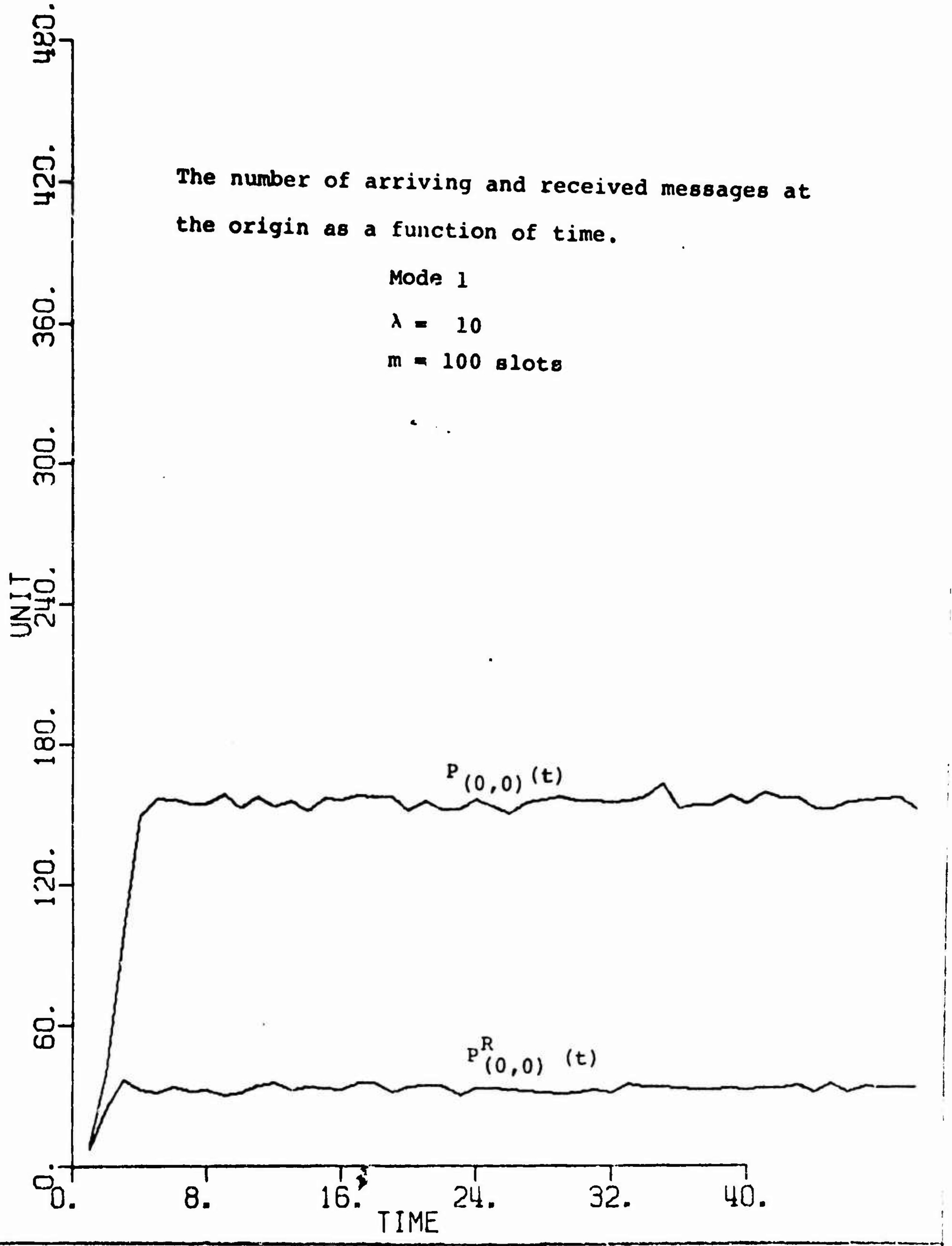
where $f_A(x)$ and $f_A^R(x)$ are frequencies on the axis of arriving and received messages. Some numerical results follow.

8.1 Summary of Initial Computer Analysis

Attached, are two curves which represent a summary of data compiled from a preliminary computer investigation of a closed grid network. The grid selected for initial analysis is the closed boundary grid at distance five. We combined computer runs with the closed form theoretical analyses of sections 4 and 5 of this report to obtain some observations of network behaviour.

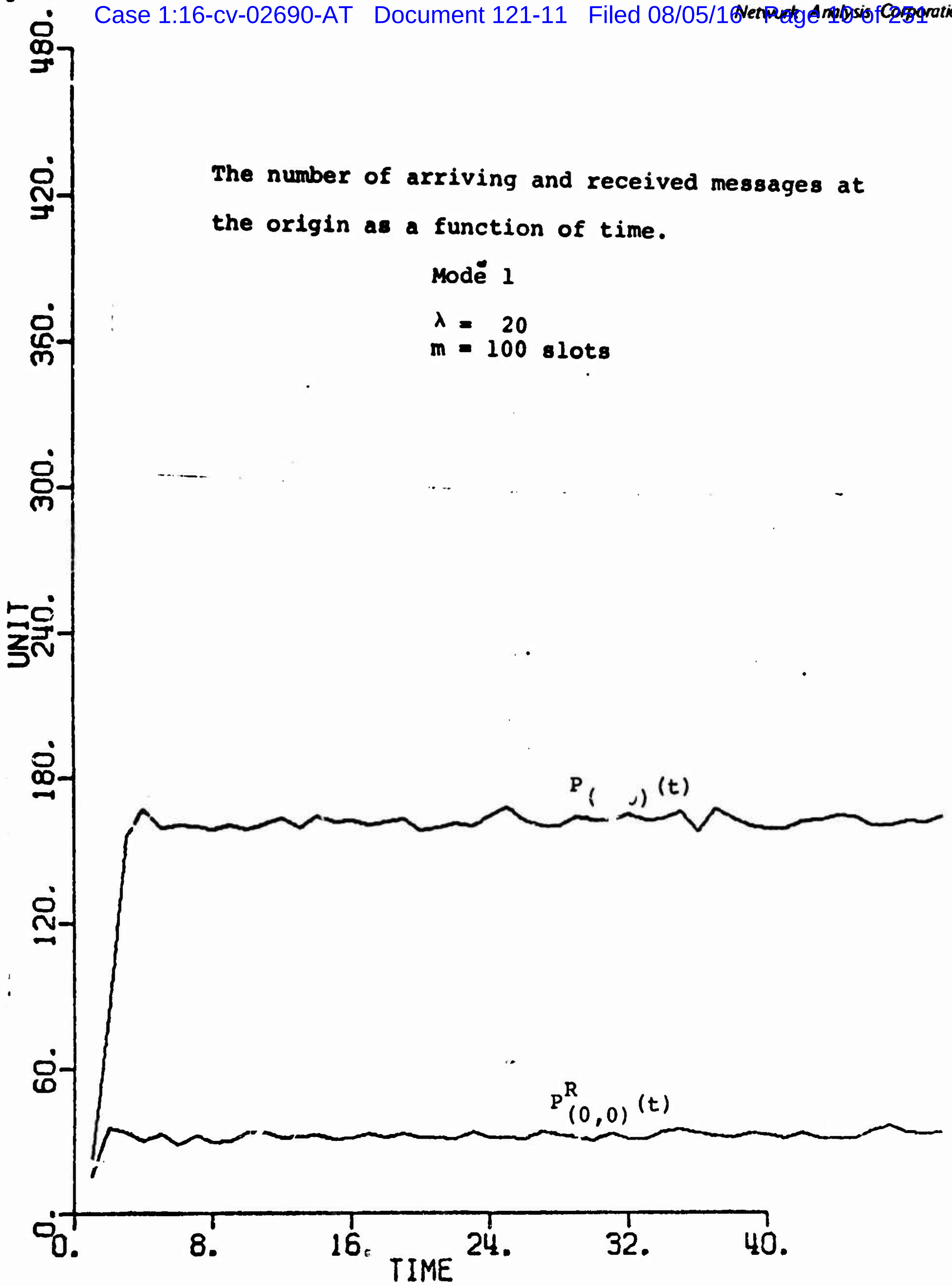
The first six curves represent a study of messages arriving and being received at the origin (fixed ground station) as a function of time. We used 20 computer runs for each of the first fifty time units. In this initial study the number of slots was kept fixed at 100, but λ(the mean number of messages originating at a given repeater) was set at 10, 20 and 30. All calculations were carried out for mode 1 and mode 2.

The message flow and reception at the origin settle down at about t=4 and remained relatively constant. For λ=10 the number of arriving messages seemed to have a mean at about 155 and the number of received messages averaged to about 31. Since the system behaviour for λ=10, m=100 settled down so quickly it seems reasonable to combine all time point data past t=10 to estimate the probability density function of arrivals and receptions at the origin in each of modes 1 and 2 when λ=10. The curves would seem to indicate asymptotic Poisson behaviour with means about 31, 155 in mode 1 and about 100, 300, in mode 2 respectively. Saturation occurs quickly in mode 2 for λ=10 or more. These results are summarized in the last four curves of probability density functions.



The number of arriving and received messages at the origin as a function of time.
Mode 1
λ = 10
m = 100 slots
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
TIME
0.
8.
16.
24.
32.
40.
P(0,0) (t)
PR(0,0) (t)

FIGURE 3



The number of arriving and received messages at the origin as a function of time.
Mode 1
λ = 20
m = 100 slots
$P_{(\ \ )}(t)$
$P^R_{(0,0)}(t)$
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 4

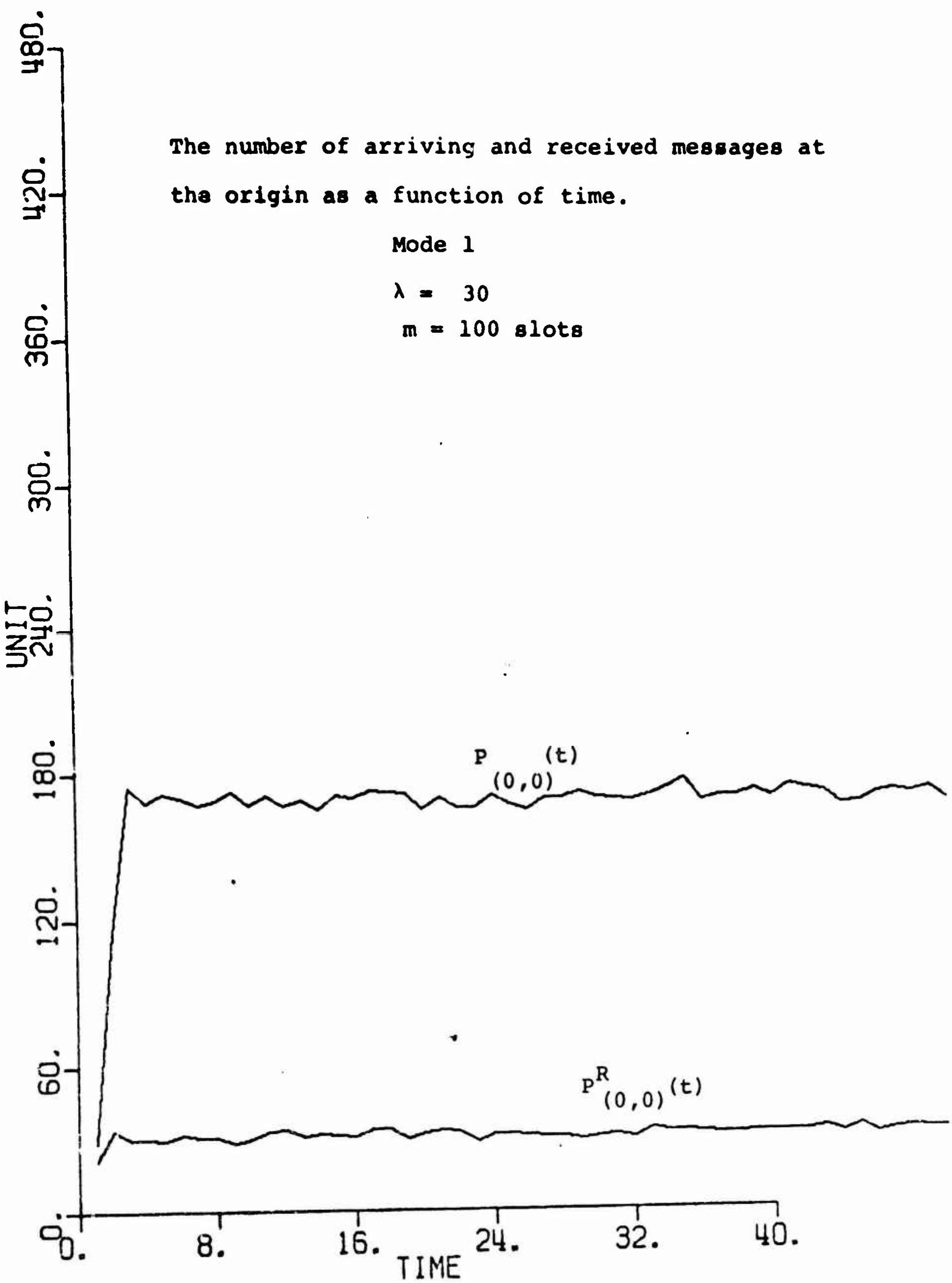

The number of arriving and received messages at
the origin as a function of time.
Mode 1
λ = 30
m = 100 slots
P (0,0) (t)
P R (0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 5

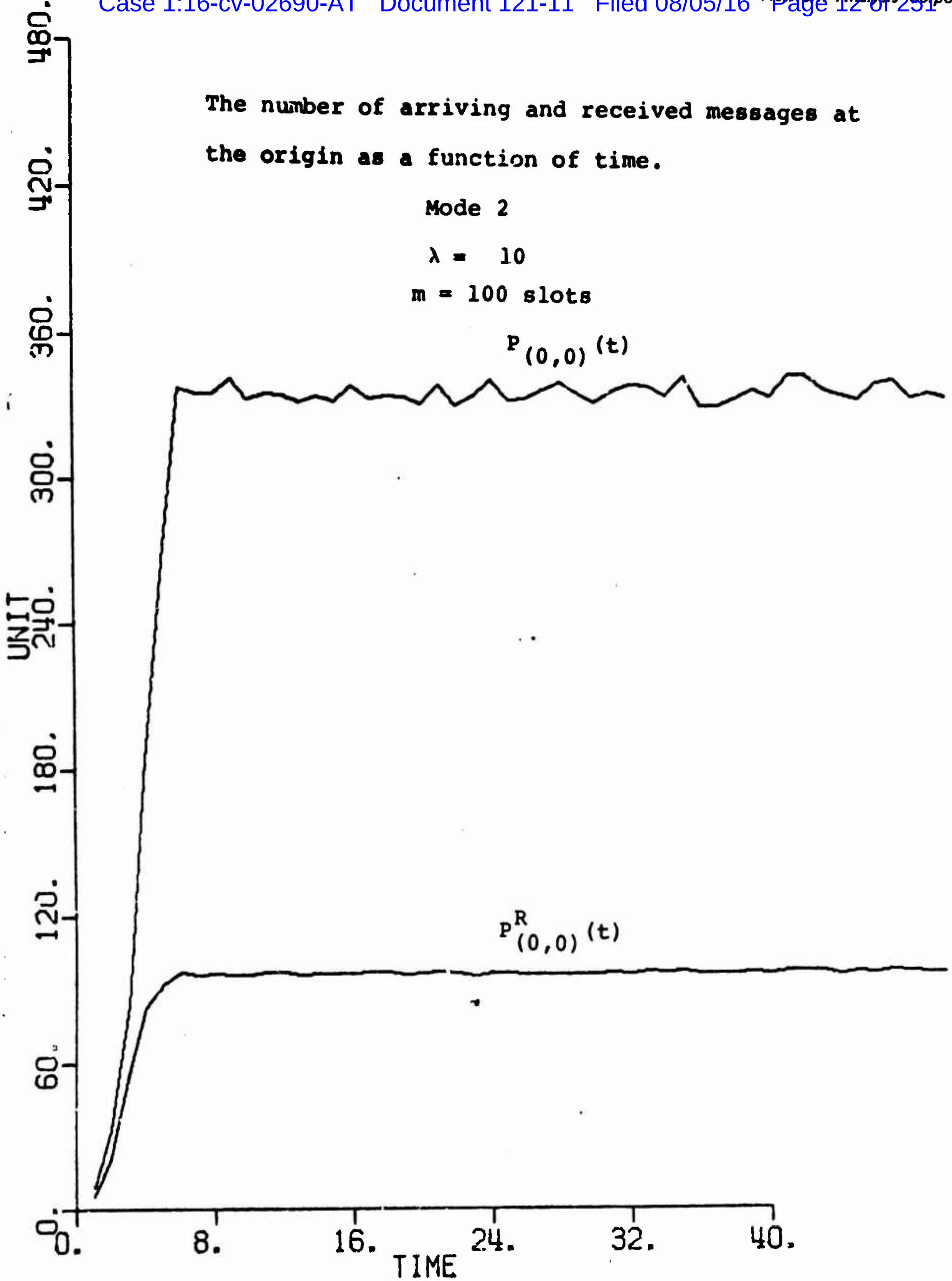

The number of arriving and received messages at the origin as a function of time.
Mode 2
λ = 10
m = 100 slots
P(0,0)(t)
P^R(0,0)(t)
UNIT
480.
420.
360.
300.
240.
180.
120.
60.
0.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 6

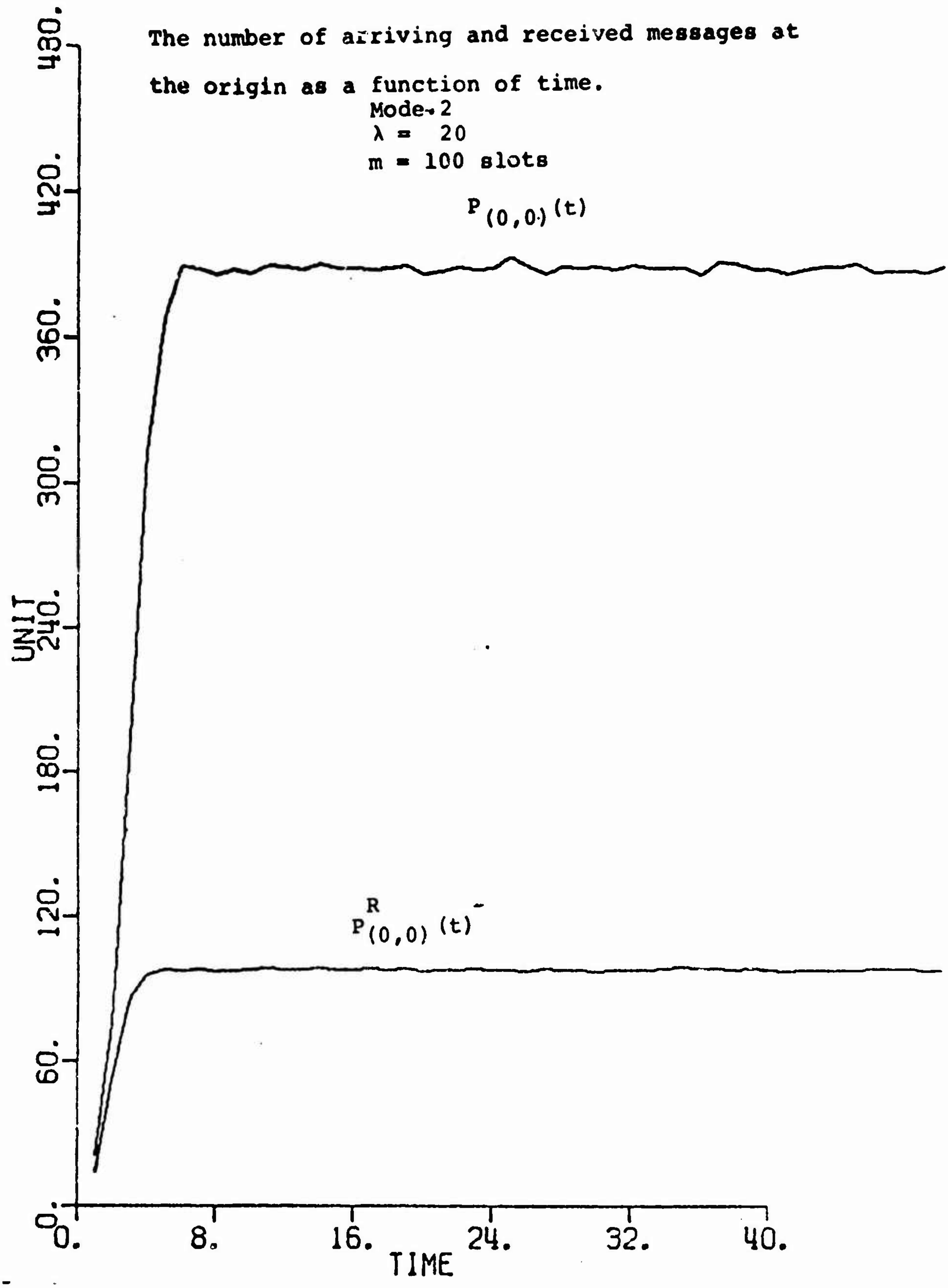

The number of arriving and received messages at
the origin as a function of time.
Mode-2
λ = 20
m = 100 slots
P(0,0)(t)
R
P(0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
430.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 7

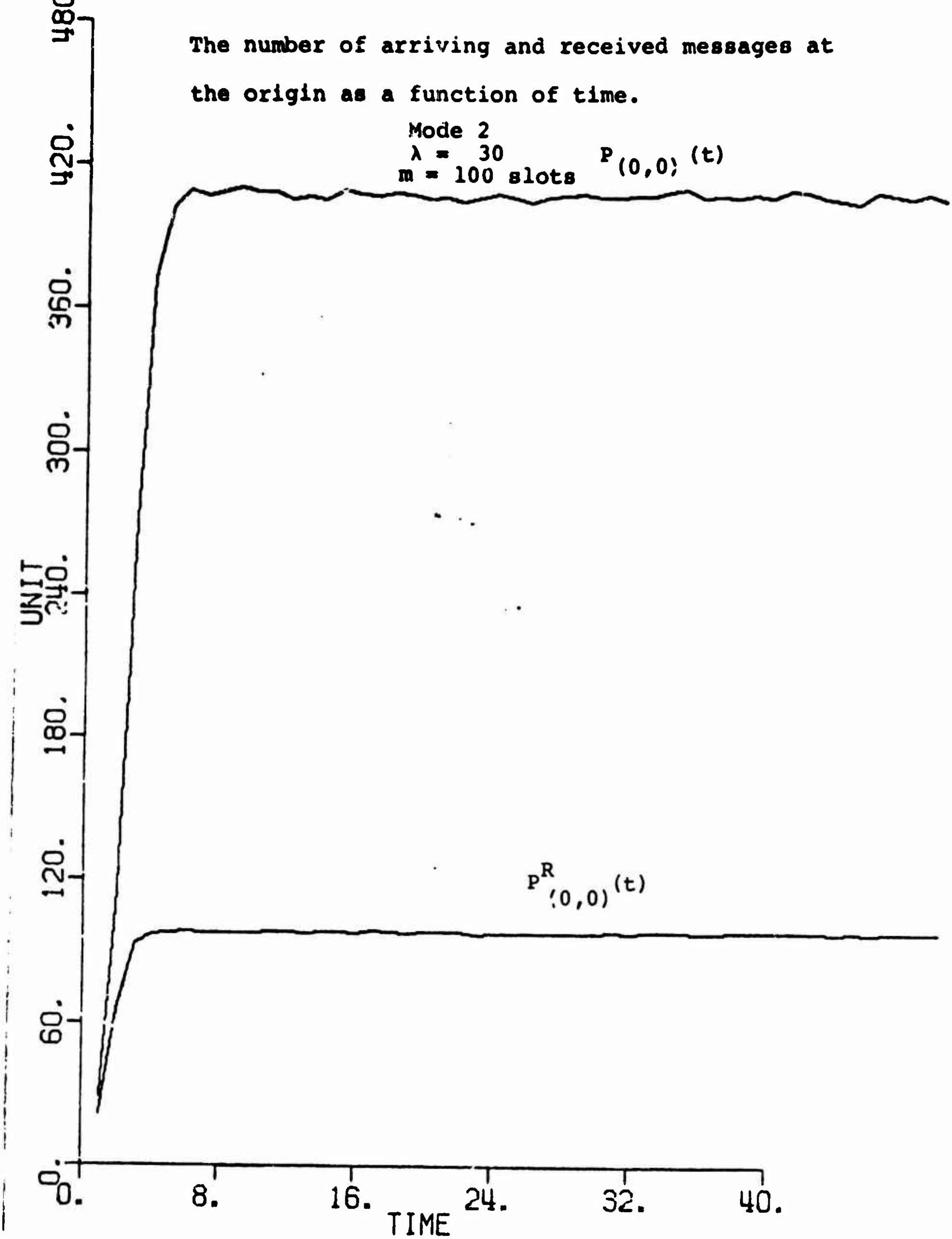

The number of arriving and received messages at
the origin as a function of time.
Mode 2
λ = 30
m = 100 slots
P(0,0) (t)
P^R(0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 8

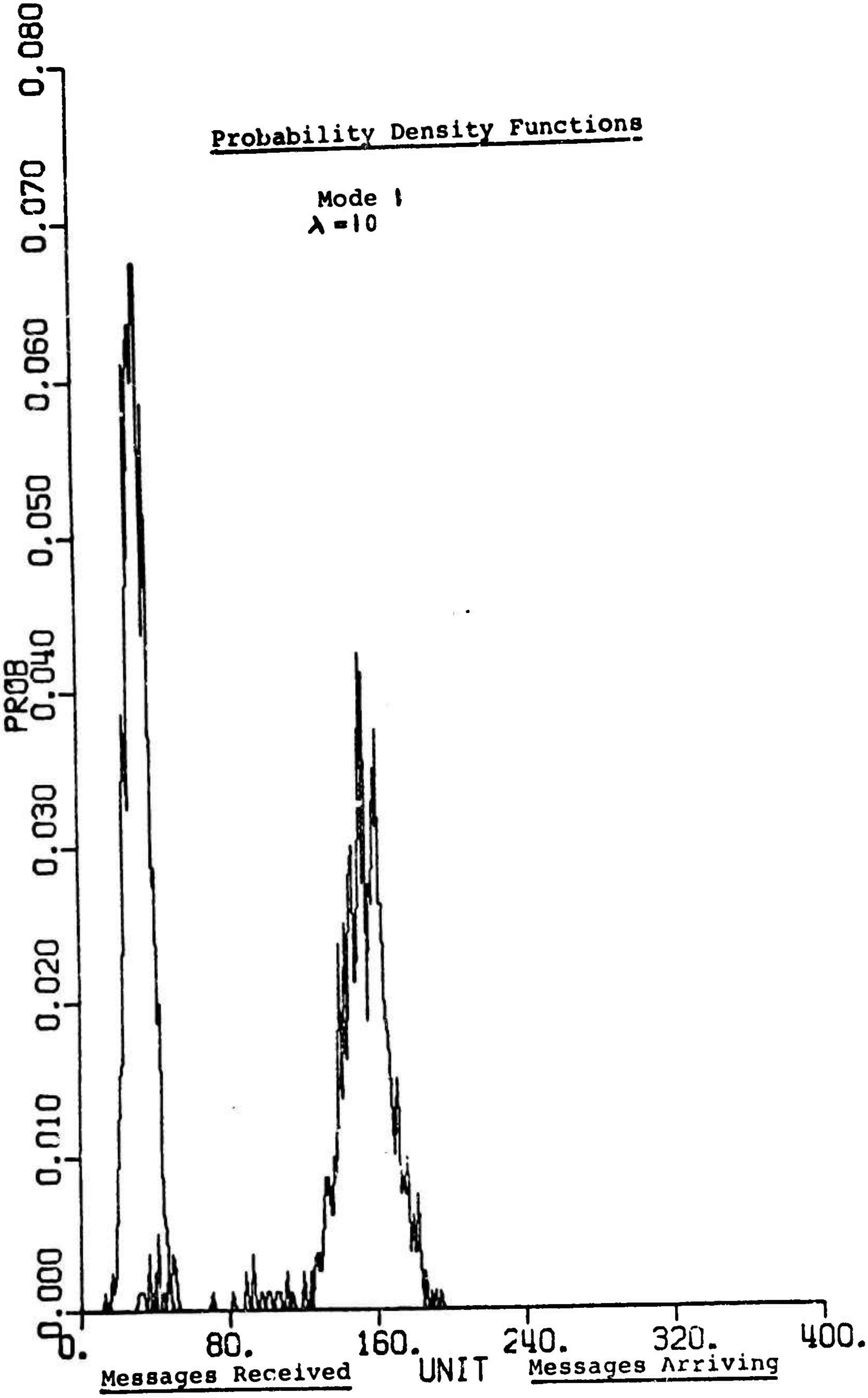

Probability Density Functions
Mode 1
λ =10
PROB
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
0.
80.
160.
240.
320.
400.
Messages Received
UNIT
Messages Arriving

FIGURE 9

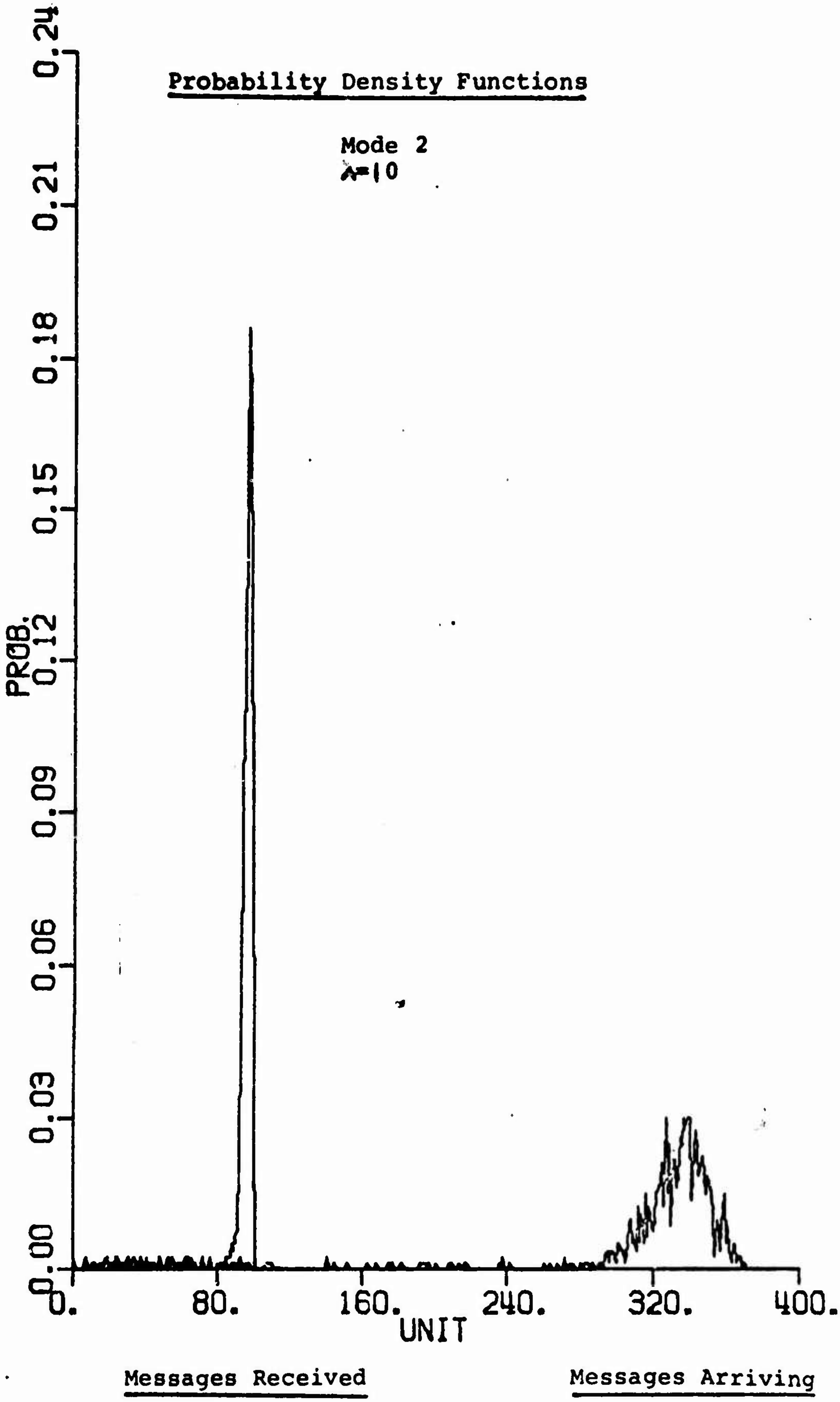

Probability Density Functions
Mode 2
λ=10
PROB.
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
0.
80.
160.
240.
320.
400.
UNIT
Messages Received
Messages Arriving

FIGURE 10

9. DYNAMICS OF A SINGLE MESSAGE ON ROUTE

In this section, we will develop the theoretical basis for a compute analysis of the dynamics of a single message originating at a repeater in the net and attempting to reach the ground station at the origin. The equations derived are directed towards a computer analysis. Let us assume that the given message originates at a repeater with coordinates (i,j) at time t. If the incoming and acceptance numbers at (k,j) at time t are respectively $X_{(i,j)}(t)$ and $X^A_{(i,j)}(t)$, we assume the given message is one of the $X_{(i,j)}(t)$ messages. Furthermore, we assume that each of the $X_{(i,j)}(t)$ messages is equally likely to be one of the accepted messages. Under these assumptions, it follows that at (i,j), there are two types of messages which have arrived. The first type is one message (the given one), the second type are $X_{(i,j)}(t)-1$ messages. The probability of acceptance at (i,j) is given by the hypergeometric probability density function:

$$\frac{\binom{X_{(ij)}(t)-1}{X^A_{(ij)}(t)-1}}{\binom{X_{(ij)}(t)}{X^A_{(ij)}(t)}} \tag{22}$$

At each repeater on every path to the ground station the same analysis applies. At any given repeater, on the path, say with coordinates (k,e) there may be several copies of the original message which arrives.

Suppose (k,e) is on a path from (i,j) to (0,0) and the number of paths from (i,j) to (k,e) is w. Then at (k,e), at time t plus the distance from (i,j) to (k,e), either 0, 1, 2,..., up to w copies of the message may arrive. If d is the distance from (i,j) to (k,e) and at time (t + d), $X_{(k,e)}(t+d)$ and $X^A_{(k,e)}(t+d)$ messages respectively arrive and are accepted then we can compute the probability that exactly Z copies of the original messages are accepted. The computation of the required probabilities is a direct extension of

P{exactly Z copies of original message is accepted at (k,e) at time t + d/ν copies are amongst the arrivals}

$$= \frac{\binom{\nu}{Z}\binom{X_{(k,e)}(t+d)-\nu}{X^A_{(k,e)}(t+d)-Z}}{\binom{X_{(k,e)}(t+d)}{X^A_{(k,e)}(t+d)}} \quad ; \quad Z = 0, 1, 2, \ldots, \nu.$$

Equation(25) is valid at every repeater along every path from (i,j) to (0,0), and in particular at he origin. The only ingredient needed to apply the equations to a computer analysis and generate numerical values is a formula for the probability that exactly ν copies of the message arrive at each repeater. This formula can be obtained recursively using the idea of isodesic line and wedge joint density functions as developed in Section 7.

If a single copy of the given message is accepted at its origination repeater, it is:

a) repeated to each of two repeaters one unit closer to the origin if it is not on an axis;

b) repeated to the one repeater one unit closer to the origin if it is on an axis.

We will focus only on (a) since (b) is essentially identical as far as the analysis is concerned. The message, when accepted at its origination, is then repeated to repeaters at (i-1,j-1) and (i-1,j). Acceptances at (i-1,j-1) and (i-1,j) are determined according to Equation (23) The isodesic line joint density of receptions and acceptances are computed at (i-1,j-1) and (i-1,j). This joint density then determines arrivals and acceptances at (i-2,j-2), (i-2,j-1) and (i-2,j). The process then continues recursively until all computations are carried out at the origin.

9.1 Outline of Computer Analysis

In our computer analysis we used the above results to compute the probability distributions and mean value of the number of copies accepted at the origin of a single message which originates at distance of 5, 4, 3, 2, 1, 0 units from the ground station. For convenience and realism of the numerical results, we selected each originating repeater to have the maximum number of paths to the origin. The coordinate system we used for these calculations is given in Figure11, below:

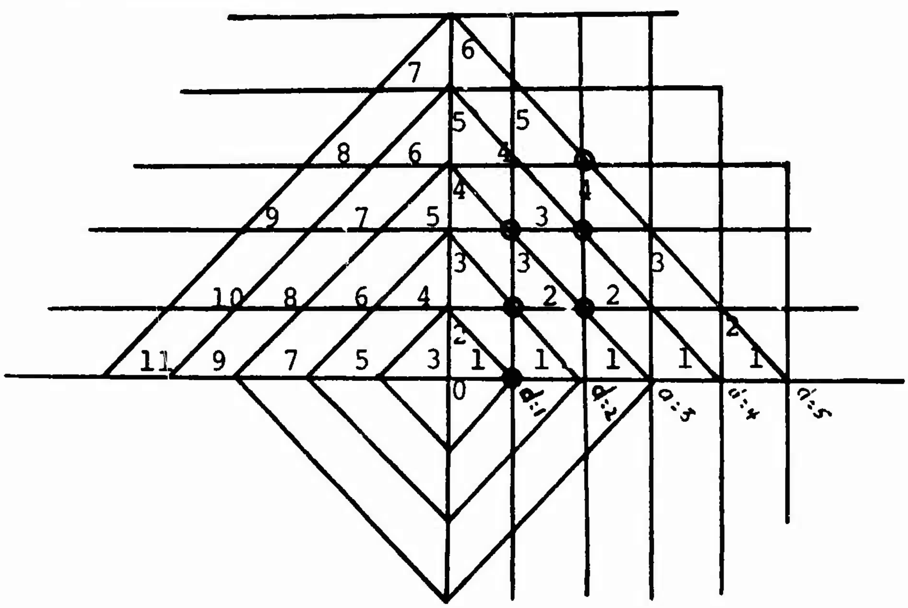


FIGURE 11

The repeaters selected for originating messages at distances 5,4,3,2,1,0 are respectively at (5,4), (4,3), (3,3), (2,2), (1,1), (0,0). The routes are designated in Figure 12, and the maximum number of copies of an originating message which can be received along each repeater on the route is given in Table 1 below. Note that the maximum number of possible copies is given by the number of paths from an originating repeater to the receiving repeater. Note in Table 1 that no copies can be received at a repeater further from the origin than the originator.

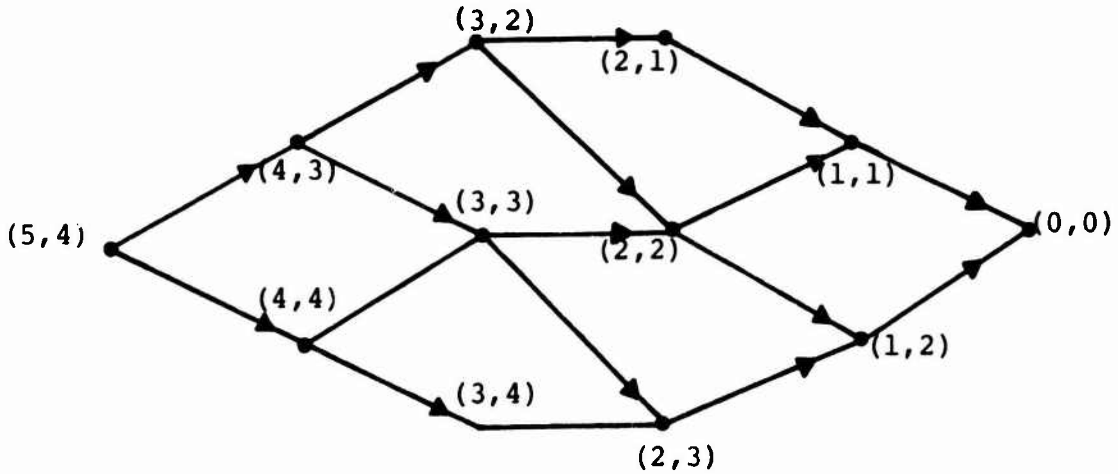


Fig. 12 Routing From (5,4) to (0,0)

	(0,0)	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(3,2)	(3,3)	(3,4)	(4,3)	(4,4)	(5,4)
(0,0)	1	1	1	1	2	1	3	3	1	6	4	10
(1,1)		1	0	1	1	0	2	1	0	3	1	4
(1,2)			1	0	1	1	1	2	1	3	3	6
(2,1)				1	0	0	1	0	0	1	0	1
(2,2)					1	0	1	1	0	2	1	3
(2,3)						1	0	1	1	1	2	3
(3,2)							1	0	0	1	0	1
(3,3)								1	0	1	1	2
(3,4)									1	0	1	1
(4,3)										1	0	1
(4,4)											1	1
(5,4)												1

Table 1 Maximum Number of Copies - Between Two Repeaters

A. The Equation for $Z_0(j;t)$

Clearly $Z_0(j;t)$ is simply given by

$$Z_0(j;t) = \frac{A_{(0,0)}(1,j;t)}{A_{(0,0)}(0,0;t)} \; ; \; j = 0,\ 1; \quad t = 0,\ 1,\ 2,\ \ldots,\ 40. \tag{24}$$

B. The Equation for $Z_1(j;t)$

$$Z_1(j;t) = \sum_{\mu=0}^{1} \binom{\mu}{j} \cdot \frac{A_{(0,0)}(\mu,j;t)}{A_{(0,0)}(0,0;t)} \cdot f_1^1(\mu;t-1) \tag{25}$$

for $j = 0,\ 1;\quad t = 1,\ 2,\ \ldots,\ 40;$ where

$$f_1^1(j;t) = \frac{A_{(1,1)}(1,j;t)}{A_{(1,1)}(0,0;t)} \; ; \; j = 0,\ 1; \quad t = 0,\ 1,\ 2,\ \ldots,\ 39.$$

C. The Equation for $Z_2(j;t)$

$$Z_2(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^2(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{26}$$

for $j = 0,\ 1,\ 2;\quad t = 2,\ 3,\ \ldots,\ 40;$ where

$$f_2^2(i,j;t) = \sum_{\mu=0}^{1} f_1^2(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 1,\ 2,\ \ldots,\ 39;\quad i = 0,\ 1;\quad j = 0,\ 1;$ where

$$f_1^2(j;t) = \frac{A_{(2,2)}(1,j;t)}{A_{(2,2)}(0,0;t)} \; ; \; t = 0,\ 1,\ 2,\ \ldots,\ 38; \quad j = 0,\ 1.$$

D. The Equation for $Z_3(j;t)$

$$Z_3(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{2} f_3^3(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{27}$$

for $t = 3,\ \ldots,\ 40;\quad j = 0,\ 1,\ 2,\ 3;$ where

$$f_3^3(i,j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^3(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\nu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for t = 2, ..., 39; i = 0, 1; j = 0, 1, 2; where

$$f_2^3(i,j;t) = \sum_{\mu=0}^{1} f_1^3(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(2,2)}(\mu,i;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu,j;t)}{A_{(2,3)}(0,0;t)} ;$$

for t = 1, 2, ..., 38; i = 0, 1; j = 0, 1; where

$$f_1^3(j;t) = \frac{A_{(3,3)}(i,j;t)}{A_{(3,3)}(0,0;t)} ; \; t = 0, 1, 2, ..., 37; \; j = 0, 1.$$

E. The Equation for $Z_4(j;t)$

$$Z_4(j;t) = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} f_4^4(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for t = 4, 5, ..., 40; j = 0, 1, 2, ..., 16; where

$$f_4^4(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^4(\nu,\mu,\rho;t-1) \binom{\nu+\mu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for t = 3, 4, ..., 39; i = 0, 1, 2, 3; j = 0, 1, 2, 3; where

$$f_3^4(i,j,k;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^4(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(2,1)}(\mu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\mu+\nu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\nu,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for t = 2, 3, ..., 38; i = 0, 1; j = 0, 1, 2; k = 0, 1; where

$$f_2^4(i,j;t) = \sum_{\mu=0}^{1} f_1^4(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu,j;t)}{A_{(3,3)}(0,0;t)} ;$$

for t = 1, 2, ..., 37; i = 0, 1; j = 0, 1; where

$$f_1^4(j;t) = \frac{A_{(4,3)}(1,j;t)}{A_{(4,3)}(0,0;t)} ; \; t = 0, 1, ..., 36; \; j = 0, 1. \qquad (28)$$

The numbers in Table 1 give the upper limits of the summation for the possible copies of messages which can be received at each repeater of a single message originating at a repeater further from the origin but within the net of Figure 12. With the selected net and the numbers of Table 1, we can use the results of section 12 to obtain numerical data.

At time zero a random number of messages has arrived at each repeater. To compute the distribution of copies arriving at (0,0) from (5,4) we assume one of the messages arriving at (5,4) is singled out and followed along the route using the hypergeometric analysis of section 12. The procedure was used for $t = 0, 1, 2, \ldots, 40$ in conjuction with the random Poisson number generator developed and discussed earlier.

Specifically we seek to compute the five numbers:

$Z_0(j;t)$; $j = 0, 1$; $t = 0, 1, 2, \ldots, 40$;

$Z_1(j;t)$; $j = 0, 1$; $t = 1, 2, \ldots, 40$;

$Z_2(j;t)$; $j = 0, 1, 2$; $t = 2, 3, \ldots, 40$;

$Z_3(j;t)$; $j = 0, 1, 2, 3$; $t = 3, 4, \ldots, 40$;

$Z_4(j;t)$; $j = 0, 1, 2, 3, 4, 5, 6$; $t = 4, 5, 6, \ldots, 40$;

$Z_5(j;t)$; $j = 0, 1, 2, 3, \ldots, 10$; $t = 5, 6, \ldots, 40$;

where $Z_k(j;t)$ is the probability that exactly j copies of a message originating at a repeater at distance k at time t-k, are accepted at the origin at time t. For the computer analysis we considered one repeater at each of the distances, as in Figure 11. The maximum j values are given by the first row of Table 1. Using the hypergeometric analyses the following equation can be used to compute each of the $Z_k(j;t)$; as a function of:

1) λ = mean number of originations at each repeater.
2) m = number of slots fixed at 100.
3) Each of two capture modes 1 and 2.

For ease of notation we denote:

$$A_{(ij)}(w,X;t) = \binom{X_{(ij)}(t)-w}{X^A_{(ij)}(t)-X}.$$

F. The Equation for $Z_5(j;t)$

(6.6) $$Z_5(j;t) = \sum_{\mu=0}^{4} \sum_{\nu=0}^{6} f_5^5(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for $t = 5, 6, \ldots, 40$; $j = 0, 1, 2, \ldots, 10$; where

$$f_5^5(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{3} \sum_{\rho=0}^{3} f_4^5(\nu,\mu,\rho;t-1) \binom{\mu+\nu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for $t = 4, 5, 6, \ldots, 39$; $i = 0, 1, 2, 3, 4$; $j = 0, 1, 2, \ldots, 6$;

where

$$f_4^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^5(\nu,\mu,\rho;t-1) \binom{\nu}{i} \binom{\nu+\mu}{j} \binom{\mu+\rho}{k} \cdot \frac{A_{(2,1)}(\nu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\nu+\mu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu+\rho,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 3, \ldots, 38$; $i = 0, 1$; $j = 0, 1, 2, 3$; $k = 0, 1, 2, 3$;

where

$$f_3^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{1} f_2^5(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu+\nu,j;t)}{A_{(3,3)}(0,0;t)} \cdot \frac{A_{(3,4)}(\nu,k;t)}{A_{(3,4)}(0,0;t)} ;$$

for $t = 2, 3, \ldots, 37$; $i = 0, 1$; $j = 0, 1, 2$; $k = 0, 1$; where

$$f_2^5(i,j;t) = \sum_{\mu=0}^{1} f_1^5(\mu,t-1) \binom{\mu}{i} \binom{\nu}{j} \cdot \frac{A_{(4,3)}(\mu,i;t)}{A_{(4,3)}(0,0;t)} \cdot \frac{A_{(4,4)}(\mu,j;t)}{A_{(4,4)}(0,0;t)} ;$$

for t = 1, 2, 3, ..., 36; i = 0, 1; j = 0, 1; where

$$f_1^5(j;t) = \frac{A_{(5,4)}(1,j;t)}{A_{(5,4)}(0,0;t)}; \quad t = 0, 1, 2, \ldots, 35; \quad j = 0, 1.$$

9.2 Probability of at Least One Message Getting Through

The first set of curves, Figures13-18, plot the probability of at least one message getting through as a function of the mean number of originations at each repeater. There is one set of curves for each unit of distance d ranging from 0 to 5. Each figure contains one curve for mode 1 and one curve for mode 2. The number of slots was fixed at 100. The data for the curves is summarized in Table 2 below.

	λ=1		λ=3		λ=5	
distance	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.398	.589	.285	.421	.264	.431
1	.243	.434	.192	.273	.117	.321
2	.355	.614	.166	.341	.129	.326
3	.428	.695	.164	.358	.119	.285
4	.613	.874	.250	.534	.159	.271
5	.740	.967	.341	.692	.157	.198

Table 2: Probability That at Least One Message Gets Through

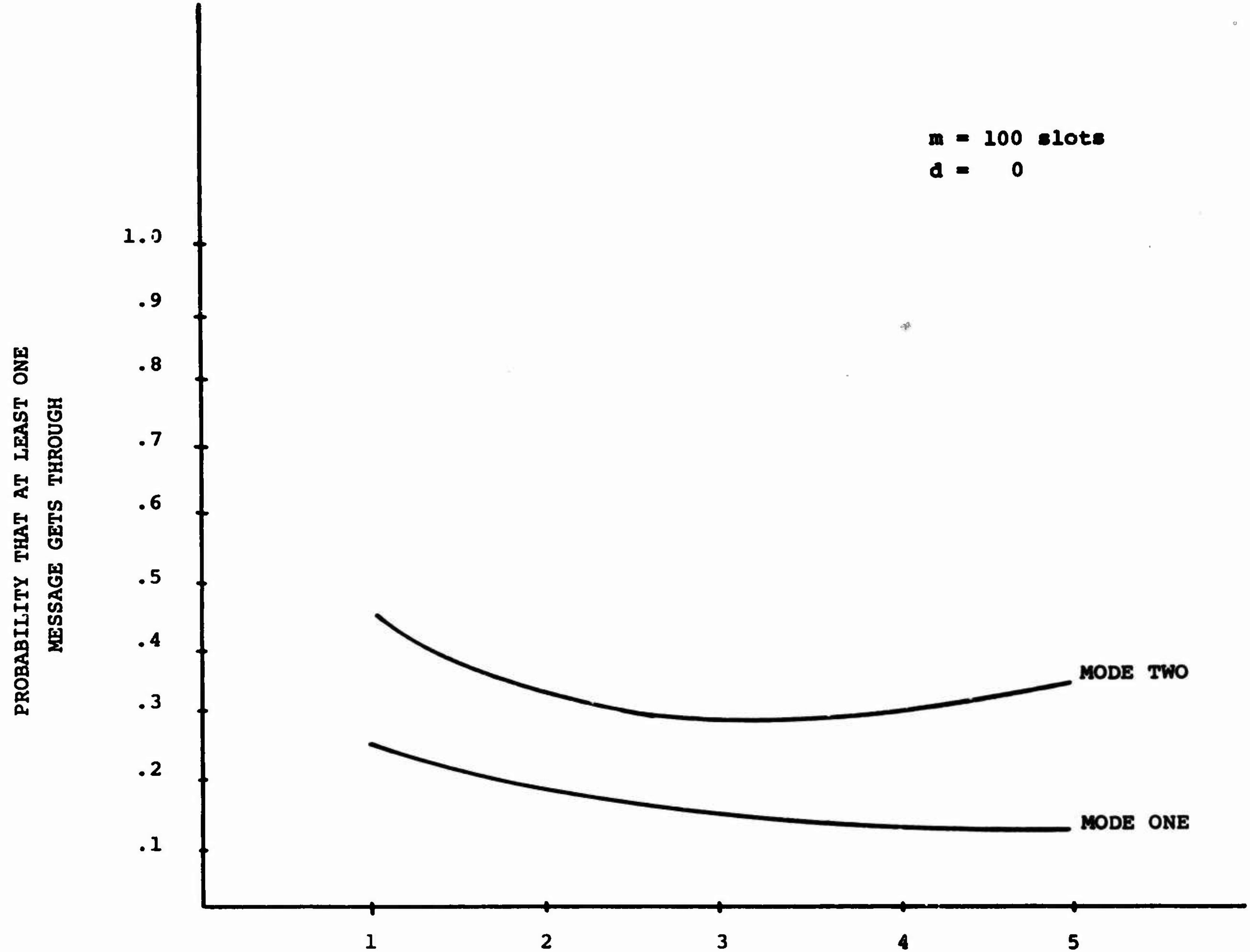

m = 100 slots
d = 0
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
MODE TWO
MODE ONE
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 0)

FIGURE 13

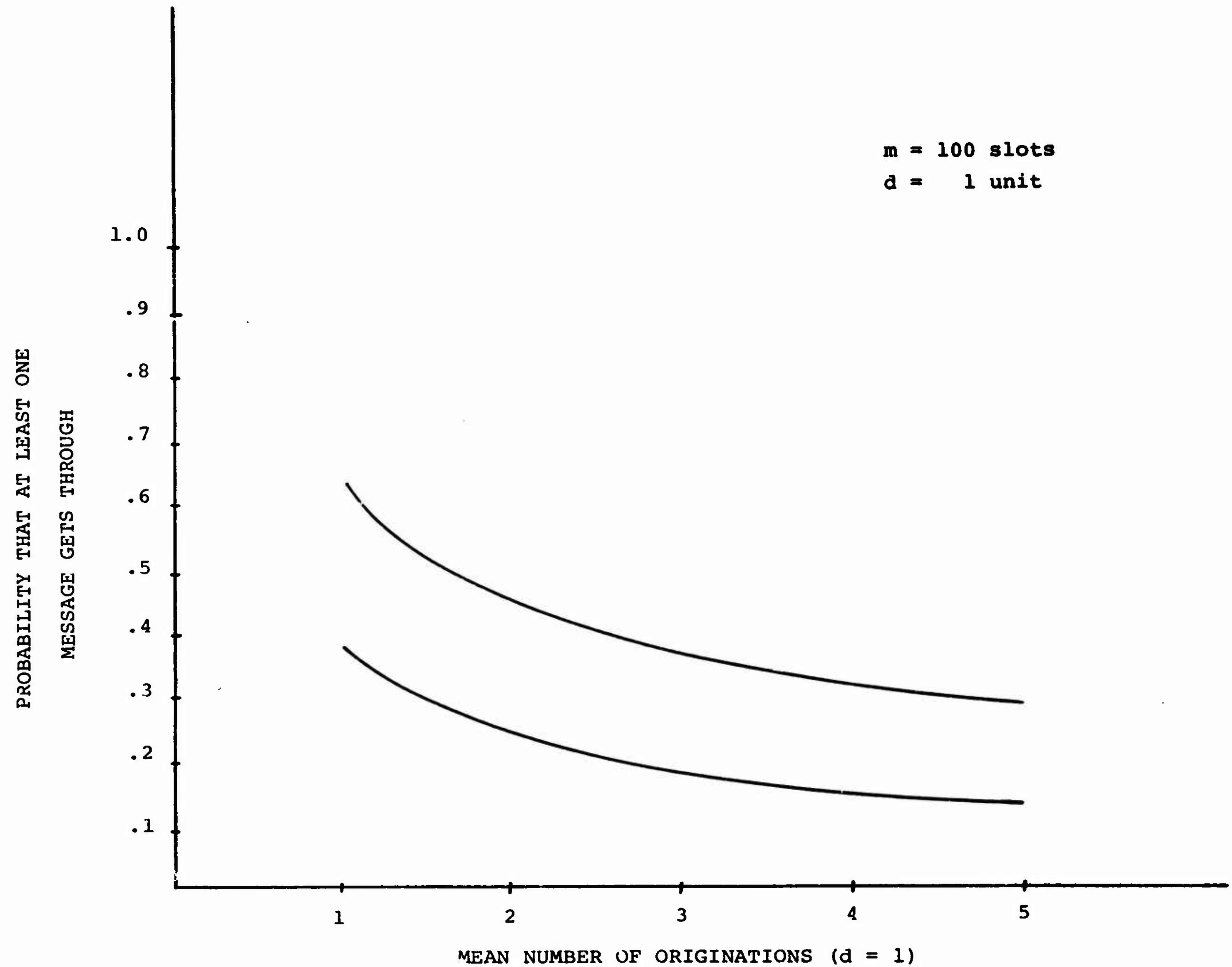

m = 100 slots
d = 1 unit
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 1)

FIGURE 14

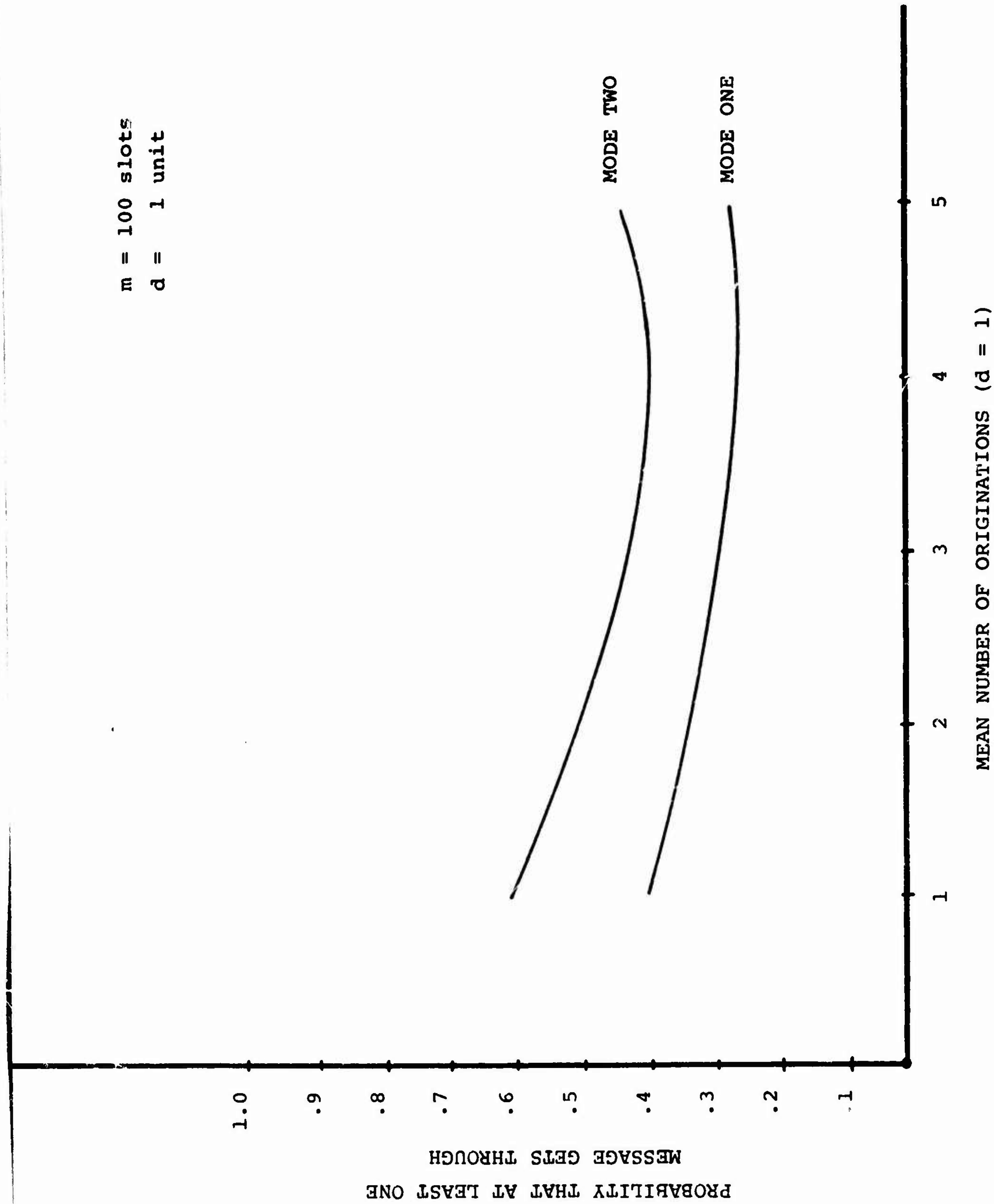

m = 100 slots
d = 1 unit
MODE TWO
MODE ONE
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 1)

FIGURE 15

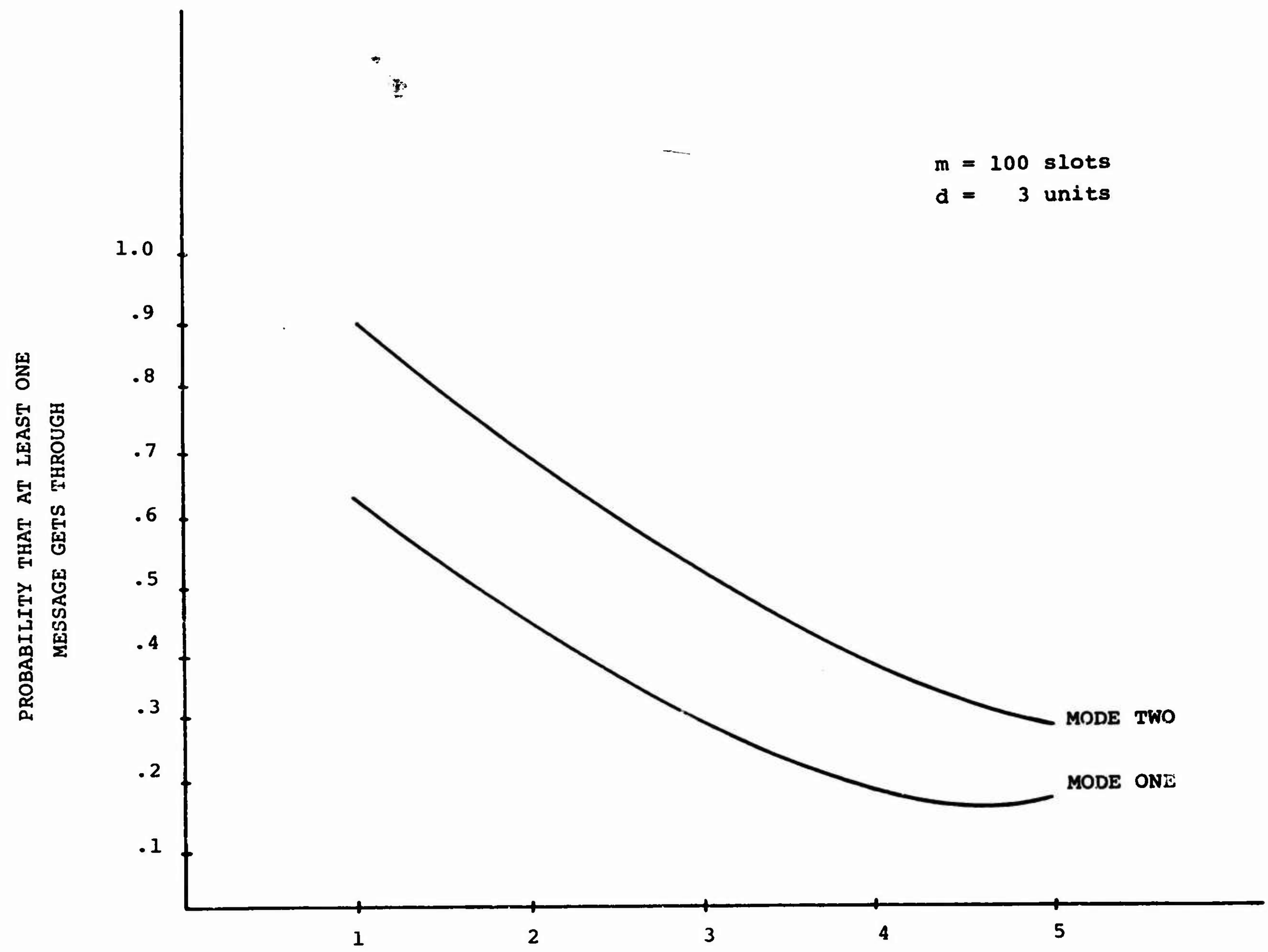

m = 100 slots
d = 3 units
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
MODE TWO
MODE ONE
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 3)

FIGURE 16

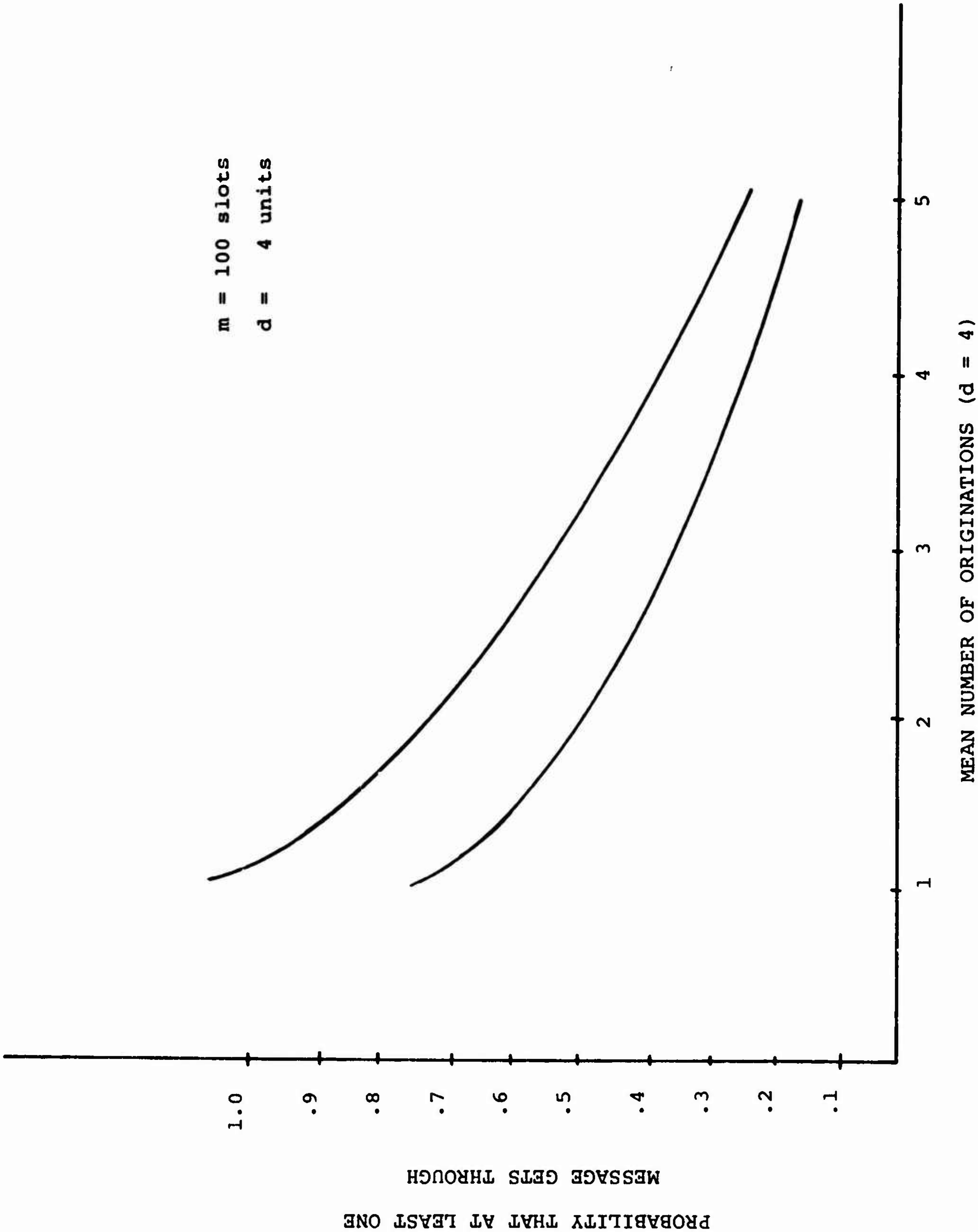

m = 100 slots
d = 4 units
MEAN NUMBER OF ORIGINATIONS (d = 4)
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

FIGURE 17

Network Analysis Corporation

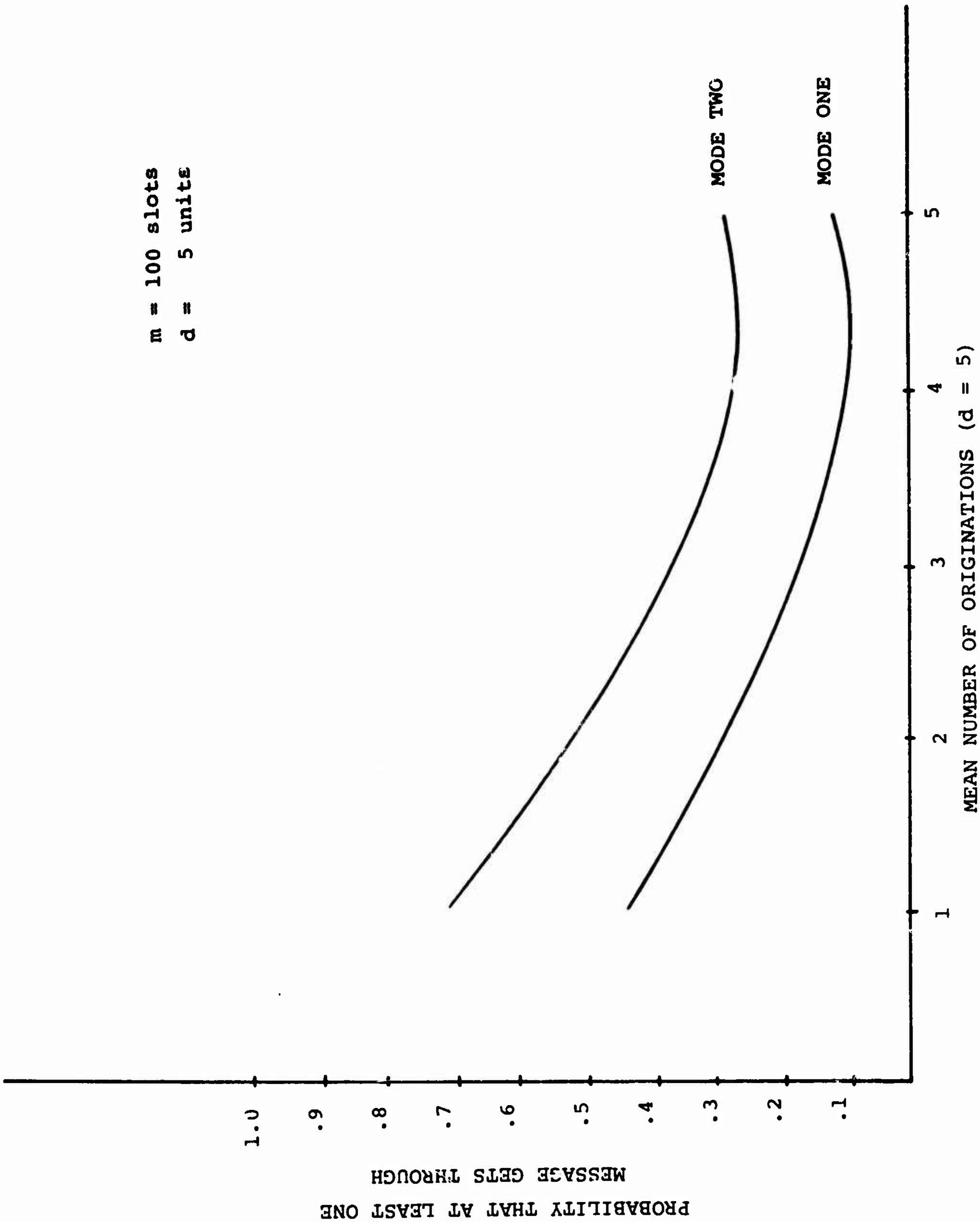


FIGURE 18

9.3 Distribution of Message Explosion as a Function of Slot Size and Mean Number of Originations

The equations for message explosion derived earlier in the report were used to obtain numerical data for message explosion. The results of the numerical analysis follow in Tables 3 through 26 and Figures 19,20 , and 21.

9.4 Distributions of Copies Getting Through as a Function of Slot Size and Mean Originations

Tables 3-26 contain the probability distributions for the number of copies of a single message which are received at the origin (ground stations) for each distance (d=0,2,3,4,5,) of origination of the message. The tables vary according to mode (each of two modes), mean number of messages originating at each repeater (λ=1,3,5), and each of four slot sizes (m=25, 50, 75, 100). This produces a total of 4 x 3 x 2 = 24 tables.

In table 27, we summarize the results of the twenty-four tables by considering only the probability that at least one copy of the message gets through as a function of distance and the three parameters; mode, mean, and slot size.

The results of table 27 are presented pictorially in figures 19, 20, and 21 for distances of zero, two and four respectively of origination of the message.

Mode 1, λ= 1, m = 25

distance \ #	0	1	2	3
0	.786	.214		
1	.921	.079		
2	.900	.096	.004	
3	.897	.097	.005	
4	.831	.153	.015	.001
5				

Table 3

Mode 2, λ= 1, m = 25

distance \ #	0	1	2	3	4
0	.689	.311			
1	.834	.166			
2	.759	.219	.021		
3	.716	.245	.037	.002	
4	.554	.324	.103	.018	.002
5					

Table 4

Mode 1, $\lambda = 3$, $m = 25$

distance \ #	0	1	2
0	.805	.195	
1	.942	.058	
2	.952	.047	.001
3	.966	.033	.001
4	.953	.045	.002
5			

Table 5

Mode 2, $\lambda = 3$, $m = 25$

distance \ #	0	1	2	3
0	.739	.261		
1	.890	.102		
2	.887	.107	.006	
3	.892	.101	.007	
4	.826	.153	.019	.001
5				

Table 6

Mode 1, λ = 5, m = 25

distance \ #	0	1	2
0	.813	.187	
1	.951	.049	
2	.963	.036	.001
3	.979	.021	.001
4	.977	.022	.001
5			

Table 7

Mode 2, λ = 5, m = 25

distance \ #	0	1	2
0	.753	.247	
1	.914	.086	
2	.916	.080	.004
3	.930	.066	.004
4	.900	.091	.008
5			

Table 8

Mode 1, λ = 1, m = 50

distance \ #	0	1	2	3
0	.755	.245		
1	.878	.122		
2	.823	.166	.001	
3	.796	.185	.019	.001
4	.661	.272	.059	.007
5				

Table 9

Mode 2, λ = 1, m = 50

distance \ #	0	1	2	3	4	5
0	.605	.395				
1	.736	.264				
2	.602	.338	.060			
3	.531	.356	.103	.010		
4	.306	.360	.232	.083	.017	.002
5						

Table 10

Mode 1, λ = 3, m = 50

distance \ #	0	1	2
0	.788	.212	
1	.926	.074	
2	.921	.076	.003
3	.926	.070	.006
4	.882	.108	.009
5			

Table 11

Mode 2, λ = 3, m = 50

distance \ #	0	1	2	3	4
0	.709	.291			
1	.860	.140			
2	.816	.170	.014		
3	.800	.178	.021	.001	
4	.875	.258	.058	.008	.001
5					

Table 12

Mode 1, $\lambda = 5$, $m = 50$

distance \ #	0	1	2
0	.794	.206	
1	.937	.063	
2	.441	.057	.002
3	.954	.044	.002
4	.935	.062	.004
5			

Table 13

Mode 2, $\lambda = 5$, $m = 50$

distance \ #	0	1	2	3
0	.733	.267		
1	.890	.110		
2	.868	.124	.008	
3	.870	.120	.010	
4	.791	.180	.027	.002
5				

Table 14

Mode 1, $\lambda = 1$, m = 75

distance \ #	0	1	2	3	4
0	.711	.289			
1	.831	.169			
2	.742	.235	.023		
3	.868	.269	.043	.002	
4	.512	.342	.121	.023	.003
5					

Table 15

Mode 2, $\lambda = 1$, m = 75

distance \ #	0	1	2	3	4	5	6
0	.526	.474					
1	.655	.345					
2	.486	.408	.106				
3	.392	.408	.175	.025			
4	.184	.304	.295	.158	.049	.008	.001
5							

Table 16

Mode 1, $\lambda = 3$, $m = 75$

distance \ #	0	1	2	3
0	.782	.218		
1	.913	.087		
2	.895	.100	.005	
3	.891	.103	.006	
4	.818	.161	.019	.001
5				

Table 17

Mode 2, $\lambda = 3$, $m = 75$

distance \ #	0	1	2	3	4
0	.674	.326			
1	.815	.185			
2	.750	.225	.025		
3	.725	.233	.040	.002	
4	.561	.313	.103	.020	.002
5					

Table 18

Mode 1, $\lambda = 5$, $m = 75$

distance \ #	0	1	2
0	.788	.212	
1	.929	.071	
2	.924	.074	.003
3	.932	.065	.003
4	.894	.098	.008
5			

Table 19

Mode 2, $\lambda = 5$, $m = 75$

distance \ #	0	1	2	3
0	.711	.289		
1	.863	.137		
2	.819	.168	.013	
3	.818	.163	.019	.001
4	.703	.239	.051	.006
5				

Table 20

Mode 1, λ = 1, m = 100

distance \ #	0	1	2	3	4	5	6
0	.602	.398					
1	.757	.242					
2	.645	.302	.053				
3	.572	.325	.093	.013			
4	.387	.369	.184	.050	.008		
5	.260	.318	.250	.122	.039	.008	.003

Table 21

Mode 2, λ = 1, m = 100

distance \ #	0	1	2	3	4	5	6	7	8
0	.411	.589							
1	.566	.434							
2	.386	.433	.181						
3	.305	.399	.240	.056					
4	.126	.244	.308	.215	.087	.019	.002		
5	.003	.110	.200	.248	.211	.126	.054	.014	.001

Table 22

Mode 1, λ = 3, m = 100

distance \ #	0	1	2	3	4
0	.715	.285			
1	.808	.192			
2	.834	.143	.023		
3	836	.147	.016		
4	.750	.211	.035	.004	
5	.659	.267	.063	.010	.001

Table 23

Mode 2, λ = 3, m = 100

distance \ #	0	1	2	3	4	5	6
0	.579	.421					
1	.727	.273					
2	.659	.279	.062				
3	.642	.276	.073	.008			
4	.466	.338	.149	.040	.007	.001	
5	.308	.333	.223	.098	.030	.007	.001

Table 24

Mode 1, λ = 5, m = 100

distance \ #	0	1	2	3
0	.736	.264		
1	.803	.117		
2	.871	.119	.010	
3	.881	.110	.009	
4	.841	.140	.018	.001
5	.843	.133	.022	.002

Table 25

Mode 2, λ = 5, m = 100

distance \ #	0	1	2	3	4	5
0	.569	.431				
1	.679	.321				
2	.674	.283	.043			
3	.715	.214	.064	.007		
4	.729	.169	.076	.021	.004	
5	.802	.115	.056	.020	.005	.001

Table 26

	λ = 1								λ = 3								λ = 5							
	M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100	
d	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.214	.311	.245	.395	.289	.474	.398	.587	.195	.261	.212	.291	.218	.326	.285	.421	.187	.247	.206	.267	.212	.289	.264	.431
1	.079	.166	.122	.264	.269	.345	.243	.434	.058	.110	.074	.140	.087	.185	.192	.273	.049	.086	.063	.110	.071	.137	.197	.321
2	.100	.241	.177	.398	.258	.514	.355	.614	.048	.113	.079	.184	.105	.250	.166	.341	.037	.084	.059	.132	.076	.181	.129	.326
3	.102	.284	.204	.469	.304	.608	.428	.695	.034	.108	.074	.200	.109	.275	.164	.358	.021	.070	.046	.130	.068	.182	.119	.285
4	.169	.446	.339	.694	.488	.816	.613	.874	.047	.174	.118	.325	.182	.437	.250	.534	.023	.100	.065	.207	.106	.297	.159	.271
5							.740	.997							.341	.692							.157	.198

TABLE 27

THE PROBABILITY OF AT LEAST ONE MESSAGE GETTING THROUGH

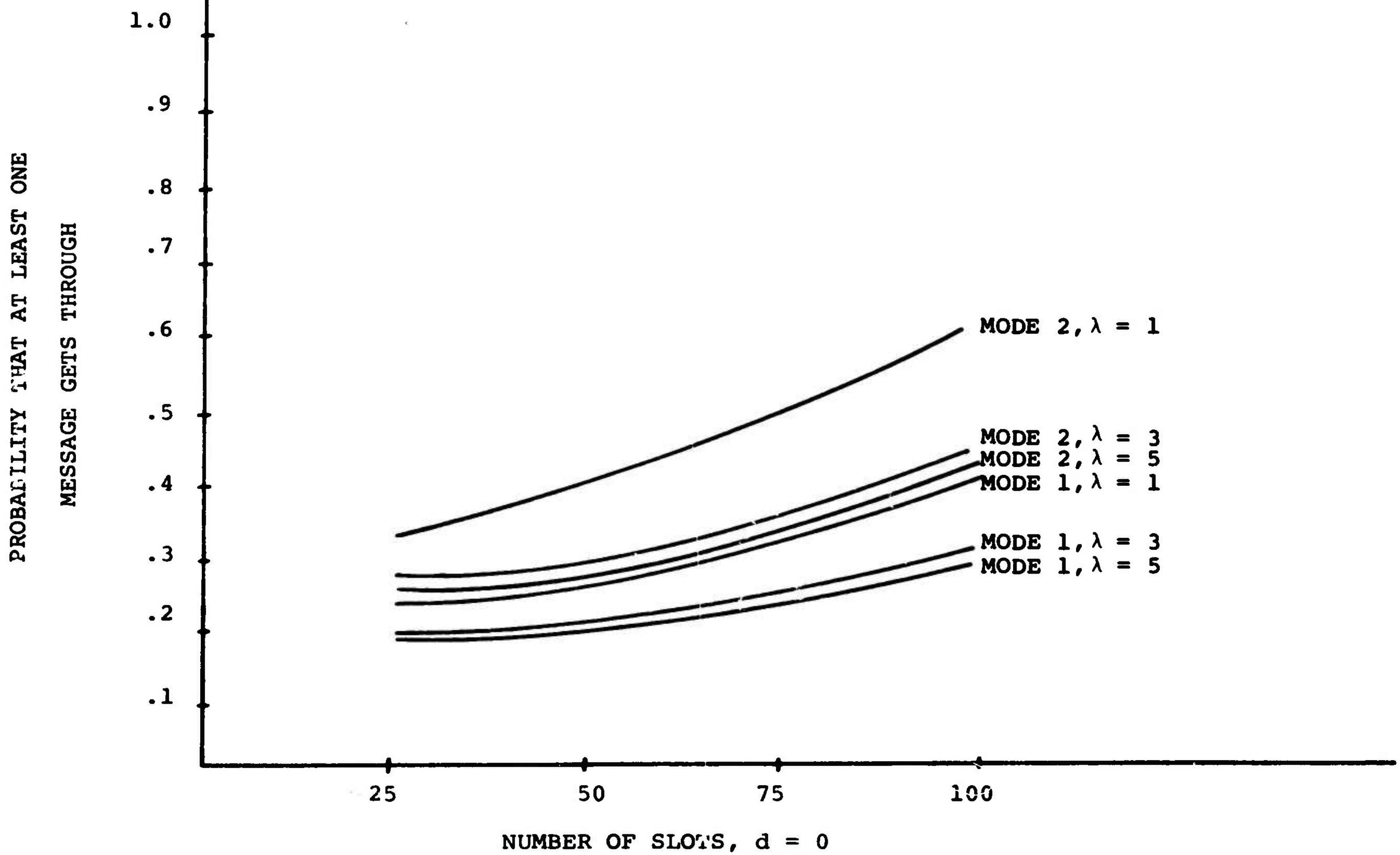

PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
MODE 2, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 1
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
NUMBER OF SLOTS, d = 0

FIGURE 19

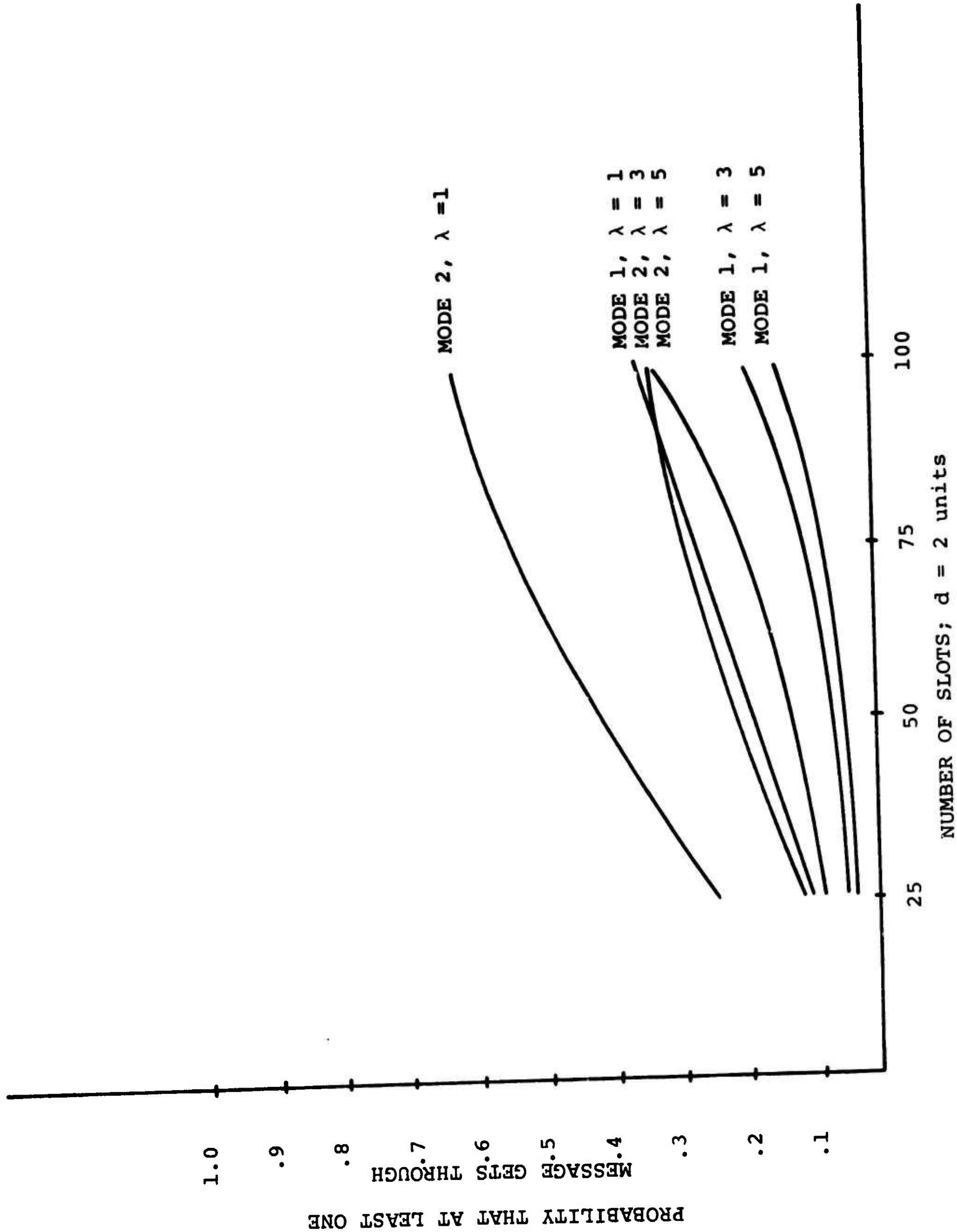

MODE 2, λ =1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
25
50
75
100
NUMBER OF SLOTS; d = 2 units

FIGURE 20

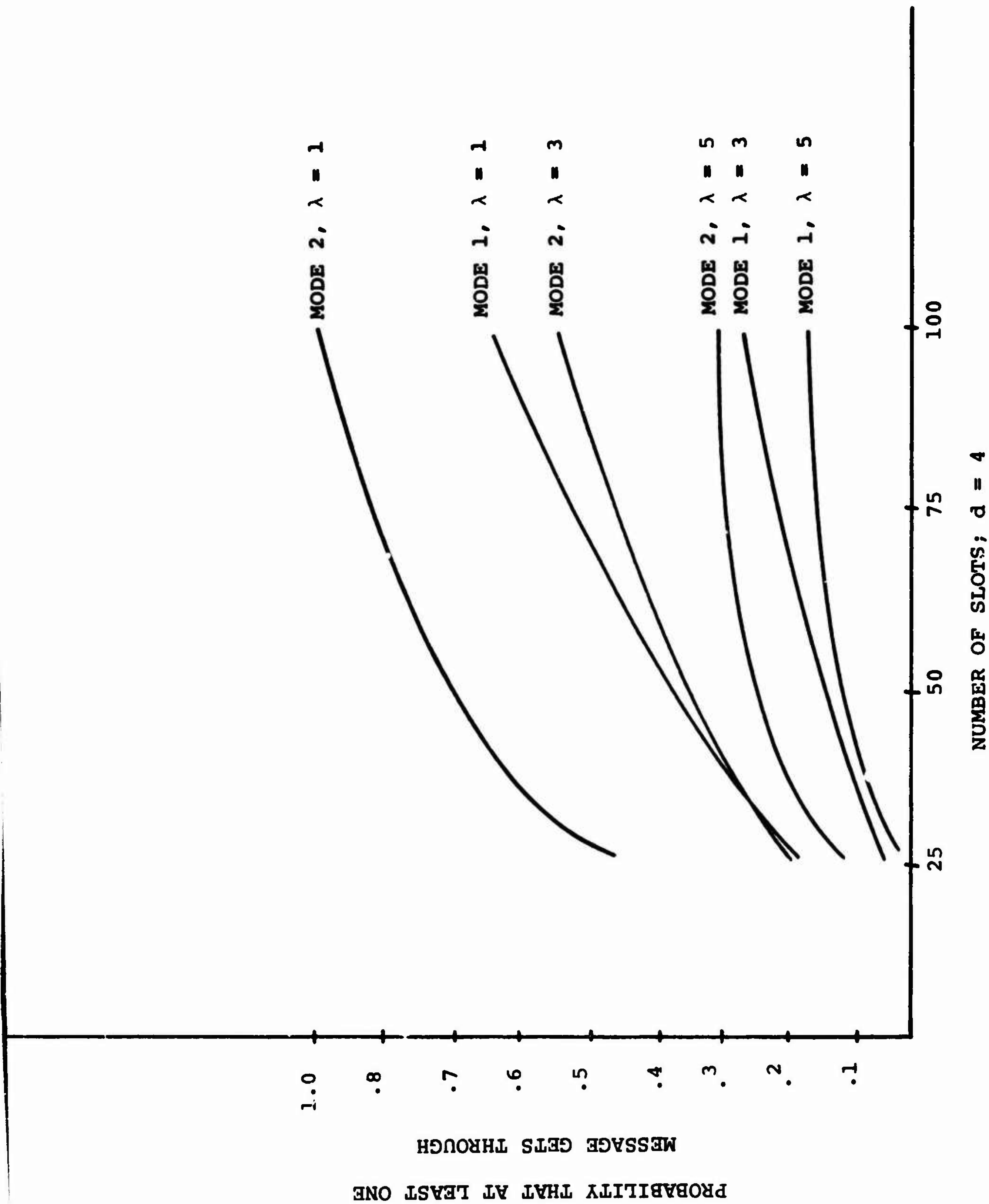

MODE 2, λ = 1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
NUMBER OF SLOTS; d = 4
1.0
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 21

10. SYSTEMS WITH RETRANSMISSIONS

10.1 Retransmission From Source of Origination

We now can extend the scope and generality of the basic model by including the possibility of retransmission of messages which are erased in random slotting or in technical language, "not captured". The notion of retransmission can be modeled in at least two ways. The first way, considered in this section is that when a message is wiped out, it is retransmitted from its source of origination after a fixed delay time J(d) which depends on the distance of origination from the ground station. The second type of retransmissions which we shall consider are retransmissions which occur at the point (repeater) of erasure at one time unit after wipeout. The latter type will be analyzed in Section 7.

To begin to develop programmable equations, we need some notation:

Let $X_{(i,j),(u,v)}(t)$ be the number of messages arriving at (i,j) at time t which originated at (u,v) at time t - (u-i). (recall that the first coordinate refers to distance from the origin).

Let $Y_{(i,j),(u,v)}(t)$ be the number of messages accepted at (i,j) at time t which originated at (u,v) at time t - (u-i).

Let $X_{(i,j)}(t)$ be the number of messages arriving at (i,j) at time t.

Let $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t.

Let $Z_{(i,j)}(t)$ be the number of retransmissions at (i,j) at time t.

We develop equations to compute $Z_{(i,j)}(t)$. To begin, we have the following assumptions.

$$X_{(i,j),(i,j)}(t) = \text{Poisson Variate} + Z_{(i,j)}(t) \quad (30)$$

$Y_{(i,j),(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. (31)

Obviously;

$$X_{(i,j)}(t) = \sum_{(u,v)\in I(i,j)} X_{(i,j),(u,v)}(t) \quad (32)$$

Where $I_{(i,j)}$ is the set of all repeaters which arein the input set to (i,j), i.e., all repeaters for which there exists <u>directed</u> path <u>to</u> (i,j).

If we assume that each arriving message is equally likely to be accepted, it follows that:

$$Y_{(i,j),(u,v)}(t) = Y_{(i,j)}(t)\ \frac{X_{(i,j),(u,v)}(t)}{X_{(i,j)}(t)}, \quad (33)$$

and that

$$Z_{(i,j)}(t) = \sum_{(u,v)\text{ in } O(i,j)} \left[X_{(u,v),(i,j)}(t-J(i)) - Y_{(u,v),(i,j)}(t-J(i))\right], \quad (34)$$

where $O_{(i,j)}$ is the "outward set of (i,j)" defined as the set of all repeaters which receive messages from (i,j), including (i,j) itself. The quantity J(i) is the delay factor which depends on i the distancefrom the origin, but is independent of the source of message wipeout.

The only part of the equation (9.5) which is not accounted for is $X_{(u,v),(i,j)}(t)$. The next equation is obvious:

$$X_{(i,j),(u,v)}(t) = \sum_{(K,W)\text{ in } II_{(i,j)}} Y_{(K,W),(u,v)}(t-1); \quad (35)$$

where $II_{(i,j)}$ is the immediate input set to (i,j), that is those repeaters one unit of distance further than (i,j) which repeat to (i,j) in one time unit.

The equations (30) to (35) were successfully programmed for the square grid net of repeaters at the lattice points of the Euclidean plane, five units or less distance from the origin. This net has a total of 61 repeaters. We do not include numerical data since many time points must be computed to obtain meaningful steady state results. This can be done at any time since the program is available.

10.2. Retransmissions at Point of Loss

In this model we assume that retransmissions of wiped out messages occur at the point of wipeout one time unit later, independently of where the message originated. For this type of assumption, we need to compute, $Z_{(i,j),(u,v)}(s,t)$ the number of retransmissions at (i,j) at time t of messages which originated at (u,v) at time s, s=0, 1, 2,..., $t-|u-j|$. The quantity Z is computed for repeaters (u,v) in the input set to (i,j). The quantities $Z_{(i,j)}(t)$ and $Z_{(i,j),(u,v)}(t)$ are defined as in Section 13.

Since we are assuming that retransmissions occur at the point of wipeout, to compute delays we must keep track of time and place of origination of messages. We therefore define the quantities $X_{(i,j),(u,v)}(s,t)$ and $Y_{(i,j),(u,v)}(s,t)$ as the number of messages arriving and respectively accepted at (i,j) at time t which originated at (u,v) at time s.

According to our assumptions, the required quantity can be computed from:

$$Z_{(i,j),(u,v)}(s,t) = X_{(i,j),(u,v)}(s,\ t-1) - Y_{(i,j),(u,v)}(s,\ t-1) \quad (36)$$

According to (5), we need to compute X and Y which can be done recursively.

We have:

$$X_{(i,j),(u,v)}(s,t) = \sum_{\substack{(K,M)\text{ in}\\ II(i,j)}} Y_{(K,M),(u,v)}(s,t-1) + Z_{(i,j),(u,v)}(s,t) \qquad (37)$$

and

$$Y_{(i,j),(u,v)}(s,t) = Y_{(i,j)}(t)\,\frac{X_{(i,j),(u,v)}(s,t)}{X_{(i,j)}(t)} \qquad (38)$$

As in the previous section $Y_{(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. The remaining quantity to compute is:

$$X_{(i,j)}(t) = \sum_{\substack{(u,v)\text{ in}\\ II(i,j)\text{ set}}} X_{(i,j),(u,v)}(s,t) + Z_{(i,j)}(t) \qquad (39)$$

These equations have been programmed for the grid of repeaters at the lattice points of the Euclidean plane, as earlier. The program is for the case where a repeater has a single fixed path to the origin or ground station. The program was run for ten time points using a single sample at each point. The numerical results are, therefore, subject to some variability. Some of the results of a single run are given in Tables 28 - 34. As is evident from the tables, saturation of the channel begins early for $\lambda = 5$. In Table 31 for example the probability that a message originating at d = 5 at time 1 gets to the origin with a delay of less than four time units is only about .44. For $\lambda = 5$ the situation deteriorates rapidly with time. To obtain a large set of representative data would require running the program for many time points, probably at least 15 or 20 for different values of λ and slot size. This can be done using the available program.

10.3. Delays and Average Delays as a Function of Distance

We can extend the calculations and analyses described in the previous two sections to include calculations of delay distributions and average delay. In addition to studying delays, we can develop equations to study bottlenecks in a given network. These formulae have been programmed and numerical results can be obtained.

Let $D_{(i,j)}(t)$ be the random variable delay of a message which originates at (i,j) at time t. We assume that the probability that a message is delayed by k-units of time is given by the proportion of

Retransmissions at Point of Wipeout

λ=5, m=100, Model

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	6	6	0	0	6	0
4	6	11	6	1	3	0
5	43	26	18	1	1	0
6		19	19	7	2	0
7		66	29	13	0	0
8		91	47	3	0	0
9		67	67	1	0	0
10		151	64	9	2	0

TABLE 28

λ=5, m=100, Mode 2

t \ d	0	1	2	3	4	5
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	7	2	1	0	0	0
4	4	10	1	0	2	0
5	34	20	2	2	1	0
6		37	2	1	1	0
7		19	5	0	1	0
8		28	6	2	1	1
9		33	18	1	1	0
10		41	4	0	1	1

TABLE 29

Delay Probability Tables for a Message Being

Accepted d units from the Ground Station

λ=5, m=100, Mode 1

A Message Originating at d=5 at time 0.

delay \ dist	0	1	2	3	4	5
0	.159	.271	.524	.933	1.000	1.000
1	.235	.247	.325	.059		
2	.134	.157	.084	.004		
3	.097	.076	.028	.002		
4	.089	.045	.016	.001		
5		.041	.009			
6			.007			

TABLE 30

A Message Origination at d=4 at time 0.

delay \ dist	0	1	2	3	4
0	.249	.454	.727	1.000	1.000
1	.251	.221	.153		
2	.196	.123	.083		
3	.083	.070	.021		
4	.054	.033	.007		
5	.046	.018	.004		
6		.016	.002		
7			.002		

TABLE 31

A Message Originating at d=3 at time 0

dist / delay	0	1	2	3
0	.916	.950	1.000	1.000
1	.036	.031		
2	.023	.010		
3	.013	.004		
4	.004	.002		
5	.003	.001		
6	.002	.001		
7		.001		

TABLE 32

A Message Originating at d=2 at time 0

dist / delay	0	1	2
0	.823	1.000	1.000
1	.170		
2	.004		
3	.002		
4	.001		

TABLE 33

A Message Originating at d=1, at time 0

dist / delay	0	1
0	.778	1.000
1	.183	
2	.038	
3	.001	

TABLE 34

THE NUMBER OF MESSAGES ACCEPTED AT THE ORIGIN

$\lambda = 5$, $m = 100$, mode 2

Originating at d = 5 at t = 0: 3 messages

Time of Acceptance at Origin	Number	Delay
t = 5	.814	0
t = 6	1.130	1
t = 7	.502	2
t = 8	.253	3
t = 9	.161	4
	2.860	

TABLE 35

Originating at d = 5 at t = 1: 6 messages

Time of Acceptance at Origin	Number	Delay
t = 6	2.010	0
t = 7	1.238	1
t = 8	.988	2
t = 9	.805	3
	5.061	

TABLE 36

Originating at d = 5 at t = 2: 2 messages

Time of Acceptance at Origin	Number	Delay
t = 7	.115	0
t = 8	.249	1
t = 9	.256	2
	.620	

TABLE 37

DELAY PROBABILITY TABLES FOR A MESSAGE

BEING ACCEPTED d UNITS FROM THE GROUND STATION

$\lambda = 5$, m = 100, mode 2

A Message Originating at d = 5 at time 0

Dist / Delay	0	1	2	3	4	5
0	.271	.496	.854	1.000	1.000	1.000
1	.377	.360	.140	-	-	-
2	.167	.071	.003	-	-	-
3	.084	.041	.003	-	-	-
4	.054	.018	-	-	-	-
5		.007	-	-	-	-

TABLE 38

A Message Originating at d = 4 at time 0

Dist / Delay	0	1	2	3	4	
0	.407	.452	.792	1.000	1.000	
1	.189	.300	.178	-	-	
2	.216	.178	.029	-	-	
3	.092	.035	.001	-	-	
4	.045	.020	.001	-	-	
5	.028	.008	-	-	-	
6	-	.003	-	-	-	

TABLE 39

A Message Originating at d = 3 at time 0

Dist / Delay	0	1	2	3
0	.511	.962	1.000	1.000
1	.425	.022	-	-
2	.031	.009	-	-
3	.020	.005	-	-
4	.007	.001	-	-
5	.003	.001	-	-
6	.002	-	-	-

TABLE 40

A Message Originating at d = 2 at time 0

Dist / Delay	0	1	2	3
0	.883	.923	1.000	
1	.061	.074	-	
2	.050	.002	-	
3	.003	-	-	
4	[illegible]	-	-	
5	.001	-	-	

TABLE 41

A Message Originating at d = 1 at time 0

Dist Delay	0	1		
0	.800	1.000		
1	.191	-		
2	.005	-		
3	.004	-		

TABLE 42

A Message Originating at d = 0 at time 0

Dist Delay	0	1		
0	1.000			
1	.000			

TABLE 43

A Message Originating at d = 5 at time 1

Dist Delay	0	1	2	3	4	5	
0	.335	.520	.718	.752	.846	1.000	
1	.206	.159	.116	.207	.132	-	
2	.165	.161	.125	.039	.020	-	
3	.134	.081	.024	.002	.002	-	
4	-	.036	.012	.003	-	-	
5	-	-	.003				

TABLE 44

PROBABILITY OF ZERO DELAY VS. DISTANCE OF ORIGINATION

MODE 1

t \ d	0	1	2	3	4	5
0	1.000	.778	.823	.916	.349	.159
1	.346	.329	.458	.125	.091	.123
2	.242	.229	,045	.085	.053	.047
3	.207	.043	.013	.058	.021	.024
4	.177	.033	.025	.021	.013	.037
5	.092	.019	.011	.006	.015	
6	.021	.005	.002	.008		
7	.029	.002	.001			
8	.031	.001				
9	.047					

TABLE 45

MODE 2

t \ d	0	1	2	3	4	5
0	1.000	.800	.883	.511	.407	.271
1	.373	.136	.118	.356	.170	.335
2	.333	.118	.102	.030	.180	.115
3	.100	.044	.033	.154	.066	.218
4	.112	.026	.061	.045	.109	.161
5	.062	.032	.005	.100	.073	
6	.021	.011	.022	.041		
7	.045	.022	.019			
8	.005	.018				
9	.021					

TABLE 46

messages which originate at time t which are delayed by k-units. If enough random samples are taken, this estimate becomes quite good. In notation, define $Y_{(o,o)\ (i,j)\ (s,t)}$ as the number of messages which are accepted at the ground station at time s, which originated at (i,j) at time t. Then,

$$P\{D_{(i,j)}(t)=k\} = \frac{Y_{(o,o),(i,j)}(t+k+i, t)}{O_{(i,j)}(t)}; \ k=0,\ 1,\ 2,\ldots, \qquad (40)$$

When doing computer trials to determine 40 we can average over all 4i repeaters at distance i and compute the delay distribution as a function only of distance to the ground station. i.e.,

$$P\{D_i(t)=k\} = \frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(t)=k\}, \text{ where} \qquad (41)$$

$D_i(t)$ is the random variable delay of a message originating at distance i at time t.

To obtain a time invariant measure, we can average 41 over time and obtain the probability distribution of the random variable D_i and its expectation, given by:

$$E(D_i) = \sum_{k=o}^{\infty} k\ P\{D_i=k\} \qquad (42)$$

The same kind of analysis can be used to study "bottlenecks." Let $D_{(i,j)}(w,t)$ be the random variable "delay of a message" originating at (i,j) at time t in getting w units from the ground station for w=0, 1, 2,..., i. As earlier we have:

$$P\{D_{(i,j)}(w,t)=k\} = \frac{Y_{(w,u),(i,j)}(t+k+i-w,\ t)}{O_{(i,j)}(t)}; \ k=0,\ 1,\ 2,\ldots, \qquad (43)$$

where (w,u) is the unique repeater on the path from (i,j) to (o,o) at distance w. Similarly as in 41 and 42 :

$$P\{D_i(w,t)=k\} = \frac{1}{4i}\sum_{j=1}^{4i} P\{D_{(i,j)}(w,t)=k\} \text{ and} \qquad (44)$$

$$E\left[D_i(w,t)\right] = \sum_{k=1}^{\infty} K P\{D_i(w,t)=k\}. \qquad (45)$$

11. MESSAGES OUTWARD FROM THE ORIGIN

11.1. A Single Message Originates at the Ground Station.

The next major part of our study is to model the situation when message flow is outward from the origin. We begin our study with the dynamics of the simple model where the repeaters are at the lattice points of the plane. A single message originates at the ground station at time t=0. Every message received by a repeater is accepted and perfectly retransmitted to each of its four nearest neighbors. We determine:

a) The number of repeaters which receive the message for the first time at time t: t = 0, 1, 2,....

b) The number of repeaters which have seen the single message by time t.

c) The number of copies of the single message received by any repeater at time t.

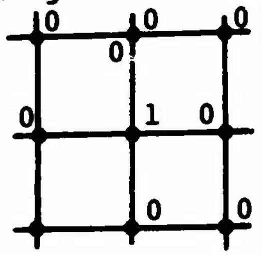
The assumptions are:

1) A single message arrives at a given node at time t = 0, no other messages are introduced into the network. We assume the message originated at the origin,(Cartesian coordinates (0,0)).

2) Message transmission is perfect, i.e., after one time unit each of the four neighbors to any repeater receive all messages transmitted by the repeater at the previous time point.

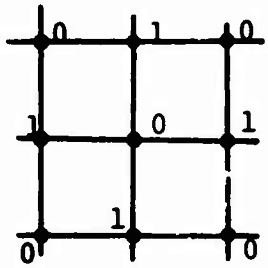
Some diagrams and numbers are helpful to fix ideas.

t = 0,

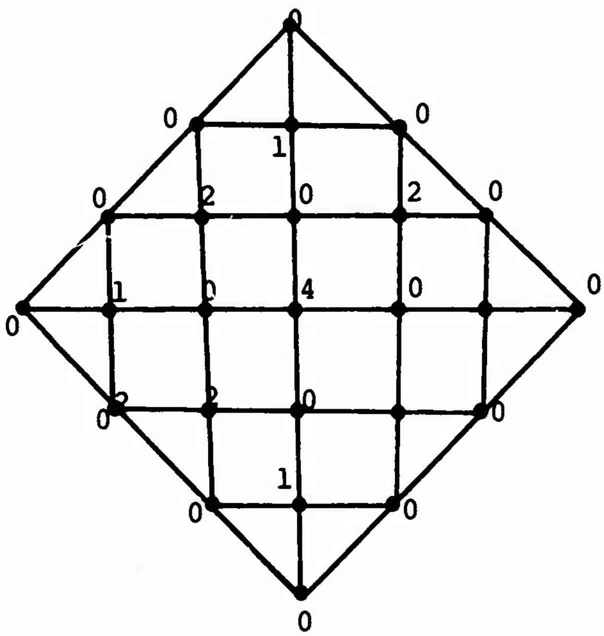


(one message at origin, no messages elsewhere)

t = 1,



(a single message at each of the four neighbors, no messages elsewhere)

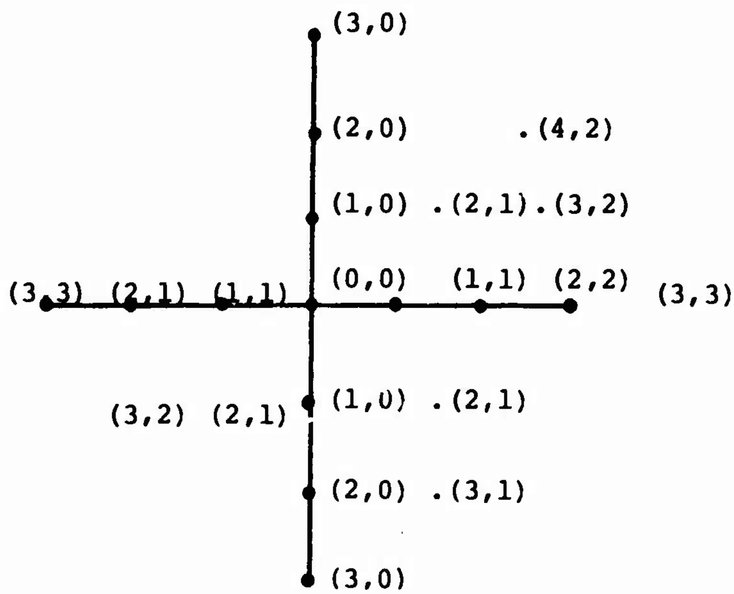


(4 messages at the origin,
0 Message at all repeaters
1 step from origin, 1 message
at each of four repeaters
2 steps in either the horizontal
or vertical directions, 2 messages
at repeaters, 1 unit in horizontal
direction and 1 unit in vertical
direction.

By examining the diagrams we are led to introduce a coordinate system based on distance as measured in steps to reach a repeater and horizontal distance of the repeater from the origin. The quadrant symmetry of the model also indicates use of these coordinates.

The coordinates of a repeater are denoted by (d,j) where d is the distance of the repeater from the origin measured in minimum time units a message needs to arrive at the repeater from the origin; the second coordinate j is the horizontal distance of the repeater from the origin again measured in time units but only in the horizontal direction.

For example, we give some coordinates:



Some further notation which is necessary;

$B(t)$ = the number of repeaters which receive the message for the first time at time t.

Clearly $B(t)$ is the number of repeaters whose first coordinate is t. i.e. that are at distance t from the origin.

$A(t)$ = the number of repeaters which have seen the message by time t. Clearly $A(t) = \sum_{j=0}^{t} B(j)$.

$N_j^d(t)$ = number of copies of the message received by a given repeater at coordinates (d,j) at time t. Clearly

a) $N_j^d(t) = 0$ for $d > t$

b) $N_j^d(d+2k+1) = 0$ for $k = 1, 2, \ldots.$

Thus it is necessary to compute $N_j^d(d+2k)$ $k = 0, 1, 2, \ldots.$

A) Calculation of B(t):

The quantity B(t), (the number of repeaters at distance t from the origin is the number of repeaters which receive the message for the first time t, is easy to compute. This quantity is given by the number of integer solutions to $|i| + |j| = t$. Since a repeater is at distance d from the origin if and only if its' Cartesian coordinates (i,j) satisfy $|i| + |j| = t$, we can solve this equation and count solutions. Note that

i=0,	j=t	or -t	2 solutions
i=1,	j=t-1	or -t+1	2 solutions
i=-1,	j=t-1	or -t+1	2 solutions
i=2,	j=t-2	or -t+2	2 solutions
i=-2,	j=t-2	or -t+2	2 solutions
⋮	⋮	⋮	
i=t-1,	j=1	or j=-1	. . .
i=-t+1,	j=1	or j=-1	. . .
i=t,	j=0		
i=-t,	j=0		

The number of solutions is, $B(0) = 1$

$$B(t) = 2 + 4(t-1) + 2 = 4t \text{ for } t \geq 1.$$

To compute A(t), we sum B(t) and obtain

$$A(t) = \sum_{j=0}^{t} B(j) = 1 + \sum_{j=1}^{t} 4(j) = 1 + 4\sum_{j=1}^{t} j = 1 + 4\frac{t(t+1)}{2}$$

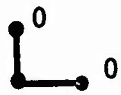
$$= 1 + 2t^2 + 2t = 2t^2 + 2t + 1$$

The rate at which A(t) the number of repeaters which receive the message by time t grows as $A'(t) = 4t + 2$ which is linear in t.

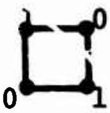
B. <u>The quantity $N_j^d(d + 2k)$, k = 0, 1, 2,....</u>.

To compute the number of copies of the message received at a repeater with coordinates (d,j) at time d + 2k, k = 0, 1, 2,... we first draw some diagrams. Due to symmetry it suffices to examine only the first quadrant.

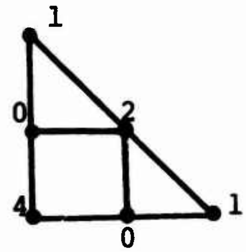
t = 0



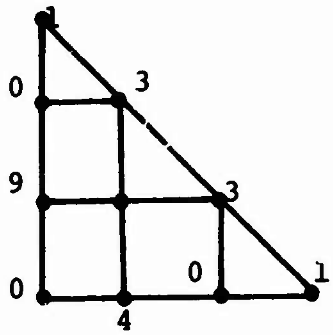
t = 1



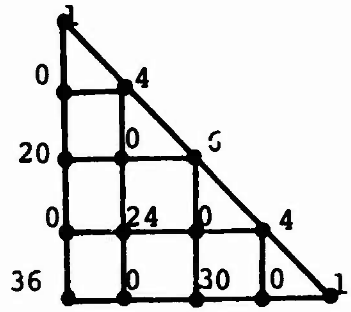
t = 2,



t = 3,



t = 4,



It seems clear from the diagram that a repeater with coordinates (d,j) will receive at t=d the number of messages which is given by the binomial coefficient $\binom{d}{j}$. From the diagram we note the relationship of the outer edge to the d[th] now of a Pascal triangle:

```
          1
         1 1
        1 2 1
       1 3 3 1
      1 4 6 4 1
     1 5 1010 5 1
```

This result is also apparent from an argument based on the number of paths of a message from (0,0) to (d,j). The number of messages received at a repeater with coordinates (d,j) is given by the number of paths from (0,0) to (d,j) which is obviously $\binom{d}{j}$. To determine a general formula for $N_j^d(d + 2k)$ for $k > 1$, we can write and solve the appropriate difference equation.

$$N_j^d(d + 2k) = N_{j-1}^{d-1}(d + 2k-1) + N_{j+1}^{d+1}(d + 2k-1) + N_j^{d+1}(d + 2k-1) + N_j^{d-1}(d + 2k-1)$$

for $k = 1, 2, \ldots; d = 0, 1, 2, \ldots; j = 0, 1, 2, \ldots, d$.

The initial conditions are

$N_j^d(t) = 0$ if $t < d$,

$N_j^d(d) = \binom{d}{j}$.

The solution to this equation is given by $N_j^d(d + 2k) = \binom{d + 2k}{k}\binom{d + 2k}{k+j}$

To check its validity note that the initial conditions are satisfied, and apply the well known definition of binomial coefficients;

$$\binom{n}{j} = \binom{n-1}{j-1} + \binom{n-1}{j}, \qquad n \geq 1 \quad j \geq 1.$$

$$N_{j-1}^{d-1}(d-1 +2k) + N_{j+1}^{d+1}(d+1+2(k-1)) + N_j^{d+1}(d+1+2(k-1)) + N_j^{d-1}(d-1+2k)$$

$$= \binom{d+2k-1}{k}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k}\binom{d+2k-1}{k+j}$$

$$= \binom{d+2k-1}{k}\left[\binom{d+2k-1}{k+j-1} + \binom{d+2k-1}{k+j}\right] + \binom{d+2k-1}{k-1}\left[\binom{d+2k-1}{k+j} + \binom{d+2k-1}{k+j-1}\right]$$

$$= \binom{d+2k-1}{k}\binom{d+2k}{k+j} + \binom{d+2k-1}{k-1}\binom{d+2k}{k+j}$$

$$= \binom{d+2k}{k+j}\left[\binom{d+2k-1}{k} + \binom{d+2k-1}{k-1}\right] = \binom{d+2k}{k+j}\binom{d+2k}{k}$$

For k very large with respect to d we can use a stirling approximation to note the $N_j^d(d+2k) \sim 2^{4k}$ i.e. grows as 2^{4k}.

To summarize:

a) $B(t) = 4t\ ,\ t \geq 1 \quad B(0) = 1,$

b) $A(t) = 2\,t^2 + 2t+1 \quad t \geq 0,$

c) $N_j^d(d+2k) = \binom{d+2k}{k+j}\binom{d+2k}{k} \sim 2^{4k}$ for large k .

11.2. A Fixed Number of Messages Originate at the Ground Station and Subject to Non-Capture

The model of message flow from the ground station out to repeaters can be extended to allow the possibility of erasure or non-capture of messages.

We assume that at each point of time t, τ messages are being generated outward from the origin. Of messages accepted at each repeater, a fixed proportion k_o are addressed to that repeater and hence are not repeated. We study the distributions of the number of messages received and accepted at each repeater at each point in time, assuming an infinite net.

Recall $X_{(i,j)}^{(1)}(t)$ = number of messages arriving at a repeater with coordinates (i,j) at time t with Mode 1 capture.

$X_{(i,j)}^{A,(1)}(t)$ = number of messages accepted at a repeater with coordinates (i,j) at time t in Mode 1 capture. In Mode 2 capture, we use the same notation except that (1) as a superscript is replace by a (2).

A. In Mode One:

In Mode 1 capture, the relationship between arriving and accepted messages is described by the transfer function:

$$P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k,m)-j} (-1)^{\nu} \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \qquad (46)$$

when $j \leq \min(k,m)$, and zero otherwise.

We can study $X^{(1)}_{(ij)}(t)$ and $X^{A,(1)}_{(ij)}(t)$ recursively.

At the origin for each $t = 0, 1, 2, \ldots;$ $X^{(1)}_{(0,0)}(t) = \tau$, a fixed constant. Furthermore:

$$P\{X^{(1),A}_{(0,0)}(t) = w\} = P_{\tau,w} = \frac{\binom{m}{w}}{m^{\tau}} \sum_{\nu=0}^{\min(\tau,m)-w} (-1)^{\nu} \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} \qquad (47)$$

If we assume $\tau \leq m$;

$$P\{X^{(1),A}_{(0,0)}(t) = w\} = \begin{cases} \frac{\binom{m}{w}}{m^{\tau}} \sum_{\nu=0}^{\tau-w} (-1)^{\nu} \binom{m-w}{\nu} \frac{\tau!}{(\tau-w-\nu)!} (m-w-\nu)^{\tau-w-\nu} & \text{if } w \leq \tau \\ 0 & \text{if } w > \tau. \end{cases} \qquad (48)$$

At coordinates (1,1), (1,0) the distribution of $X_{(1,0)}(t)$ and $X_{(1,1)}(t)$ are identical, hence we write only $X^{(1)}_{(1,0)}(t)$.

We have:

$$X^{(1)}_{(1,0)}(t) = \begin{cases} X^{(1),A}_{(0,0)}(t-1) & t = 1, 2, \ldots; \\ 0 & \text{if } t = 0. \end{cases} \qquad (49)$$

For the acceptance at $t = 1, 2, \ldots;$

$$P\{X^{A,(1)}_{(1,0)}(t) = j\} = \sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1)}_{(1,0)}(t) = k\} \qquad j = 0, 1, 2, \ldots;$$

or recursively:

$$P\{X^{A,(1)}_{(1,0)}(t) = j\} = \begin{cases} \sum_{k=j}^{\tau} P_{kj} \cdot P\{X^{(1),A}_{(0,0)}(t-1) = k\} & \text{if } j \leq \tau; \\ 0 & \text{otherwise.} \end{cases} \qquad (50)$$

Equation(50) is recursively solvable since $P\{X^{(1),A}_{(0,0)}(t-1) = k\}$ is given by (48) and P_{kj} is given by (46).

Now more generally, at a repeater at distance d with coordinates (d,j); $j \neq 0,d$. We have for d = 2, 3, ...;

$$X^{(1)}_{(d,j)}(t) = (1-k_0)\left[X^{(1),A}_{(d-1,j-1)}(t-1) + X^{(1),A}_{(d-1,j)}(t-1)\right]. \qquad (51)$$

The acceptances are given by:

$$P\{X^{A,(1)}_{(d,j)}(t) = r\} = \sum_{k=r}^{(2m)} P_{k,r} \cdot P\{X^{(1)}_{(d,j)}(t) = k\}; \qquad (52)$$

where P_{kr} is given by (1) and $P\{X^{(1)}_{(d,j)}(t) = k\}$ can be computed recursively from (51) using the notion of isodesic line joint densities. The equations for mode 2 analysis are identical except that P_{kj} is replaced by P^*_{kj}.

When j = 0 or d, i.e. the repeater is on the axis at distance d a simpler analysis unfolds. Since the random variables $X_{(d,0)}(t)$ and $X_{(d,d)}(t)$ have the same probability distribution, we write equations only for $X_{(d,0)}(t)$; $d \geq 2$ since $X_{(0,0)}(t)$ and $X_{(1,0)}(t)$ have already been determined.

$$X^{(1)}_{(d,0)}(t) = (1-k_0)\, X^{(1),A}_{(d-1,0)}(t-1); \qquad t = d, d+1, \ldots; \quad d = 2, 3, \ldots \qquad (53)$$

For the acceptances;

$$P\{X^{(1),A}_{(d,0)}(t) = j\} = \sum_{k=j}^{\infty} P_{kj}\, P\{X^{(1)}_{(d,0)}(t) = k\}, \qquad = 0, 1, 2, \ldots, m \qquad (54)$$

The equations (46) through (54) can be used with computer generated data to study message arrivals and acceptances at each repeater.

In particular, we can write equations such as 46-54 for the closed net under consideration and obtain numerical data for flow from the ground station.

Let $X_{(i,j)}(t)$ be the number of messages received at (i,j) at time t and $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t. We assume $X_{(o,o)}(t)=J$ a fixed constant for all time points. As in the inward model, $Y_{(i,j)}(t)$ is obtained from $X_{(i,j)}(t)$ by randomizing over either mode 1 or mode 2 slotting. When performing the calculations on the computer, we assumed a finite grid of 61 repeaters as earlier. However, now a repeater repeats messages to those repeaters which are one unit of distance further from the origin or ground station. Thus for example a repeater in a quadrant repeats to its two further neighbors, while a repeater on the axis repeats to the one repeater which is one unit further.

The specific equations used for the first quadrant calculations follow, we assume that $\frac{1}{61}$ of those messages accepted are addressed to each repeater and hence not repeated. The calculations were carried out in each of the two slotting modes.

Step 1. Set $X_{(o,o)}(t) = J = 80$, $t = 1, 2, \ldots, 35$.

Step 2. Compute $Y_{(o,o)}(t)$ by randomizing over the transfer distribution in each of two modes.

Step 3. Compute $X_{(1,1)}(t)$ and $X_{(1,2)}(t)$ from:

$$X_{(1,1)}(t) = X_{(1,2)}(t) = \frac{60}{61} Y_{(o,o)}(t-1).$$

Step 4. Compute $Y_{(1,1)}(t)$ and $Y_{(1,2)}(t)$ in each of two modes.

Step 5. For t = 2, 3, 4,...,37; Compute $X_{(2,1)}(t)$, $X_{(2,2)}(t)$ and $X_{(2,3)}$ from:

$$X_{(2,1)}(t) = \frac{60}{61} Y_{(1,1)}(t-1)$$

$$X_{(2,2)}(t) = \frac{60}{61} (Y_{(1,1)}(t-1) + Y_{(1,2)}(t-1))$$

$$X_{(2,3)}(t) = \frac{60}{61} Y_{(1,2)}(t-1).$$

Step 6. Compute $Y_{(2,1)}(t)$, $Y_{(2,2)}(t)$ and $Y_{(2,3)}(t)$ by randomizing in each slotting mode.

Step 7. Compute $X_{(3,2)}(t)$, $X_{(3,3)}(t)$, $X_{(3,4)}(t)$ from:

$$X_{(3,2)}(t) = \frac{60}{61}\left[Y_{(2,1)}(t-1) + Y_{(2,2)}(t-1)\right]$$

$$X_{(3,3)}(t) = \frac{60}{61}\left[Y_{(2,2)}(t-1) + Y_{(2,3)}(t-1)\right]$$

$$X_{(3,4)}(t) = \frac{60}{61} Y_{(2,3)}(t-1).$$

Step 8. Compute $Y_{(3,2)}(t)$, $Y_{(3,3)}(t)$, $Y_{(3,4)}(t)$ by randomizing in mode 1 and mode 2.

Step 9. Compute $X_{(4,3)}(t)$ and $Y_{(4,4)}(t)$ by randomizing.

Step 10. Compute $X_{(5,4)}(t)$ from:

$$X_{(5,4)}(t) = \frac{60}{61}\left[Y_{(4,3)}(t-1) + Y_{(4,4)}(t-1)\right].$$

Step 11. Compute $Y_{(5,4)}(t)$ by randomizing and print out X's and Y's for t = 1, 2,...,40. The numerical data is summarized in Table 26 and the accompanying Figure 10.

11.3 Messages Coming Outward From the Origin

It is assumed that at each point in time eighty (80) messages originate at the origin (ground station). The messages are repeated outwards to the various repeaters. At each point in time each repeater sends all but a fixed proportion of its accepted message to each neighboring repeater, one unit of distance further from the ground station. The fixed proportion not repeated is 1/61 of the number of accepted messages, which are assumed to be addressed to the given repeater. The number of slots is fixed at 100.

In table 47 we summarize the result of this calculation by giving the average number of messages accepted and arriving as a function of distance and mode. The numerical data is displayed graphically in figure 22.

DISTANCE	MODE 1 ARRIVING	MODE 1 ACCEPTED	MODE 2 ARRIVING	MODE 2 ACCEPTED
0	80	33	80	52
1	32	21	51	36
2	27	19	47	33
3	32	22	57	41
4	46	30	87	58
5	58	31	114	66

TABLE 47

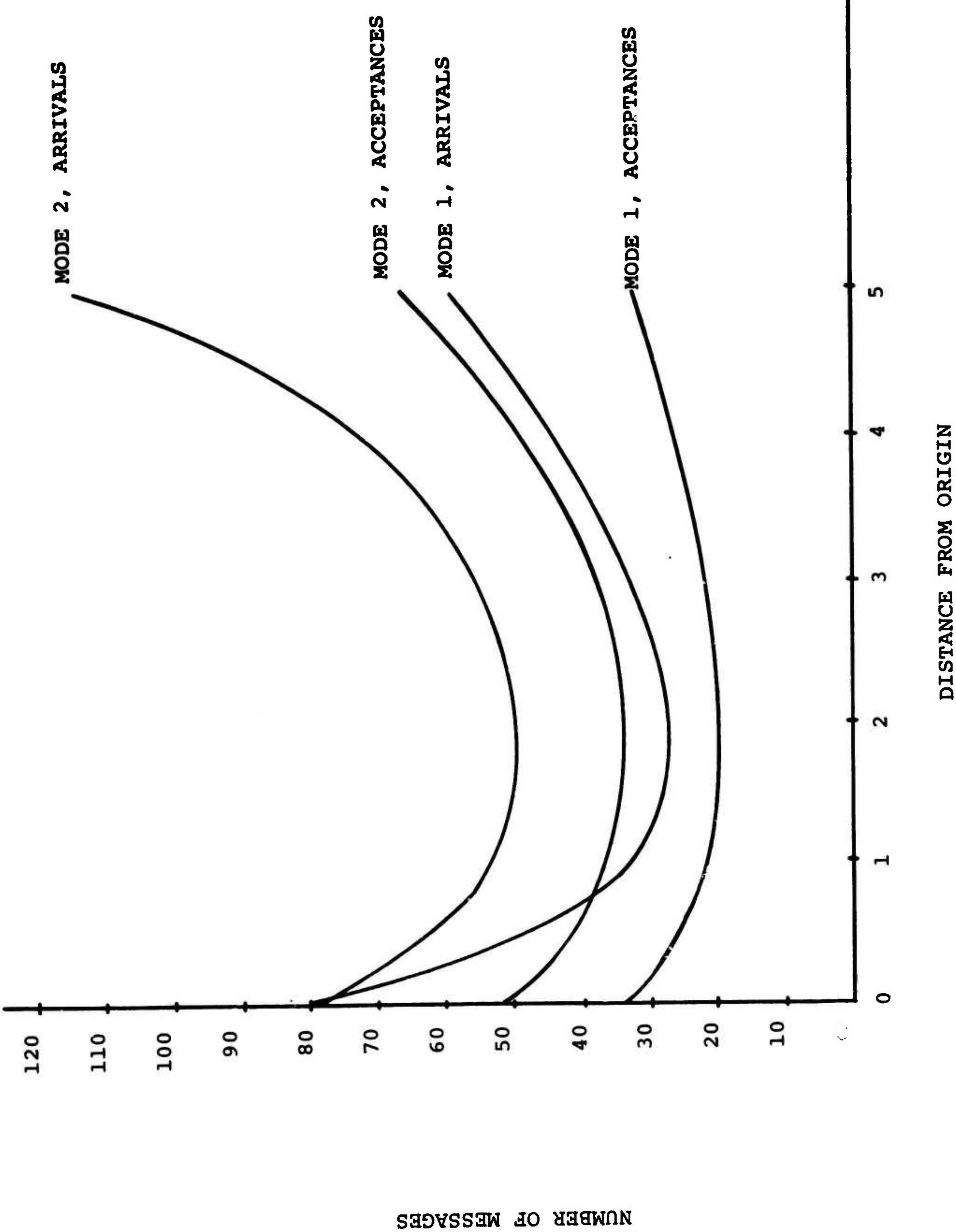

MODE 2, ARRIVALS
MODE 2, ACCEPTANCES
MODE 1, ARRIVALS
MODE 1, ACCEPTANCES
NUMBER OF MESSAGES
120
110
100
90
80
70
60
50
40
30
20
10
DISTANCE FROM ORIGIN
0
1
2
3
4
5

FIGURE 22

12. OTHER MODELS

A number of models other than the "basic model" were considered. The results for quantities of interest were obtained in closed forms under the assumption of an infinite grid. One example of such a model was described in Section 3 for messages repeated only toward the ground station. Of course that was part of our "basic model". In the process of developing the results of Section 3, we assumed that a single message originated at each repeater at each point in time and multiplied the results by the mean, τ, to obtain average flows. We will now "justify" that calculation and study uninhibited passage of messages in each direction in an infinite grid. Our new assumptions are:

1) At each point in time, starting at t=0, messages originate at each repeater according to a Poisson distribution with mean λ.

2) The arrivals (originations) at each repeater are independent over time and different repeaters.

The probability that exactly j new messages originate at any time point at any repeater is

$$\frac{e^{-\lambda}\lambda^j}{j!}, \quad j = 0, 1, 2, \ldots.$$

We compute formulas for;

a) $N_o(t)$ = average number of messages which arrive at the origin at time t. Since all repeaters are statistically identical there is no loss in generality in studying message flow at the origin.

b) $N'_o(t)$ = Average number of distinct messages which arrive at the origin at time t for the first time.

c) Ieff(t)=Inefficiency of the network defined by;

$$\text{Ieff}(t) = \frac{No(t)}{N'_o(t)} = \frac{\text{average number of messages}}{\text{average number of new messages for the 1st time.}}$$

This is a measure of inefficiency since the larger Ieff, the more inefficient the system.

The actual number of messages which arrive at the origin at time t is a random variable. In fact it is a sum of a large number (when t is fairly large) of independent random variables. The summand random variables can loosely be described as the contribution to message flow at the origin arising out of some number of messages originating at each repeater at each point in time.

To compute $N_o(t)$ we can sum up all the contributions. This is interesting but tedious. A simpler method is to compute $X_o(t)$ which we define as the number of messages arriving at the origin at time t in a deterministic model obtained by assuming that at each point in time, at each repeater, exactly one new message originates. It will then follow that

$$N_o(t) = \lambda X_o(t).$$

Similarly, if we define $X'_o(t)$ to be the number of distinct messages that arrive at the origin at time t in the deterministic model it will follow that

$$N'_o(t) = \lambda X'_o(t).$$

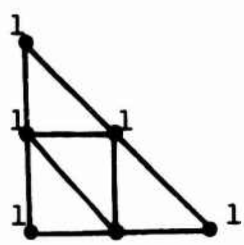
We indicate an armwaving proof of the first assertion. The quantity No(t) is a sum of average contribution to the flow at the origin at time t, as a result of messages originating at repeaters less than t units in distance and times earlier than t. The average contribution from each repeater is a constant (not with time) at each fixed time point and repeater, multiplied by λ, the average generation rate. (the constant is given by the calculations in section II and depends on the coordinates (d,j) and time. Thus λ factors from the sum and $N_O(t)$ is λ multiplied by the total flow resulting from a single message originating at each repeater at each point in time.

We now make the specific assumption.

At each point in time and at each repeater, a single new message is originated.

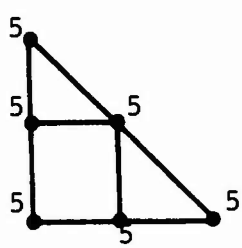
Under this model to compute $X_O(t)$ and hence $N_O(t)$ is trivial. To fix ideas we depict the situation at three time points

t = 0



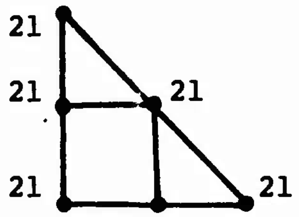
At time zero one message originates at each repeater, hence the message flow is $X_O(0)=1$.

t=1



At time 1 each repeater receives 5 messages, one from each of four nearest neighbors and one new message.

$t = 2,$



At time $t = 2$ each repeater receives 5 messages from each of its four nearest neighbors and one new message for a total of 21.

To compute $X_o(t)$ in general we note that each repeater is statistically identical in terms of message flow. Hence,

$$X_o(t) = 4X_o(t-1) + 1 \quad t \geq 1$$

$$X_o(0) = 1.$$

This difference equation is trivial to solve and hence,

$$X_o(t) = \frac{4^{t+1}-1}{3} \quad t \geq 0.$$

Thus

$$N_o(t) = \frac{\lambda}{3} (4^{t+1}-1)$$

To compute $N_o'(t)$ we consider the same deterministic model and compute $X_o'(t)$, the number of distinct messages which arrive at the origin for the first time at time t. It is easy to compute $X_o'(t)$ from the following table by summing contributions.

Time of Origination	Distance from Origin	Number of Messages
o	t	4t
1	t-1	4(t-1)
2	t-2	4(t-2)
⋮	⋮	⋮
t-1	1	4
t	0	1

The first column is the time the message first appeared in the system if it is received by the origin for the first time at time t. The second column indicates the distance from the origin that the message originated. The third column indicates the number of message originated at that time and distance which are received at the origin at time t. Thus,

$$X'_o(t) = 4t + 4(t-1) + .. +4+1 = \sum_{j=1}^{t} 4j+1$$
$$= 2t^2 + 2t + 1, \quad t \geq 0 .$$

Thus,

$$N'_o(t) = \lambda(2t^2 + 2t + 1)$$

The inefficiency of this "undamped" network is

$$Ieff(t) = \frac{\frac{\lambda}{3}(4^{t+1}-1)}{\lambda(2t^2 + 2t+1} = \frac{4^{t+1}-1}{3(2t^2+2t+1)} \sim \frac{4^{t+1}}{6t^2}$$

The inefficiency grows rapidly with time for this undamped system.

We can now put restrictions on the operating characteristics of the repeaters and message flow.

12.1. No Message Can Be Transmitted More Than k Times

In this mode it is assumed that each message has a counting feature whereby each time the message is repeated the counter is updated by one unit. When the counter reaches the number k the message is no longer repeated and disappears from the system.

In this mode we compute;

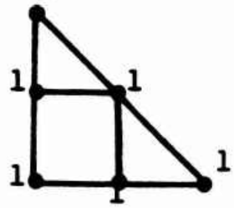
a) $N_o(t)$ = average number of messages received by the origin at time t.

b) $N_o'(t)$ = average number of distinct messages received at the origin for the first time at time t.

$$Ieff(t) = \frac{N_o(t)}{N_o'(t)}$$

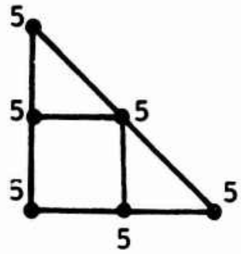
Once again by the argument presented in the previous section it suffices to consider the deterministic model and to compute $X_o(t)$ and $X_o'(t)$. To fix ideas we diagram the first 6 time points, inherently assuming $k \geq 5$.

t = 0,



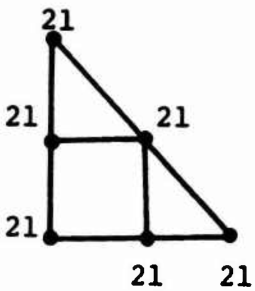
one message at each repeater,
all of age zero

t = 1,



five messages each repeater:
one of age zero
four of age one

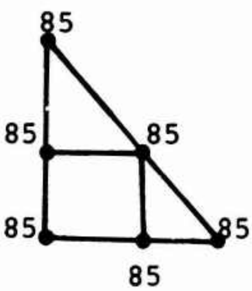
t = 2,



21 messages at each repeater

1 of age zero
4 of age one
16 of age two

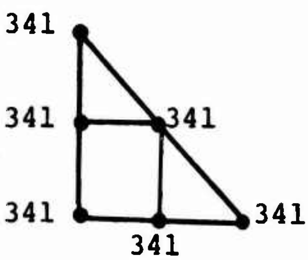
t = 3,



85 messages at each repeater

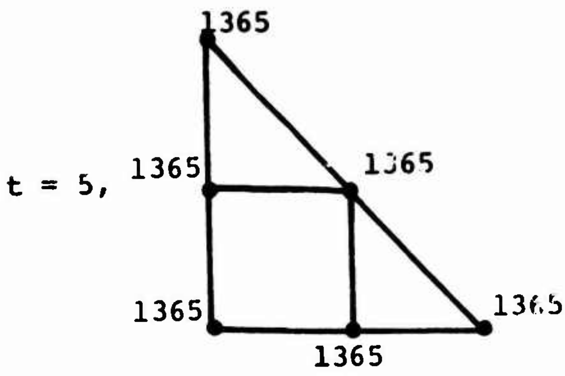
64 of age three
16 of age two
4 of age one
1 of age zero

t = 4,



341 messages at each repeater

256 of age four
64 of age three
16 of age two
4 of age one
1 of age zero



1365 Messages	
1024	of age 5
256	of age 4
64	of age 3
16	of age 2
4	of age 1
1	of age 0
1365	TOTAL

Let $X_o^{>k}(t)$ be the number of messages arriving at the origin at time t whose age is less than k. That is, those that will be transmitted. Since the flow at all repeaters is statistically identical:

$$X_o(t) = 4X_o^{>k}(t-1) + 1 , \qquad t \geq 1 .$$

The number of messages received at the origin at time t is four times the number that will be transmitted by any of the four nearest neighbors plus one new one. Once again this is trivial difference equation to solve in k. We obtain as a solution:

$$X_o(t) = \frac{4^{k+1}-1}{3} \qquad t \geq k,$$

$$X_o(t) = \frac{4^{t+1}-1}{3} \qquad t \leq k .$$

Hence by the arguments above;

$$N_o(t) = \begin{cases} \lambda\dfrac{[4^{k+1}-1]}{3} & t \geq k \\ \lambda\dfrac{[4^{t+1}-1]}{3} & t < k, \end{cases}$$

A similar analysis shows that,

$$N_o'(t) = \begin{cases} 2k^2 + 2k + 1, & t \geq k, \\ 2t^2 + 2t + 1 & t \leq k. \end{cases}$$

Thus:

$$I_{eff}(t) = \frac{\frac{\lambda}{3}[4^{k+1} - 1]}{\lambda(2k^2 + 2t + 1)} \qquad t \geq k.$$

$$= \frac{4^{k+1} - 1}{3(2k^2 + 2k + 1)} \sim \frac{4^{k+1}}{6k^2} = \frac{2 \cdot 4^k}{3k^2}.$$

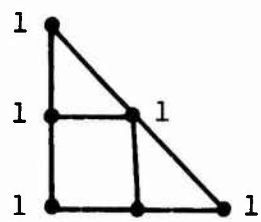
In tabular form for k=1, 2, 3, 4, 5 the inefficiencies are given

Ieff	1	1.62	3.40	8.3	22.4	....
k	1	2	3	4	5	....

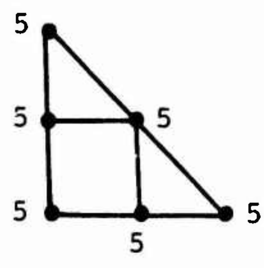
12.2 <u>If the Same Message Arrives From Different Sources Only one Transmission is Made.</u>

In this mode of operation a repeater has the ability to compare messages which arrive at the same instant. We may consider this mode to be a "memory" type system of length of time "one unit" or instantaneous. We seek to compute $N_o(t)$ and $N_o'(t)$. Again we consider the deterministic model. To fix ideas let us examine some early time points and compute multiplicities. All repeaters in this case also have statistically identical flows.

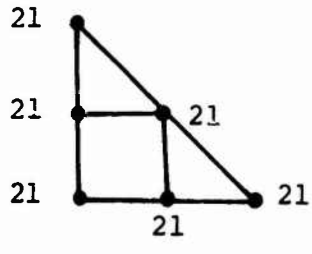
t = 0,



t = 1,



t = 2,



At time $t = 2$ all repeaters receive 21 messages, and they are statistically identical. Of the 21 total messages received at the origin only 14 are distinct.

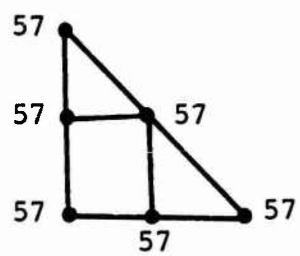
We can partition this total using a table as follows:

Origin of Message	Time of Origin	Number Distinct
(0,0)	t=2	1
(1,1), (1,0)	t=1	4
(2,0),(2,1), 2,2)	t=0	8
(0,0)	t=0	1

The total distinct is 14.

Hence the diagram for t=3 is,

t = 3,



of the $57 = 4 \cdot 14 + 1$, one is new and 14 came from each of four nearest neighbors.

The same method can be used to compute $X_o(2t)$ and $X_o(2t-1)$ in general. Let $X_o^d(2t)$ and $X_o^d(2t-1)$ be the number of distinct messages received at the origin at time $(2t)$ and $(2t-1)$ respectively. Clearly, these quantities satisfy;

$$X_o(2t) = 4X_o^d(2t-1) + 1 \quad \text{and}$$

$$X_o(2t+1) = 4X_o^d(2t) + 1.$$

Thus it suffices to compute $X_o^d(2t)$ and $X_o^d(2t-1)$.

To compute x_o^d (2t) and x_o^d (2t-1) we decompose each as follows:

$$x_o^d(2t) = x_o^{d,1}(2t) + x_o^{d,2}(2t) + x_o^{d,3}(2t) + \ldots + x_o^{d,t+1}(2t),$$

$$x_o^d(2t-1) = x_o^{d,1}(2t-1) + x_o^{d,2}(2t-1) + \ldots + x_o^{d,t}(2t-1).$$

Where $x_o^{d,v}(t)$ = number of distinct messages received at the origin at time t for the v^{th} time. The quantities $x_o^{d,v}(t)$ are computed by the following table, which essentially decomposes $x_o^{d,v}(t)$ by distance and time of origination of each message contributing to $x_o^{d,v}(t)$. For $x_o^d(2t-1)$;

1^{st} time -	time of origin.	0	1	• • •	2t-2	2t-1
	dist. at origin	2t-1	2t-2	• • •	1	0
2^{nd} time -	time of origin.	0	1	• • •	2t-4	2t-3
	dist. of origin.	2t-3	2t-4	• • •	1	0
. . .	. . .	. . .	. . .	. . .	. . .	. . .
$(t-1)^{th}$ time-	Time of origin.	0	1	2	3	
	dist. at origin.	3	2	1	0	
t^{th} time -	time of origin.	0	1			
	dist at origin.	1	0			

The grand sum is;

$$x_o^{d,1} = 1+4\sum_{j=1}^{2t-1}(2t-j), \quad x_o^{d,2} = 1+4\sum_{j=3}^{2t-1}(2t-j), \ldots, \quad x_o^{d,t} = 4\sum_{j=2t-1}^{2t-1}(2t-j)$$

since the number of messages originating at distance d is 4d as we have seen earlier. Therefore,

$$x_o^d(2t-1) = t+4\sum_{k=1}^{t}\sum_{j=2k-1}^{2t-1}(2t-j)$$

After summing we obtain:

$$X_o^d(2t-1) = \frac{t(4t+1)(2t+1)}{3},$$

By a similar analysis we can show that

$$X_o^d(2t) = \frac{(4t+3)(2t+1)(t+1)}{3},$$

Thus,

$$X_o(2t) = 4 \cdot X_o^d(2t-1) + 1 = \frac{4t(4t+1)(2t+1)}{3} + 1, \quad \text{or}$$

$$X_o(2t) = \frac{(4t+3)(8t^2+1)}{3}.$$

Similarly,

$$X_o(2t+1) = 4 \cdot X_o^d(2t) + 1$$

$$= \frac{4(4t+3)(2t+1)(t+1)}{3} + 1$$

$$= \frac{32t^3+72t^2+52t+15}{3}.$$

Hence:

$$N_o(2t) = \frac{\lambda(4t+3)(8t^2+1)}{3},$$

$$N_o(2t+1) = \frac{\lambda(32t^3+72t^2+52t+15)}{3},$$

$$N_o^1(2t) = \lambda(2t^2+2t+1) = \lambda(8t^2+4t+1).$$

The efficiency of this mode is:

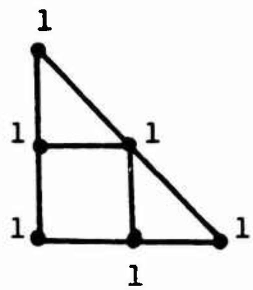
$$\text{Eff}(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3}.$$

12.3 A Repeater Never Transmits The Same Message More Than Once Except Upon Initial Reception

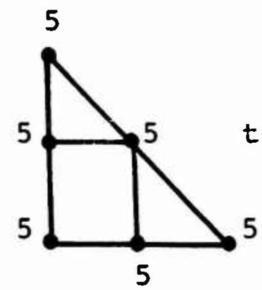
This mode of operation implies infinite but not instantaneous memory in the repeater. We analyze the deterministic case with no loss of generality.

Pictorially the first few time points appear as follows;

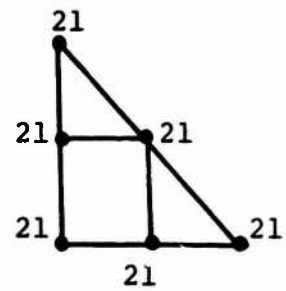
t = 0,



t = 1,



t = 2,



Of the 21 messages received at t=2 there are 17 which are received for the first time. These can be enumerated by point and time of origin: 1 new one at (t=2,d=0), 4 at (t=1, d=1), 12 at (t=0, d=2). Thus there will be 69 messages received at each repeater at t=3.

In general let:

$X_o(t)$ be the number of messages received at the origin at time t, and $X_o^1(t)$ be the number of messages received at the origin for the first time at time t. Then,

$$X_o(t) = 4\, X_o^1(t-1) + 1 .$$

If a message was received for the first time at any repeater at time t-1 it originated at distance t-1 at time 0, or at d=t-2 at time 1 or, ..., at distance 0 at time t-1. Summing these by their appropriate multiplicities we obtain,

$$X_o^1(t-1) = 4 \sum_{k=2}^{t} \sum_{j=0}^{t-k} \binom{t-k+1}{0} \binom{t-k+1}{j} + 1,$$

or,

$$X_o^1(t-1) = 2^{t+2}-4t-3 \qquad \text{and,}$$

$$Xo(t) = 2^{t+4}-16t-11 \text{ .}$$

Similarly;

$$No(t) = \lambda (2^{t+4}-16t-11) \quad \text{and as before}$$

$$N_o^1(t) = \lambda (2t^2+2t+1)$$

The efficiency of this mode is:

$$Eff = \frac{No(t)}{No^1(t)} = \frac{2^{t+4}-16t-11}{2t^2+2t+1} \sim \frac{2^{t+3}}{t^2} \text{ .}$$

12.4 Mixture of Modes 12.2 and 12.3

In this mixed mode a repeater has infinite memory for comparison of messages and instant memory. The result (operational) of this mixed mode is that when multiple reception of the same message is received only one transmission is made. Furthermore if a message is received for the second time it is not transmitted at all. The result of this combined mode is to reduce the transmission in the instantaneous mode to only messages received for the first time.

Therefore:

$$X_o(t) = 4X_o^{',d}(t-1) + 1$$

where

$X_o^{',d}(t)$ is the number of distinct messages received at the origin for the first time at time t. This quantity has been previously computed to be,

$$X_o^{',d}(t-1) = 2(t-1)^2 + 2(t-1) + 1 = 2t^2 - 2t - 1.$$

It follows that,

$$X_o(t) = 8t^2-8t-3, \text{ and}$$

$$N_o(t) = \lambda(8t^2-8t-3).$$

The efficiency of this mixed mode is seen to be

$$\text{Eff}(t) = \frac{\lambda(8t^2-8t-3)}{\lambda(2t^2+2t+1)} \quad \frac{8t^2-8t-3}{2t^2+2t+1} \sim 4 \quad .$$

12.5 Mixture of Modes 12.1 and 12.2

In this mixture of modes no message can be transmitted more than k-times and each repeater has instantaneous memory but not finite memory. This mode also provides an upper bound to the case where each repeater has zero capture and messages are dropped after k transmissions.

A little analysis will show that for $t \leq k$ there is no change in the message flow from the instantaneous memory case. For $t \geq k$ the exact same analysis as in the instantaneous memory only case shows that the formulas are exactly those except that t should be replaced by k.

Therefore in this mixed mode, for memory of k.

$$X_o(2t) = \frac{(2k+3)(8k^2+1)}{3}, \quad t \geq [\tfrac{k}{2}]$$

$$= \frac{(2t+3)(8t^2+1)}{3}, \quad t \leq [\tfrac{k}{2}]$$

$$X_o(2t+1) = \frac{32k^3+72k^2+52k+15}{3} \qquad t \geq k$$

$$= \frac{32t^3+72t^2+52t+15}{3} \qquad t \leq k.$$

Furthermore,

$$N_o(2t) = X_o(2t),$$

$$N_o(2t+1) = X_o(2t+1).$$

We arrive at the same result with t replaced by k.

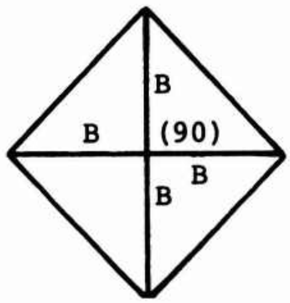
$$_I\text{Eff}(2t) = \frac{(4k+3)(4k^2+1)}{3(8k^2+4k+1)} \sim \frac{4}{3}k.$$

In tabular form;

$_I$Eff	1.33	2.67	4.0	5.3	6.67	8.0
k	1	2	3	4	5	6

12.6 A Closed Boundary Model

In this part of the model it is assumed that each repeater is less than B units from the origin. As in earlier cases, the units are measured in steps. The region is now closed and appears as follows:



Furthermore, in the initial stages of this model, it is assumed that a single message originates at the origin at time t=0. All message flow is generated as a result of this single message. Unfortunately, this model causes a loss of symmetries which facilitated the ease of obtaining closed form solutions for message flow in the previous models. However, some closed form analysis can be obtained. In particular it is not difficult to obtain an algorithm which will supply a complete analysis of message flow at any point in time on any repeater or station.

Let $N^d_{B,j}(2t+d)$ be the number of messages received at a station or repeater which is d-units from the origin and j-units from the y-axis at time (2t+d) where $d=0, 1, 2,\ldots,B-1$, $j=0, 1, 2,\ldots B$.

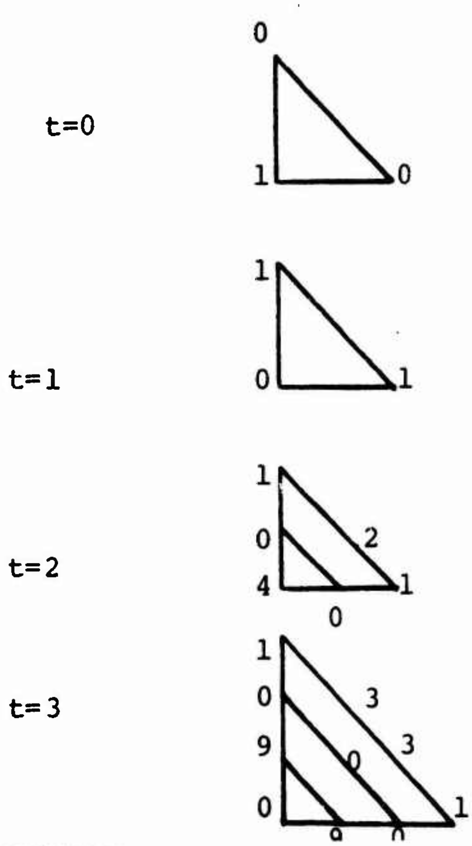
The equations for $N^{d}_{B,j}(2t+d)$ are identical to the equations for the open boundary case when $0 \le t \le B-d$ since the closing of the boundary does not affect message flow at a repeater at distance B-d until t=B+d. Therefore;

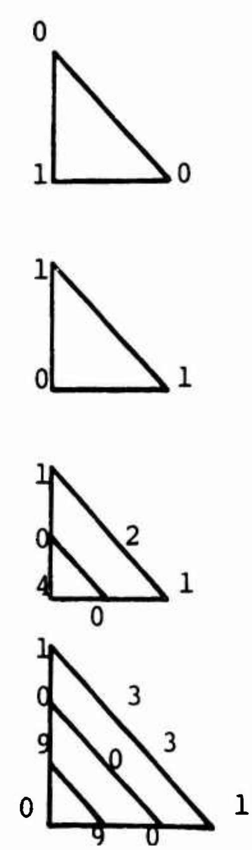
$$N^{d}_{B,j}(2t+d) = \binom{2t+d}{t}\binom{2t+d}{t+j}; \quad t \le \frac{B}{2}.$$

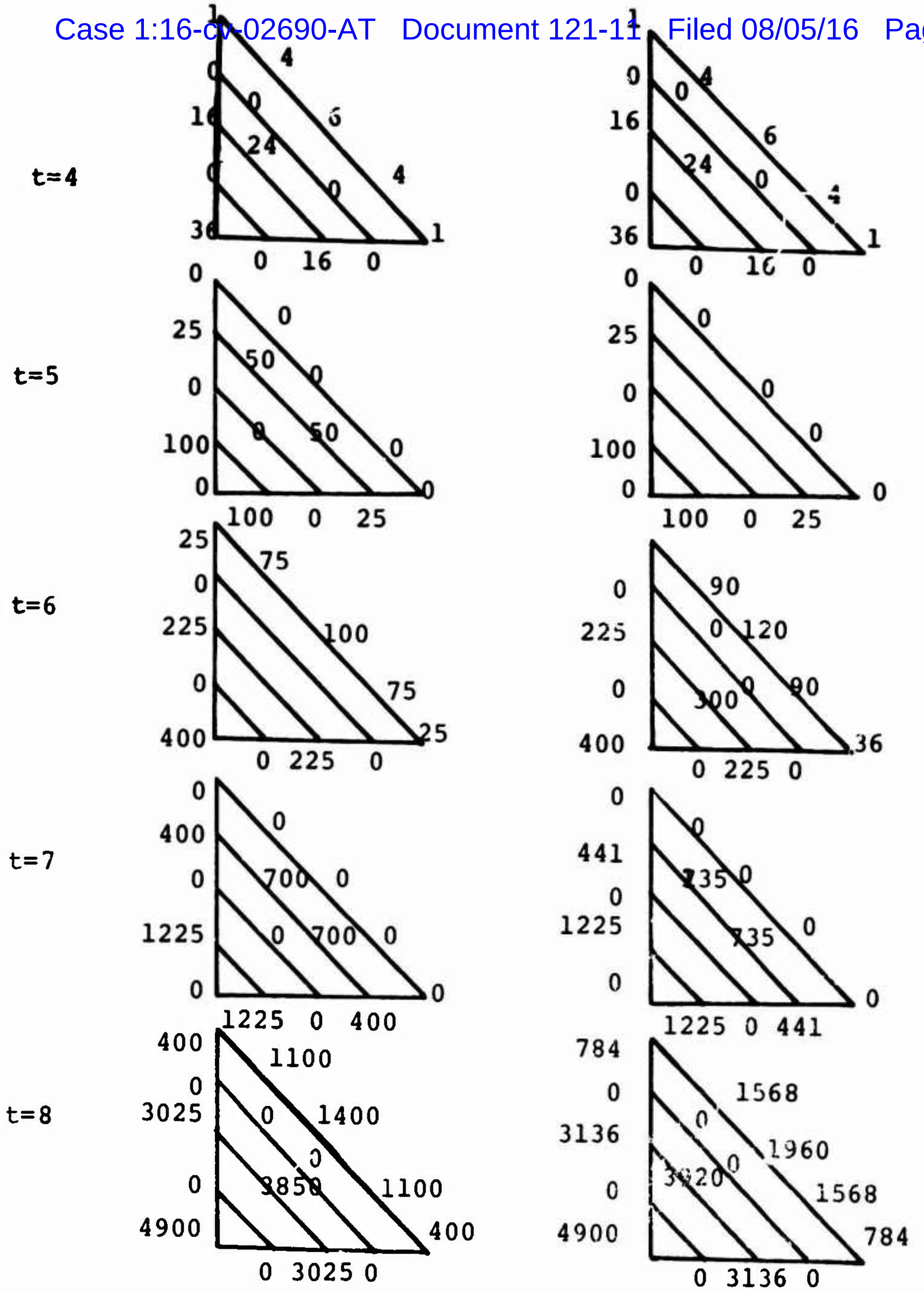
We can use this equation to compute message flow at all time points up to and including t=B. To compute message flow after time B, we can work backwards from the boundary and successively compute message flow at each station or repeater for <u>any</u> time point using the equation we will develop. First however, it is helpful to examine a particular case with a diagram and compare the closed boundary with the open boundary. We let B=4.

<u>Closed Boundary at B=4</u> <u>Open Boundary</u>

The closing of the boundary will not affect any repeaters or stations until t=6.







It is obvious how to continue for as long as one wishes to obtain the message flow. Since all message flow can be computed by the formulas of the preceding section for $t \leq B$, the message flow with closed boundary can be computed by backward iteration for all times larger than B. The initial conditions at time t=B are given by the following relations,

<u>At time t=B</u>

$$N^{d}_{B,j}(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise.} \end{cases}$$

since $N^{d}_{B,j}(B)$ is zero unless B and d are both odd or even numbers.

<u>At time t=B+1</u>

$N^{B}_{B,j}(B+1)=0$, i.e. no messages at the boundary,

$$N^{d}_{B,j}(B+1)=\begin{cases}\binom{B+1}{k}\binom{B+1}{k+j} & \text{if } B=2k+d-1,\\ 0 & \text{otherwise.}\end{cases}$$

<u>At time t=B+2</u>

On the boundary:

$$N^{B}_{B,0}(B+2) = N^{B-1}_{B,0}(B+1) = \binom{B+1}{1}^2 = \binom{B+1}{B}^2 = N^{B}_{B,B}(B+2).$$

$$N^{B}_{B,j}(B+2) = N^{B-1}_{B,j}(B+1) + N^{B-1}_{B,j-1}(B+1)$$
$$=\binom{B+1}{B}\binom{B+2}{j+1} = N^{B}_{B,B-j}(B+2)$$
for $j=1, 2,\ldots,B-1$.

for $d < B$

$$N^{d}_{B,j}(B+2) = \begin{cases}\binom{B+2}{k}\binom{B+2}{k+j} & \text{if } B=2k+d-2,\\ 0 & \text{otherwise}\end{cases} \qquad j=0, 1, 2,\ldots,d.$$

<u>At time t=B+3</u>

On the boundary

$N^{B}_{B,j}(B+3) = 0$: $j=0, 1, 2,\ldots,B$.

<u>At distance d=B-1</u>

$$N^{B-1}_{B,0}(B+3) = N^{B}_{B,0}(B+2) + N^{B-2}_{B,0}(B+2) + 2N^{B}_{B,1}(B+2)$$
$$=\left[\binom{B+1}{B}+\binom{B+2}{B}\right]^2 = N^{B-1}_{B,B-1}(B+3).$$

$$N^{B-1}_{B,j}(B+3) = N^{B-2}_{B,j-1}(B+2)+N^{B}_{B,j+1}(B+2) + N^{B-2}_{B,j}(B+2)$$
$$+ N^{B}_{B,j}(B+2)$$
$$=\binom{B+3}{j+2}\left[\binom{B+1}{B} + \binom{B+2}{B}\right] = N^{B-1}_{B,B-1-j}(B+3).$$
$$j=1, 2,\ldots,B-2.$$

At distance $d < B-1$

$$N^{d}_{B,j}(B+3) = \begin{cases} \binom{B+3}{k}\binom{B+3}{k+j} & B=2k+d-3 \\ 0 & \text{otherwise} \end{cases} \qquad j=0, 1,\ldots,d.$$

At time $t=B+4$

On the boundary

$$N^{B}_{B,0}(B+4) = N^{B-1}_{B,0}(B+3) = \left[\binom{B+1}{B}+\binom{B+2}{B}\right]^2 = N^{B}_{B,B}(B+4).$$

$$N^{B}_{B,1}(B+4) = N^{B-1}_{B,0}(B+3) + N^{B-1}_{B,1}(B+3)$$

(Note that $N^{B}_{B,1}(B+4)$ must be computed separately since $N^{B-1}_{B,0}(B+3)$ and $N^{B-1}_{B,j}(B+3)$ for $j > 1$ do not share a common formula, i.e. are not special cases of a general formula.)

$$N^{B}_{B,1}(B+4) = N^{B}_{B,B-1}(B+4) = \left[\binom{B+1}{B}+\binom{B+2}{B}\right]\left[\binom{B+1}{B}+\binom{B+2}{B}+\binom{B+3}{B}\right]$$

$$N^{B}_{B,j}(B+4) = N^{B-1}_{B,j-1}(B+3) + N^{B-1}_{B,j}(B+3)$$

(Note the formula is the same for $j=1$ as above.)

$$= \left[\binom{B+1}{B}\binom{B+2}{B}\right]\left[\binom{B+4}{j+2}\right] = N^{B}_{B,B-j}(B+4)$$
$$j=2,\ldots,B-2.$$

At distance $d=B-2$

$$N^{B-2}_{B,0}(B+4) = N^{B-3}_{B,0}(B+3) + N^{B-1}_{B,0}(B+3) + 2\,N^{B-1}_{B,1}(B+3)$$
$$= \left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}\right]^2 = N^{B-2}_{B,B-2}(B+4)$$

$$N^{B-2}_{B,j}(B+4) = N^{B-3}_{B,j-1}(B+3) + N^{B-1}_{B,j+1}(B+3) + N^{B-1}_{B,j}(B+3)$$

$$+ N^{B-3}_{B,j}(B+3).$$

$$= \binom{B+4}{j+3}\left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$= N^{B-2}_{B,B-j-2}(B+4).$$

for $d < B-2$

$$N^{d}_{B,j}(B+4) = \begin{cases} \binom{B+4}{k}\binom{B+4}{k+j} & \text{if } B=2k+d-4 \\ 0 & \text{otherwise} \end{cases}.$$

<u>At time t=B+5</u>

<u>at distance d=B-1</u>

$$N^{B-1}_{B,0}(B+5) = N^{B-2}_{B,0}(B+4) + N^{B}_{B,0}(B+4) + 2\,N^{B}_{B,1}(B+4)$$

$$= \left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]^2 = N^{B-1}_{B,B-1}(B+5)$$

$$N^{B-1}_{B,1}(B+5) = N^{B-2}_{B,0}(B+4) + N^{B}_{B,2}(B+4) + N^{B-2}_{B,1}(B+4)$$

$$+ N^{B}_{B,1}(B+4)$$

$$= \left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$\cdot\left[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}\right] = N^{B-1}_{B,B-1}(B+5).$$

$$N^{B-1}_{B,j}(B+5) = N^{B-2}_{B,j-1}(B+4) + N^{B}_{B,j+1}(B+4) + N^{B-2}_{B,j}(B+4)$$

$$+ N^{B}_{B,j}(B+4)$$

$$= \binom{B+5}{j+3}\left[2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}\right]$$

$$= N^{B-1}_{B,B-j-1}(B+5).$$

$$j=2,\ldots,B-3.$$

At distance B-3

$$N_{B,0}^{B-3}(B+5) = N_{B,0}^{B-4}(B+4) + N_{B,0}^{B-2}(B+4) + 2N_{B,1}^{B-2}(B+4)$$

$$= [\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]^2 = N_{B,B-3}^{B-3}(B+5).$$

$$N_{B,j}^{B-3}(B+5) = N_{B,j-1}^{B-4}(B+4) + N_{B,j+1}^{B-2}(B+4)$$

$$+N_{B,j}^{B-4}(B+4) + N_{B,j}^{B-2}(B+4)$$

$$= \binom{B+5}{j+4}[\binom{B+1}{B} + \binom{B+2}{B} + \binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N_{B,B-j-3}^{B-3}(B+5) \qquad j=1,\ 2,\ldots,B-4.$$

for d < B-3

$$N_{B,j}^{d}(B+5) = \begin{cases} \binom{B+5}{k}\binom{B+5}{k+j} & \text{if } B = 2k+d-5 \\ 0 & \text{otherwise.} \end{cases}$$

$$j=0,\ 1,\ 2,\ldots,d.$$

We will continue for three more time points to get some idea as to how the solution behaves with time. We will omit the general equations and give only the results since it is clear that the expressions become too cumbersome for easy comprehension.

For time t=B+6

at the boundary

$$N_{B,0}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}]^2 = N_{B,B}^{B}(B+6)$$

$$N_{B,1}^{B}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][3\binom{B+1}{B} + 3\binom{B+2}{B}+2\binom{B+3}{B}+\binom{B+4}{B}]$$

$$N^{B}_{B,2}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+1}{B} + \binom{B+2}{B}+\cdots+\binom{B+5}{B}]$$

$$N^{B}_{B,j}(B+6) = [2\binom{B+1}{B} + 2\binom{B+2}{B} + \binom{B+3}{B}][\binom{B+6}{J+3}]$$

$j=3,\ 4,\ldots,B-3.$

At distance $d=B-2$

$$N^{B-2}_{B,0}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]^2 = N^{B-2}_{B,B-2}(B+6)$$

$$N^{B-2}_{B,1}(B+6) = [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}]$$

$$\cdot\ [\binom{B+1}{B} + \binom{B+2}{B} +\cdots+\binom{B+5}{B}] = N^{B-2}_{B,B-3}(B+6)$$

$$N^{B-2}_{B,j}(B+6) = \binom{B+6}{j+4}\ [3\binom{B+1}{B} + 3\binom{B+2}{B} + 2\binom{B+3}{B} + \binom{B+4}{B}] = N^{B-2}_{B,B-2-j}(B+6)$$

$j=2,\ldots,B-4.$

At distance $d=B-4$

$$N^{B-4}_{B,0}(B+6) = [\binom{B+1}{B} + \binom{B+2}{B} +\cdots+ \binom{B+5}{B}]^2 = N^{B-4}_{B,B-4}(B+6)$$

$$N^{B-4}_{B,j}(B+6) = \binom{B+6}{j+5}[\binom{B+1}{B} + \binom{B+2}{B} +\cdots+ \binom{B+5}{B}]^2 = N^{B-4}_{B,B-4-j}(B+6)$$

$j=1,\ 2,\ldots,B-5.$

For $d < B-4$

$$N^{d}_{B,j}(B+6) = \begin{cases} \binom{B+6}{k}\binom{B+6}{k+j} & \text{if } B=2k+d-6 \\ 0 & \text{otherwise.} \end{cases}$$

At time $t=B+7$

At distance $d=B-1$

$$N^{B-1}_{B,0}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]^2 = N^{B-1}_{B,B-1}(B+7).$$

$$N^{B-1}_{B,1}(B+7) = [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$\cdot\ [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]$$

$$= N^{B-1}_{B,B-2}(B+7).$$

$$N^{B-1}_{B,2}(B+7) = (\sum_{j=1}^{6}\binom{B+j}{B})\ [5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$= N^{B-1}_{B,B-3}(B+7).$$

$$N^{B-1}_{B,j}(B+7) = \binom{B+7}{j+4}[5\binom{B+1}{B} + 5\binom{B+2}{B} + 3\binom{B+3}{B} + \binom{B+4}{B}].$$

$$j=3,\ldots,B-4.$$

At distance d=B-3

$$N^{B-3}_{B,0}(B+7) = [4\binom{B+1}{B} + 4\binom{B+2}{B} + 3\binom{B+3}{B} + 2\binom{B+4}{B} + \binom{B+5}{B}]^2$$

$$N^{B-3}_{B,1}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{B}][4\binom{B+1}{B}+4\binom{B+2}{B}+3\binom{B+3}{B}+2\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B-3}_{B,j}(B+7) = \binom{B+7}{j+5}[4\binom{B+1}{B}+4\binom{B+2}{B}+3\binom{B+3}{B}+2\binom{B+4}{B}+\binom{B+5}{B}]$$

$$j=2,\ldots,B-5.$$

At distance B-5

$$N^{B-5}_{B,0}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{j}]^2 = N^{B-5}_{B,B-5}(B+7)$$

$$N^{B-5}_{B,j}(B+7) = [\sum_{j=1}^{6}\binom{B+j}{j}]\binom{B+7}{j+6}. \quad j=1, 2,\ldots,B-6.$$

At d < B-5

$$N^{d}_{B,j}(B+7) = \begin{cases} \binom{B+7}{k}\binom{B+7}{k+j} & \text{if } B=2k+d-7 \\ 0 & \text{otherwise} \end{cases}$$

At time t=B+8

at the boundary

$$N^{B}_{B,0}(B+8) = N^{B}_{B,B}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]^2$$

$$N^{B}_{B,1}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+2}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B}_{B,2}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$\cdot[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N^{B}_{B,3}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B} + \binom{B+4}{B}]$$

$$N^{B}_{B,j}(B+8) = \binom{B+8}{B}[5\binom{B+1}{B}+5\binom{B+2}{B}+3\binom{B+3}{B}+\binom{B+4}{B}]$$

$$j=4,\ldots,B-4.$$

At distance d=B-2

$$N^{B-2}_{B,0}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]^2$$

$$N^{B-2}_{B,1}(B+8) = [9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$\cdot[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N^{B-2}_{B,2}(B+8) = (\sum_{j=1}^{7}\binom{B+j}{B})[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

$$N^{B-2}_{B,j}(B+8) = \binom{B+8}{B}[9\binom{B+1}{B}+9\binom{B+2}{B}+6\binom{B+3}{B}+3\binom{B+4}{B}+\binom{B+5}{B}]$$

At distance d=B-4

$$N_{B,0}^{B-4}(B+8) = [5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]^2$$

$$= N_{B,B-4}^{B-4}(B+8)$$

$$N_{B,1}^{B-4}(B+8) = [\sum_{k=1}^{7}\binom{B+k}{B}][5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$$N_{B,j}^{B-4}(B+8) = \binom{B+8}{j+6}[5\binom{B+1}{B}+5\binom{B+2}{B}+4\binom{B+3}{B}+3\binom{B+4}{B}+2\binom{B+5}{B}+\binom{B+6}{B}]$$

$j=2,\ldots,B-6.$

At distance d< B-4

$$N_j^d(B+8) = \begin{cases} \binom{B+8}{k}\binom{B+8}{k+j} & \text{if } B=2k+d-8 \\ 0 & \text{otherwise.} \end{cases}$$

This completes an analysis of the first eight time points past the time to the boundary. The expressions are quite unwieldy. However, the algorithm is clear and can compute message distribution throughout the grid at any time point. We write down the general set of equations and then indicate a "closed form" combinatorial method to make "sense" out of the unwieldy expressions.

The General Equations

The initial conditions for t=B are:

At time t=B: Initial Conditions

$$N_{B,j}^d(B) = \begin{cases} \binom{B}{k}\binom{B}{k+j} & \text{if } B=2k+d \\ 0 & \text{otherwise} \end{cases}$$

The General Equations

On the boundary $k \geq 1$

1. $N^{B}_{B,0}(B+2k) = N^{B}_{B,B}(B+2k) = N^{B-1}_{B,0}(B+2k-1).$

2. $N^{B}_{B,j}(B+2k) = N^{B-1}_{B,j}(B+2k-1) + N^{B-1}_{B,j-1}(B+2k-1)$

$j=1, 2,\ldots,B-1.$

Note that:

$$N^{B}_{B,j}(B+2k) = N^{B}_{B,B-j}(B+2k), \qquad j=0,\ldots,B.$$

Along the axes $0 < d < B$, $k=1, 2,\ldots.$

$$\begin{aligned} N^{d}_{B,0}(B+2k) &= N^{d+1}_{B,0}(B+2k-1) + N^{d-1}_{B,0}(B+2k-1) \\ &+ 2N^{d+1}_{B,1}(B+2k-1) \\ &= N^{d}_{B,d}(B+2k) \end{aligned}$$

Off the axes $0 < d < B$, $0 < j < d$

$$\begin{aligned} N^{d}_{B,j}(B+2k) &= N^{d+1}_{B,j}(B+2k-1) + N^{d-1}_{B,j}(B+2k-1) \\ &+ N^{d-1}_{B,j-1}(B+2k-1) + N^{d+1}_{B,j+1}(B+2k-1). \end{aligned}$$

At the origin:

$$N^{0}_{B,0}(B+2k) = 4N^{1}_{B,1}(B+2k-1).$$

This completes the general equations.

A Conjecture on Solutions to the General Equation

An examination of the coefficients of the $\binom{B+k}{B}$ expressions in the first eight time points indicates that the coefficients are the same as the rows in the following infinite sequence of tableaus.

T_1

	j=1	2	3	4	5	6
k=1	1					
k=2	1	1				
k=3	2	2	1			
k=4	5	5	3	1		
k=5	14	14	9	4	1	
k=6	42	42	28	14	5	1
	.	.	.	.	.	.
	.	.	.	.	.	.

T_2

k \ j	1	2	3	4	5	6	7
1	0	1					
2	1	1	1				
3	3	3	2	1			
4	9	9	6	3	1		
5	28	28	19	10	4	1	
6	90	90	62	34	15	5	1
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.

T_3

k=1	0	0	1					
2	1	1	1	1				
3	4	4	3	2	1			
4	14	14	10	6	3	1		
5	48	48	34	20	10	4	1	
6	165	165	116	69	35	15	5	1
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.

T_4

0	0	0	1					
1	1	1	1	1				
5	5	4	3	2	1			
20	20	15	10	6	3	1		
75	75	55	35	20	10	4	1	
275	275	200	125	70	35	15	5	1
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

The tableaus through T_4 cover all the coefficients which arise up to and including t=B+8. It is possible to give a solution for $S_{kj}(T_\nu)$ which is the j^{th} element in the k^{th} row of the tableau T_ν, $\nu = 1, 2, ...$ However, it is perhaps not obvious how these tableaus are generated. The j^{th} term of the k^{th} row of any tableau is obtained by summing the last $(k+\nu-j)$ terms in the previous row. That is, the third term in the fourth row of T_2 is obtained by summing the last three terms of the third row. In mathematical form the equation for the tableaus are:

Initial Condition $$S_{1,j}(T_\nu) = \begin{cases} 0, j<\nu, \\ 1, j=\nu \\ 0, j>\nu. \end{cases}$$

$$S_{k,1}(T_\nu) = S_{k,2}(T_\nu) = \sum_{j=1}^{k+\nu-2} S_{k-1,j}(T_\nu).$$

$$S_{kj}(T_\nu) = \sum_{w=j-1}^{k+\nu-2} S_{k,w}(T_\nu), \qquad k > 1$$

It is clear that the between tableau equations are:

$$S_{k,j}(T_\nu) = S_{k,j}(T_{\nu+1}) - S_{k-1,j}(T_{\nu+2}), \quad k > 1, \quad \nu = 1, 2, \ldots$$

$$S_{k,j}(T_2) = S_{k+1,j}(T_1) - S_{k,j}(T_1).$$

Thus, to solve these equations it suffices to give a general formula for $S_{k,j}(T_1)$. The other terms can then be computed recursively. It is easy to check that the solution for T_1 is given by,

$$S_{k,j}(T_1) = \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad j=1, 2, \ldots, k \quad k=1, 2, \ldots.$$

Then by recursion:

$$S_{k,j}(T_2) = \frac{j}{k+1}\binom{2k-j+1}{k} - \frac{j}{k}\binom{2k-j-1}{k-1}, \qquad k=1, 2, \ldots. \quad j=1, 2, \ldots, k+1.$$

$$S_{k,j}(T_3) = \frac{j}{k+2}\binom{2k-j+3}{k} + \frac{2j}{k+1}\binom{2k-j+1}{k}$$

$$\vdots$$

To apply these results we take a particular example to conjecture:

$$N_{B,0}^{B}(B+2k) = \left[\sum_{j=1}^{k} S_{k,j}(T_1)\binom{B+j}{B}\right]^2, \qquad k \geq 1$$

$$= \left[\sum_{j=1}^{k} \frac{j}{k}\binom{2k-j-1}{k-1}\binom{B+j}{B}\right]^2$$

This conjecture holds for time points k=1, 2, 3, 4.

$$N_{B,0}^{B-1}(B+2k+1) = [\sum_{j=1}^{k+1} S_{k+1,j}(T_1)\binom{B+j}{B}]^2$$

This conjecture holds for all k=1, 2, 3, and satisfies the general equations.

13 SUMMARY OF RESULTS OF THEORETICAL MODELS

Recall that;

$N_o(t)$ = expected number of messages received at the origin at time t.

$N'_o(t)$ = expected number of distinct messages received at the origin for the first time at time t.

$$Eff(t) = \frac{N_o(t)}{N'_o(t)}.$$

These symbols are in the model where there is Poisson input at each repeater at each time point. In all modes;

$$N'_o(t) = (2t^2+2t+1).$$

A. Deterministic The first model analyzed is the effect of a single message originally at the origin at time zero. No other messages ever enter the system. The propagation of this message can be measured by the following three quantities.

1) B(t) - the number of repeaters which receive the message for the 1st time at time t.

$$\boxed{B(t) = 4t \text{ for } t \geq 1, \quad B(0) = 1}$$

2) A(t) = the number of repeaters which have received the message by time t,

$$\boxed{A(t) = 2t^2+2t+1.}$$

3) $N_j^d(t)$ = number of copies of the message received at time t at a repeater with coordinates (d,j).

$$N_j^d(t) = 0, \quad \text{if} \quad t < d$$

$$N_j^d(d) = \binom{d}{j} \quad j=0, 1, \ldots, d; \quad d=0, 1, \ldots,$$

$$N_j^d(d+2k+1) = 0 \quad k=0, 1, 2, \ldots ,$$

$$N_j^d(d+2k) = \binom{d+2k}{k}\binom{d+2k}{k+j} \quad k=0, 1, \ldots .$$

The quantity $N_j^d(d+2k)$ can be more generally interpreted as the number of copies of a message received at a repeater which is at distance d and horizontal distance j from the originating repeater, after d+2k time units.

B. Poisson Input Results

1. With no restrictions imposed on the operation of any repeater;

$$N_o(t) = \frac{\lambda}{3}(4^{t+1}-1)$$

$$N_o'(t) = \lambda(2t^2+2t+1)$$

$$Eff(t) = \frac{4^{t+1}-1}{3(2t^2+2t+1)} \sim \frac{4^{t+1}}{6t^2}$$

2. No Message Can Be Transmitted More Than k-Times

$$N_o(t) = \frac{\lambda}{3}[4^{k+1}-1]$$

$$N_o'(t) = \lambda[2k^2+2k+1]$$

$$Eff\ (t) = \frac{4^{k+1}-1}{3(2k^2+2k+1)}$$

Eff	1	1.60	3.40	8.3	22.4
k	1	2	3	4	5

3. <u>If the Same Message Arrives From Different Sources Only One Transmission is Made</u>

This mode of operation assumes that a repeater can compare all messages arriving at the same time and transmit only one copy of duplicate messages. It is inherently assumed in this mode that there is only instantaneous memory (which we may call 0-memory)

$N_o(2t) = \lambda \frac{4t+3)(8t^2+1)}{3}$
$N'_o(2t) = \lambda(8t^2+4t+1)$
$IEff(2t) = \frac{(4t+3)(8t^2+1)}{3(8t^2+4t+1)} \sim \frac{4t}{3}$

This mode produces a substantial reduction in duplicate traffic.

4. <u>All Messages Are Transmitted in the Direction of a Fixed Ground Station</u>

In this mode it is assumed that repeater "knows" where the message should be received.

$N_o(t) = \lambda\ 2^{t+3}+4t-7)$
$N'_o(t) = \lambda(2t^2+2t+1)$
$IEff(t) = \frac{2^{t+3}+4t-7}{2t^2+2t+1} \sim \frac{2^{t+2}}{t^2}$

This mode is better than no mode, but holds little promise by itself.

5. <u>A Repeater Never Transmits The Same Message More Than Once, Except Upon Initial Reception</u>

In this mode a repeater has "infinite" memory but not instantaneous memory.

$N_o(t) = \lambda(2^{t+4}-16t-11)$
$N'_o(t) = \lambda(2t^2+2t+1)$
$IEff(t) = \frac{2^{t+4}-16t-11}{2t^2+2t+1} \sim \frac{2^{t+3}}{t^2}$

It appears that infinite memory without instant memory does not "damp" the message flow significantly:in fact it is worse than "directionality" in the repeater.

6. Infinite Memory Plus Instant Memory

If we combine the two modes of operation

$N_o(t) = \lambda(8t^2-8t-3)$
$N'_o(t) = \lambda(2t^2+2t+1)$
$IEff(t) = \frac{8t^2-8t-3}{2t^2+2t+1} \sim 4$

The effect of combining the instant memory feature with the instantaneous memory feature is to reduce the message flow in the instantaneous mode by a factor of t/3 which is significant.

7. Instant Memory Plus no Message Transmitted More Than k-Times

In this mixed mode the analysis shows that all results in the instant memory case are the same except that the flow becomes independent of time after $t \geq k$. The inefficiency is

$IEff(t) \sim \frac{4}{3}k$

IEff	1.33	2.67	4	5.33	6.67	8
k	1	2	3	4	5	6

CHAPTER 12

TIME AND SPACE CAPTURE IN SPREAD SPECTPUM RANDOM ACCESS

I. INTRODUCTION AND SUMMARY

When a receiving station for packet data communication is being accessed in a random access mode, the use of spread spectrum communication offers the possibility of time capture. That is, a packet may be distinguished and received correctly even if contending packets overlap the transmission as long as the signal strength of the contending packets is not too great. Moreover, if multiple receivers are available at the receiving station, spread spectrum coding can be used to allow the reception of several distinct packets being transmitted with overlap on a single channel.

When the transmitters are widely distributed, geometric or power capture is possible [Roberts; 1972]. With or without spread spectrum, if a competing signal is much weaker (further away), than the desired signal, there is no interference. Both types of capture can give rise to performance superior to that predicted by a simple unslotted ALOHA model. On the other hand, power capture gives rise to bias against the more distant transmitters because the close in transmitters overpower the ones further away. Thus, the probability q(r) of a successful transmission at a distance r from the station will decrease as r increases and the number of retransmissions increase as well as delay.

The purpose of this chapter is to characterize the probability of successful transmission as a function of:

1. distance from the receiver, r,

2. the multiplicity, M, of receivers available at the receiving location,

3. the packet arrival rate,

4. the time bandwidth product K (extent of spectrum spreading),

5. the required signal to noise ratio SN of the receivers,

6. the exponent α in the inverse power law (s/r^{α}) assumed for the transmission power of a transmitter at distance r from the receiver.

The situation we consider is that of a circular field of transmitters of radius R accessing a receiving station in the center with M receivers where R is the range of the receivers. Arriving traffic is assumed to have a uniform rate per unit area in the field and the arrivals plus retransmissions are assumed to be Poisson distributed.

Particularly important for the design of a Packet Radio System is the analysis of multiple receiver stations. Adding extra receivers to the Packet Radio Station adds throughput and shortens delay. This is a very attractive approach because the modification is simple and is applied only at the station. No modifications are required to the terminals, the repeaters, or their geographical location. Finally, the enhancement is applied directly where it is needed at the station bottleneck where all the information flow of the system must eventually pass. The simulation results show that multiple receivers will in fact increase performance substantially; moreover, very few additional receivers are required to realize most of the advantage (usually only one additional is required). For addition of receivers to be effective, two prerequisites must be satisfied: (i) the traffic rate must be quite high; otherwise, the chance of several messages arriving simultaneously will be small and (ii) the spread spectrum factor K must be sufficiently large compared to the required signal to noise ratio SN; otherwise, even if there are sufficient receivers to receive a number of simultaneous messages the interference created by the messages will cause none of them to be received correctly.

The q(r) function derived here will also be very useful in the location of repeaters to provide coverage for terminals. The simplest models require that each possible terminal location be "covered" by some number of repeaters in order to guarantee reliable communication. This type of model assumes a binary characterization of transmitter-repeater communication: either communication is possible or it is not. The q(r) function could be used to reflect the fact that a far away receiver does not provide as good service as a nearby one.

In Section 2, the model and the assumptions it is based upon are introduced. In Section 3, analysis is used to predict the qualitative behavior of the q(r) function especially with respect to limiting values of parameters. It was found, for example, that to a user at far distances, $r \to R$, the system performs as in unslotted ALOHA. That is, a transmission arriving at time t_o gets through if and only if no other transmission starts in the interval $[t_o-T, t_o+T]$ where T is a packet transmission time. On the other hand, for $r \to o$ for the one receiver case performance approximates slotted ALOHA in the sense that only packets preceeding, $t_o - T \leq t \leq t_o$, can interfere. Explicit bounds on the benefits of adding multiple receivers are also given. In Section 4, simulation is used to verify and extend these results. Finally, the simulation program is documented in an Appendix.

The first analysis of the geometric effect of capture for a circular field of packet transmitters accessing a receiver in a random access mode was [Roberts; 1972]. Roberts made (among others) the following assumptions:

1. the probability q of a packet being received correctly for a given transmission or retransmission is:

(a) independent of distance r from the receivers,

(b) independent of whether the transmission is the first or a retransmission,

2. the probability of more than one contending packet is negligible,

3. noise is negligible compared to competing signal strengths.

Kleinrock and Lam [1972] improved on these results by relaxing assumption (1 b) to allow different q's for the initial transmission and for retransmissions. John Leung [1973] considered the same problem where the signal power, instead of being represented as a deterministic inverse square of distance, had a Rayleigh distribution to represent the effects of multipath. One consequence of this is that the receiver has unlimited range. Leung also essentially assumes (1) and (2) (see in particular, eq. 7, where G(r) is apparently assumed proportional to r; i.e., that the rate of arrivals plus retransmissions per unit area is independent of r.

Fralick [1972] also does an analysis of the field of terminals situation, extending Roberts' results for the use of spread slotted spectrum, including propagation delays. He assumes (1), (2), and (3).

Abramson [1973] considers a central receiver in an infinite field of transmitters and determines the "Sisyphus distance" R_s which is the radius beyond which the expected number of retransmissions is infinite. He assumes (2) and (3). Thus, the critical distance R_s is defined by competing signals while the R we use is defined by noise considerations. All the above models assumed one receiver, M=1.

The model for spread spectrum reception is similar to those described in [Kaiser; 1973, p. 2], [McGuire; 1973], and [Fralick; 1973a]. Fralick [1973a] considers several models of spread spectrum random access the one closest to ours is the one using "Synch Preamble." In particular, our lower bound for q(o) is based on his analysis of this case. Also, Fralick pointed out an error in normalization in Section 2 that appeared in an earlier version of this note. For simplicity, we assume that the surface wave devices for encoding the spread spectrum code are programable and that the chip code varies from bit to bit in a pseudo-random manner so that competing signals appear more nearly like noise; this, for example, avoids many of the difficulties pointed out by Fralick [1973]. Programmable surface wave devices are discussed in [Staples and Claiborne; 1973] and [LaRosa; 1973].

2. MODEL AND ASSUMPTIONS

A central receiving station is receiving messages from an infinite number of terminals within a radius R where R is defined to be the range of the station; that is, the distance at which a terminal can just be heard. The terminals send packets according to a Poisson distribution with arrival rate density ρ packets per unit time per unit area. The principal objective is to determine the grade of service for terminals as a function of distance from the station. The grade of service q(r) is defined to be the probability that a message sent by a terminal at radius r from the station will be received by the station on any one transmission (first and subsequent retransmissions are assumed to have the same probability of success [Kleinrock & Lam; 1972].)

Having q(r) we can also define G(r) = S(r)/q(r) to be the arrival plus retransmission rate density at radius r where S(r) is the arrival rate density at radius r given by $2\pi r\rho$.

For our purposes, the receiving station can be in one of M+1 states; o receivers busy, 1 receiver busy, ..., M receivers busy. We will denote these states as S_o, S_1, ..., S_M. In state $S_i(i<M)$, i receivers are busy and one of M-i remaining is awaiting a synchronization code from the beginning of some packet. If the receiver achieves synchronization with a new packet, the receiver is captured ($S_i \rightarrow S_i+1$) and is now busy (in a receiving state) and remains in that state for one packet transmission time, T. The new packet is correctly received if the total power of contending signals does not become too high during the transmission time. The receiver is busy for the full time T in either case. When all the receivers are busy, state S_M, all arriving packets are lost until one of the receivers becomes free. We specifically ignore the possibility of false alarms; that is, the receiver going from free to busy on the basis of noise. Thus, a packet arriving at time t is assumed to be received correctly by the station if the

following three conditions are satisfied:

1. a receiver is free at t.

2. the signal strength is sufficiently stronger than ambient noise and other signals arriving in (T-t,t) so that the receiver achieves synchronization.

3. signals arriving in (t, t+T) are not strong enough to drown out the signal after synchronization is achieved.

In our model of the receiver, we assume that a signal can be heard if the desired signal energy is sufficiently greater than the competing energy which consists of two elements: ambient noise and the other undesired signals. We are allowed to use spectrum spreading to increase discrimination between signal and the competing energy.

To be more specific, assume we divide each bit into a coded sequence of K chips. Further, suppose the receiver works by integrating the received signal correlated with the chip code. If the integral of the signal amplitude over one chip time is A, then, if there is correlation with the chip code, the received energy in one bit is $K^2A^2 = s/r^\alpha$ for a signal at distance r. If the signal is uncorrelated with the chip code, the received signal is the integral of a sum of K binomially distributed signals of amplitude A which for K reasonably large approaches a Gaussian distribution with mean o and variance $KA^2 = s/Kr^\alpha$. Thus, received signals in synchronization with the receiver deliver after correlation a factor of K more power than competing signals of the same strength which are uncorrelated with the receiver.

If we let N be the ambient noise per bit, the signal to noise energy ratio is:

$$\frac{\frac{s}{r_o^\alpha}}{\frac{1}{K}\left(\sum \frac{s}{r_i^\alpha}\right) + N} \geq SN \tag{1}$$

which we require to be no less than SN for reliable reception where r_o is the radius of the desired signal which is assumed to be in synch with the receiver's chip code and r_i, $i \neq o$ correspond to competing signals not in synch which effectively look like noise added to the ambient noise N.

The range R is determined by:

$$\frac{\frac{s}{R^\alpha}}{N} = SN, \text{ or}$$

$$R = \left(\frac{1}{N}\ \frac{s}{SN}\right)^{1/\alpha} \tag{2}$$

The parameters we wish to study are SN, α, and K; so, by judicious choice of units, we may normalize the units of power and distance, so that s = 1 and R = 1. This results in the decision criterion:

$$\frac{K \frac{1}{r_o^\alpha}}{\sum \frac{1}{r_i^\alpha} + \left(\frac{K}{SN}\right)} \geq SN \tag{3}$$

$$0 \leq r_i \leq 1 \qquad i = 0, 1, \ldots$$

See [Kaiser; 1973] for a similar model.

3. PRELIMINARY ANALYSIS

The exact analysis of the probability, $q(r)$, of a message getting through on any given transmission from a radius r seems most difficult; however, we can get quite a good qualitative idea of $q(r)$ by analysis. This analysis will be supplemented by simulation results in the next section.

Since the signal power is monotone decreasing with increasing radius and the message arrivals are independent, it is clear that $q(r)$ is monotone non-increasing. A message being transmitted, starting at time t from the critical radius $R = 1$, will be successful if, and only if, there is no other transmission in the interval $(t-T, t+T)$ since any additional signal will prevent reception. But this is exactly the situation for unslotted ALOHA. Thus, if G is the rate of message arrivals and retransmissions for the entire circular area of R radius from the receiver:

$$q(R) = e^{-2\,GT}. \tag{4}$$

Unfortunately, we do not yet know G. Clearly from (4) and the fact that $q(r)$ is monotone $G \leqq G_u$ where G_u is the arrival plus retransmission rate for an unslotted ALOHA system; i.e., G_u is the solution of

$$S = G_u\, e^{-2G_u T}$$

where $S = \pi R^2 \rho$ is the total arrival rate. We can generalize (4) somewhat. For, even if $r < R = 1$, if r is sufficiently large, any other transmission can prevent it from being received. Thus, suppose we are considering a packet from r_o contending with a packet arriving from $r_1 = R = 1$, the weakest possible. The message from r_o can be heard whenever,

$$\frac{K \dfrac{1}{r_o^{\alpha}}}{1 + \dfrac{K}{SN}} \geqq SN$$

or

$$r_o \leqq \left(\frac{1}{1 + \frac{SN}{K}} \right)^{1/\alpha} = \bar{r} \; ;$$

On the other hand, if $r_o \geq \bar{r}$, any other message is sufficient to block reception. Therefore, we have,

$$q(r) = e^{-2GT} \geqq e^{-2G_u T} \quad \text{for } \bar{r} \leqq r \leqq 1. \qquad (5)$$

As r→o, the signal power becomes infinite, so that the probability of successful transmission depends only on whether a receiver is free or not. For the case where M=1 and with perfect time capture, Fralick [1973a] has shown that the probability the receiver is free is $\frac{1}{1+G}$. Since there isn't perfect time capture in our model, the probability the receiver is free is greater than that. So we have:

$$g(o) \geqq \frac{1}{1+G}.$$

Given there are one or more transmissions in (t-T, t) the probability the receiver is captured is greater than the probability that one specific transmission, say the first in the interval, captures the receiver which is, in turn, greater than the probability that that specific transmission is received which is S/G on the average.

Thus,

q(o) = 1 - (Prob. of transmission in (t-T, t) times the probability receiver is captured by one of the transmissions)

$\leq 1 - (1 - e^{-GT}) \frac{S}{G}$

yielding

$$\frac{1}{1+G} \leq q(o) \leq 1 - (1-e^{-GT}) \frac{S}{G} \qquad (6)$$

If we add additional receivers we get improved performance for small r, but not for $\bar{r} \leq r \leq 1$. In order for additional receivers to be helpful, there must be sufficient traffic to keep them busy but not so much that the interference caused by non-synchronized messages overwhelms the desired one. We first determine the maximum number of receivers which can all be correctly receiving at the same time. This clearly happens when all signals are at the same radius and that radius approaches zero. For n receivers at radius ε with $\varepsilon \rightarrow o$, (3) has the limit

$$\frac{K}{n} \geq SN$$

Thus, we see that

$$\text{Maximum number of receivers} \leq K/SN \qquad (7)$$

For example, if the spread spectrum factor K were 100 and the required signal to noise power SN were 20, then at most, five receivers could be simultaneously receiving correctly. This indicates that the number of repeaters which can profitably be added is small.

Another factor affecting the utilization of multiple receivers is the amount of traffic in the system. Suppose the gross traffic including retransmissions is a Poisson Process with arrival rate G per second. Then, the number of messages being received at a given time t_o is simply the messages which arrived in the interval $(t_o - T, T)$ where T is the transmission time. The probability P(k) of k active messages is then given by the Poisson distribution:

$$P(GT;k) = \frac{(GT)^k e^{-GT}}{k!} \qquad k = 0, 1, \ldots \qquad (8)$$

We define

$$Q(GT;k) = \sum_{i=o}^{k} P(GT;i)$$

For an M-receiver system, a new arrival at r=o will be received if less than M messages arrive in the previous T seconds (although sometimes a receiver will be free even if M or more arrived in the interval); thus,

$$q(o) \geq Q(GT, M-1).$$

Similarly for $K \to \infty$ we have

$$q(r) \equiv \frac{S}{G} \geq Q(GT, M-1).$$

We do not know G but we can define $\bar{G}$ to be the solution of

$$S = G\ Q(GT, M-1)$$

yielding the bound $\frac{S}{G} \geq \frac{S}{\bar{G}}$ in the case $K \to \infty$ and $q(o) \geq \frac{S}{\bar{G}}$ in general.

To get an upper bound, we consider a hypothetical system with perfect capture in which messages turn themselves off if they do not capture a receiver. In other words, a message is received correctly if and only if there are less than M receivers which are in the process of correctly receiving a message. For such a system,

$$q(r) \equiv \frac{S}{G} = Q\ (ST,\ M-1)$$

where we assume the packets received correctly follow a Poisson process with arrival rate S. Since messages do not turn themselves off, our performance is worse and for real systems we have,

$$q(r) \leqq Q(ST,\ M-1) \qquad (9)$$

For M=1 we obtain,

$$q(r) \leqq e^{-ST} \qquad (10)$$

Since a finite K will only make the system behave still worse, (9) and (10) also hold for finite K.

It is important to note that almost all these results depend on the processes involved being Poisson. There are three processes of interest the arrivals to the system, the starting times of first transmissions and retransmissions, and the starting times of successful transmissions. The first and third processes have arrival rates S in equilibrium while the second process has rate G. It is reasonable to assume the first process is Poisson, and by being sufficiently clever with the retransmission scheme used, the second process can probably be made Poisson to an arbitrarily close approximation, but there is no way the third process can be Poisson. For example, the probability of M+1 successful receptions starting in an interval of length T is zero for the third process and positive for a Poisson process. The Poisson approximation has been shown to be accurate for lower traffic rates and one receiver but for the high traffic rates examined here and with multiple receivers discrepancies can be expected.

For example, safer bounds for (9) and (10) respectively might be,

$$(9') \qquad q(r) \leqq \frac{Q(ST,\ M-1)}{Q(ST,\ M)}$$

and for M=1

$$(10') \qquad q(r) \leqq \frac{1}{1 + ST}$$

In the simulations of the next section only the second process is assumed Poisson.

Now let us summarize our knowledge. In general we have:

(i) $q(r_1) = q(r_2)$ for $o \leqq r_1 \leqq r_2 \leqq 1$

(ii) $e^{-2G_u T} \leqq e^{-2GT} \leqq q(r)$ for $o \leqq r \leqq 1$ where G_u satisfies

$S = G_u e^{-G_u}$

(iii) $q(r) = e^{-2GT}$ for $\bar{r} \leq r \leq 1$ where $\bar{r} = \left(\frac{1}{1 + \frac{SN}{K}}\right)^{1/\alpha}$

(iv) $q(r) \leqq Q(ST,\ M-1)$

(v) $q(o) \geqq \frac{S}{\bar{G}}$ where $\bar{G}$ satisfies $S = GQ(GT,\ M-1)$

(vi) $q(o) = \frac{1}{1+G}$ if $K \to \infty$ and $q(o) \geq \frac{1}{1+G}$ for finite K.

(vii) $q(o) \leqq 1 - (1-e^{-GT}) \frac{S}{G}$

Finally, the maximum number of receivers which can be correctly receiving messages at the same time is less than or equal to K/M.

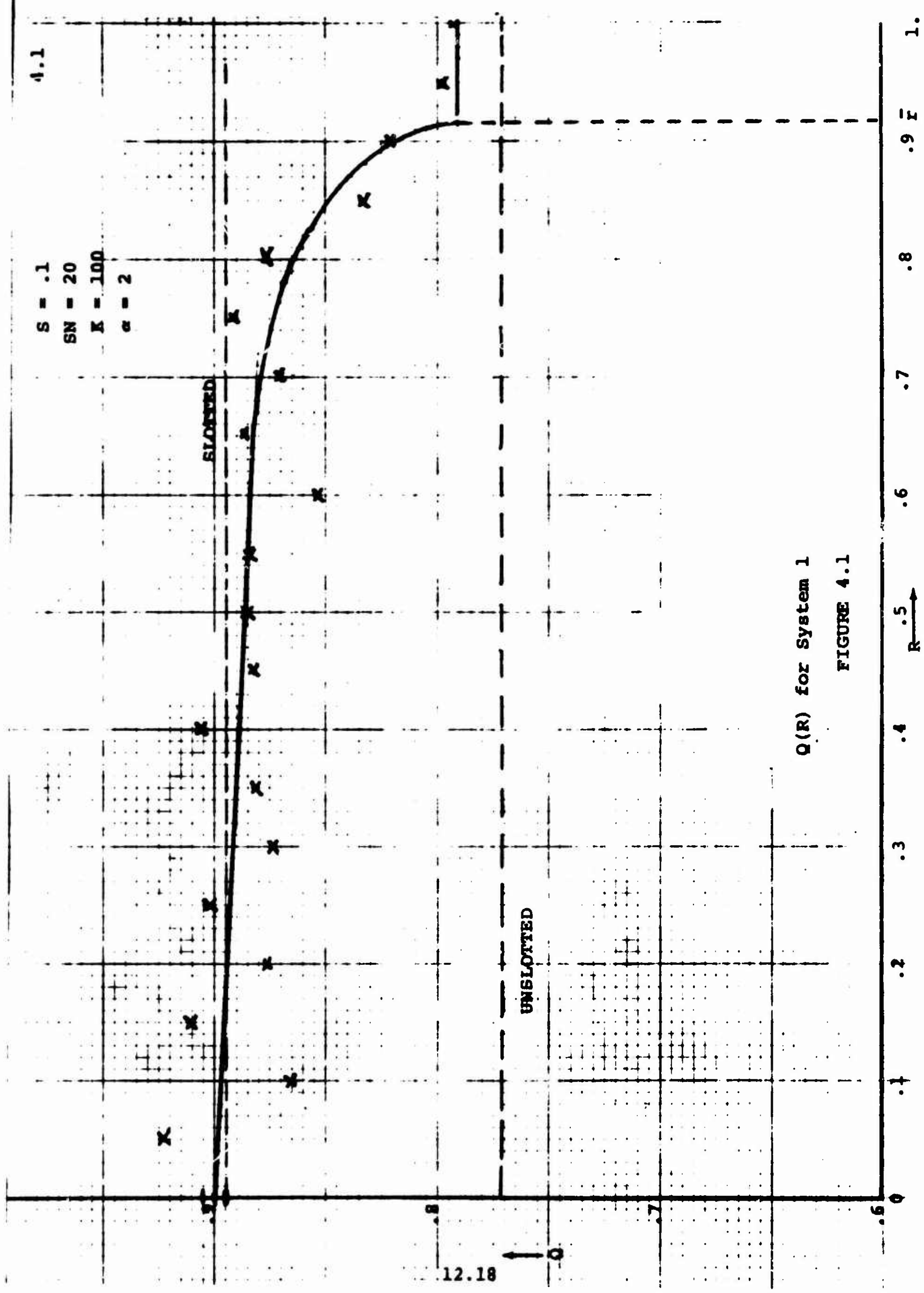
4. A SIMULATION APPROACH

Simulation was resorted to in order to verify the relations derived in Section III and to estimate more detailed attributes of the system in particular to estimate G which is required for several of the estimates in Section III. Details of the simulation method are given in the Appendix. In general, the estimates of G were quite stable, with apparent accuracies of better than 1%. The functional form of q(r) was not as successfully simulated; however, the simulations were consistent with the approximations and bounds of Section III within sampling error.

The nominal system simulated had S = .1, SN = 20, K = 100, M = 1, and α = 2; T was everywhere taken to be 1. Table 1 summarizes the systems simulated. Figures 1 - 7 are q(r) curves for the first seven systems simulated. Figure 8 shows the change in q(r) obtained by adding a second receiver (Systems 2 and 10); changes for adding more than one receiver were negligible in all cases. Figure 9 illustrates the dependence of G on K for fixed S; while Figure 10 shows the relation between G and S for K = 100 and K = 1000.

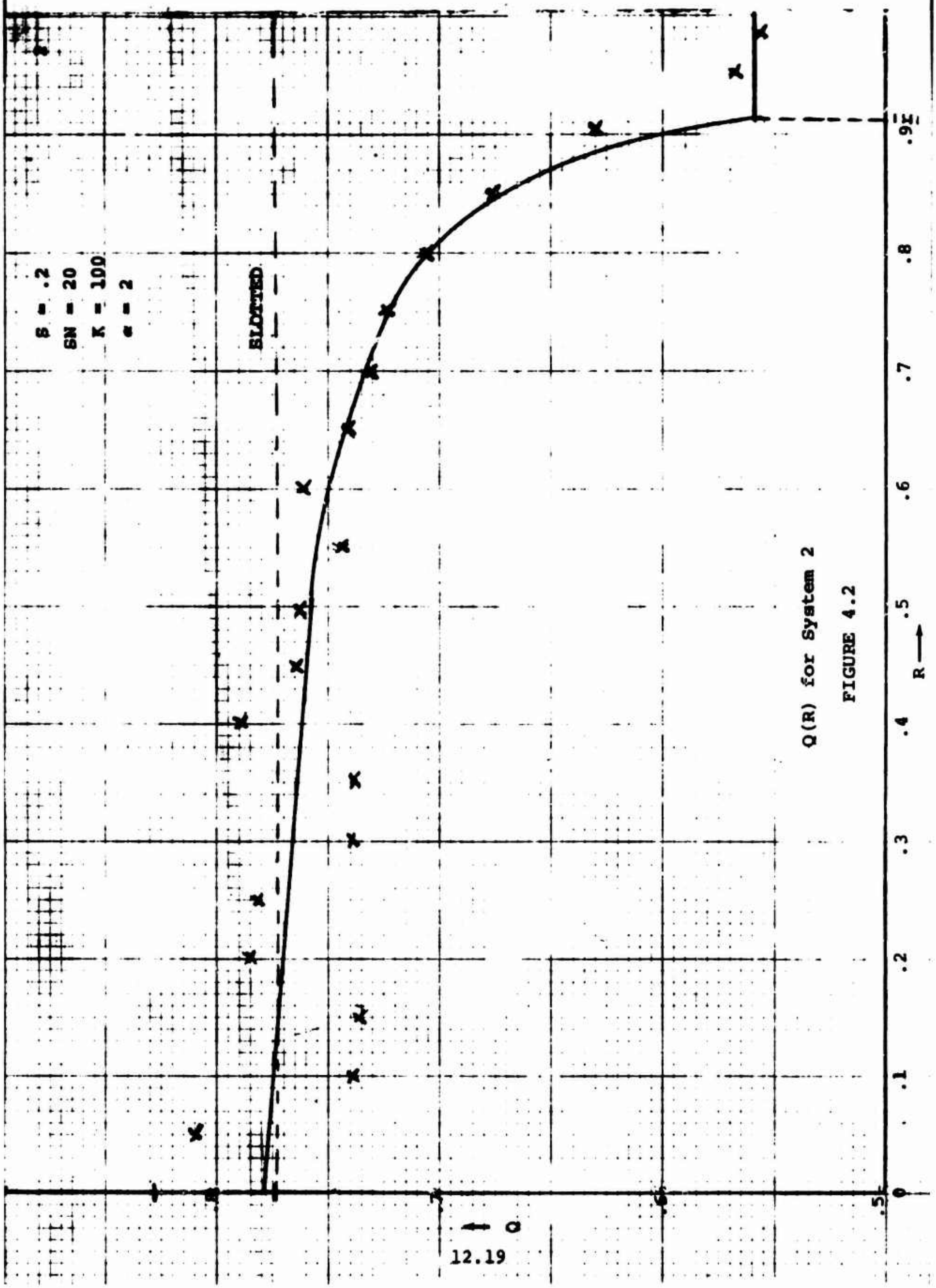
SYSTEM NUMBER	ARRIVAL RATE S	SIGNAL TO NOISE SN	SPREAD SPECTRUM FACTOR K	POWER EXPONENT α	CRITICAL RADIUS $\bar{r}$	MEASURED RATE G	NUMBER OF RECEIVERS M
1	.1	20	100	2	.913	.1170	1
2	.2	20	100	2	.913	.2920	1
3	.1	10	100	2	.953	.1152	1
4	.1	20	10	2	.577	.1255	1
5	.1	20	1000	2	.990	.1126	1
6	.1	20	1	2	.223	.1290	1
7	.1	20	100	4	.955	.1166	1
8	.1	20	100	2	.913	.1087	2
9	.1	20	100	2	.913	.1085	3
10	.2	20	100	2	.913	.2414	2
11	.2	20	100	2	.913	.2397	3
12	.2	20	100	2	.913	.2397	5
13	.333	20	1000	2	.990	.5503	1
14	.333	20	1000	2	.990	.3718	2
15	.333	20	1000	2	.990	.3589	3

TABLE 1
Simulation Runs



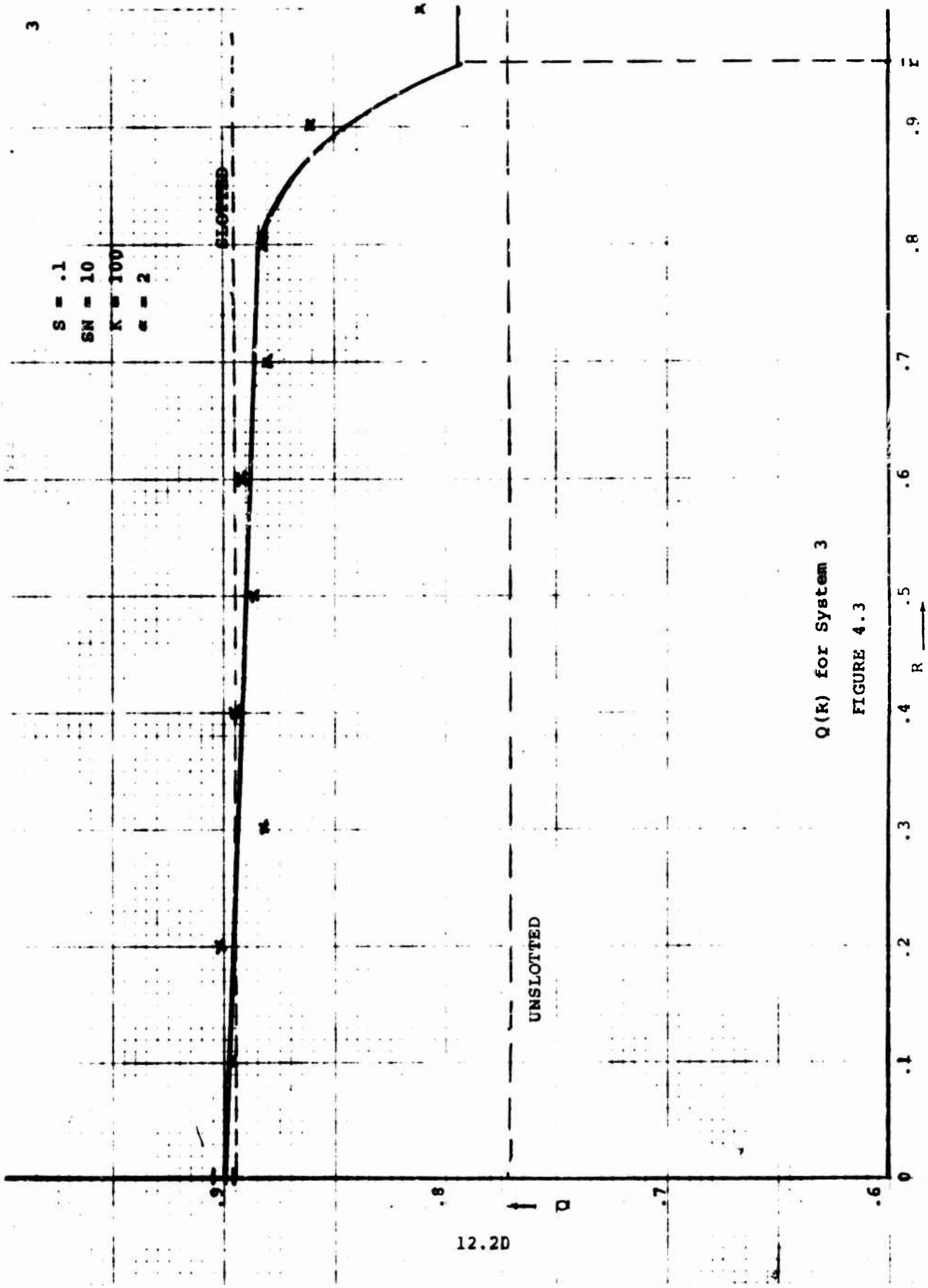
Q(R) for System 1

FIGURE 4.1



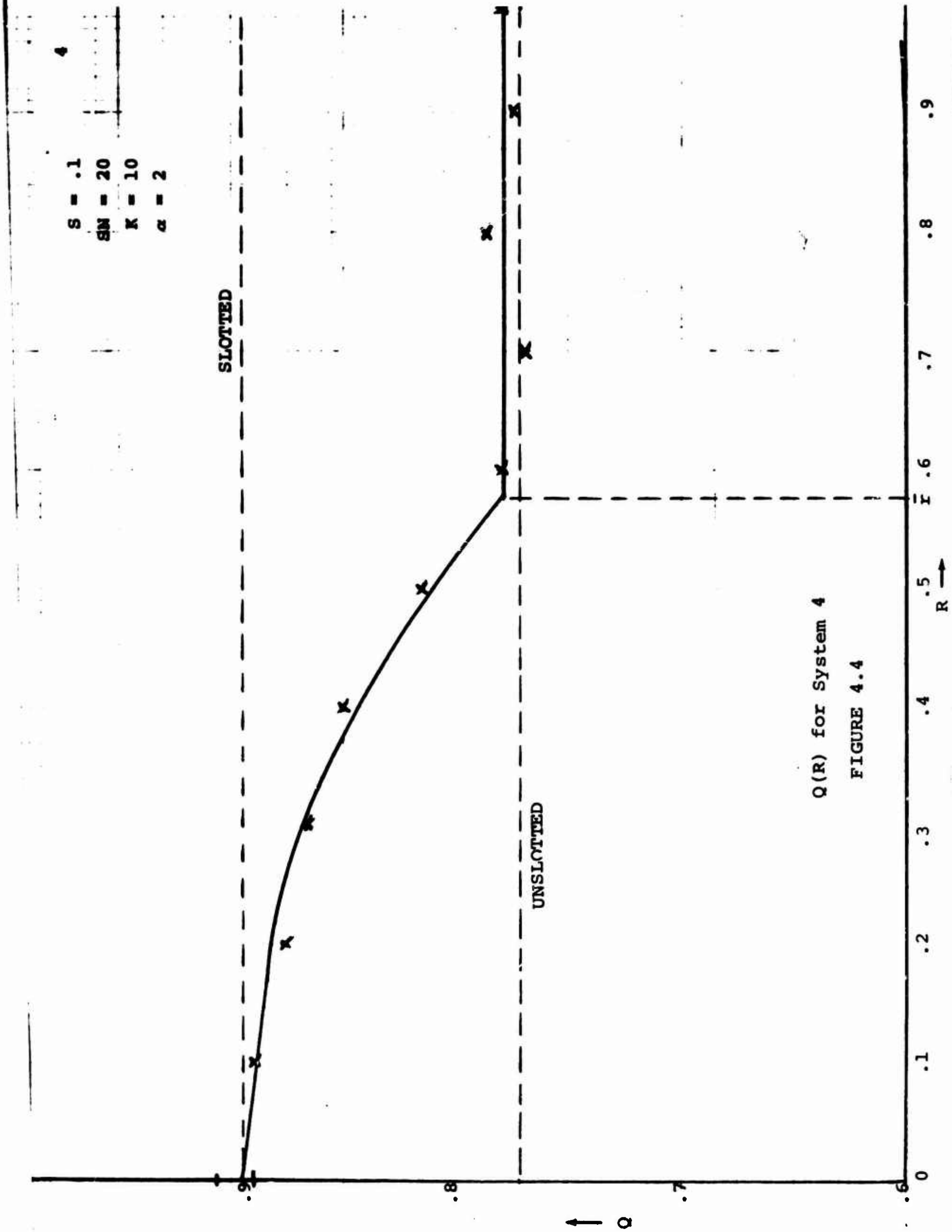
Q(R) for System 2

FIGURE 4.2



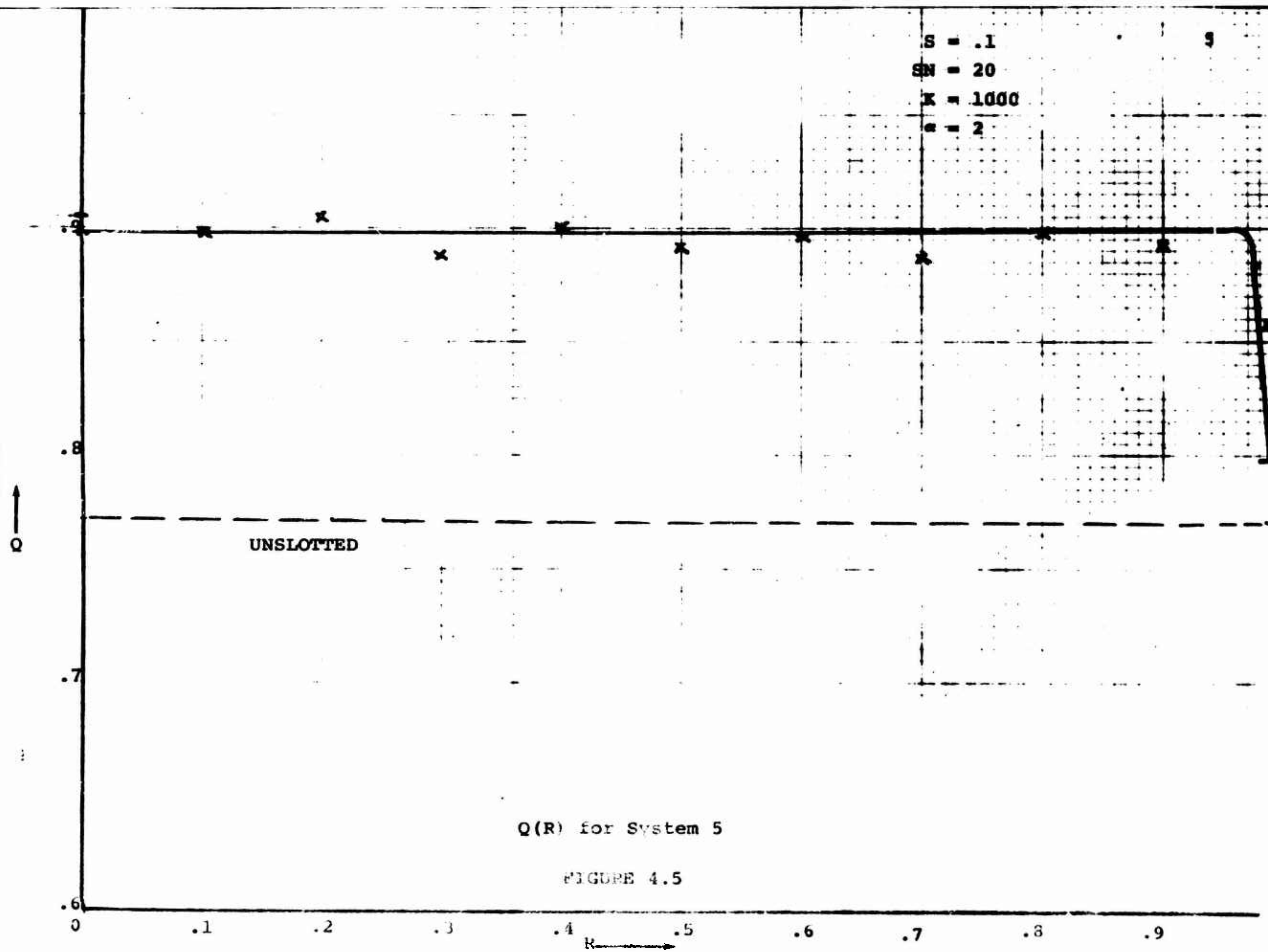
Q(R) for System 3

FIGURE 4.3



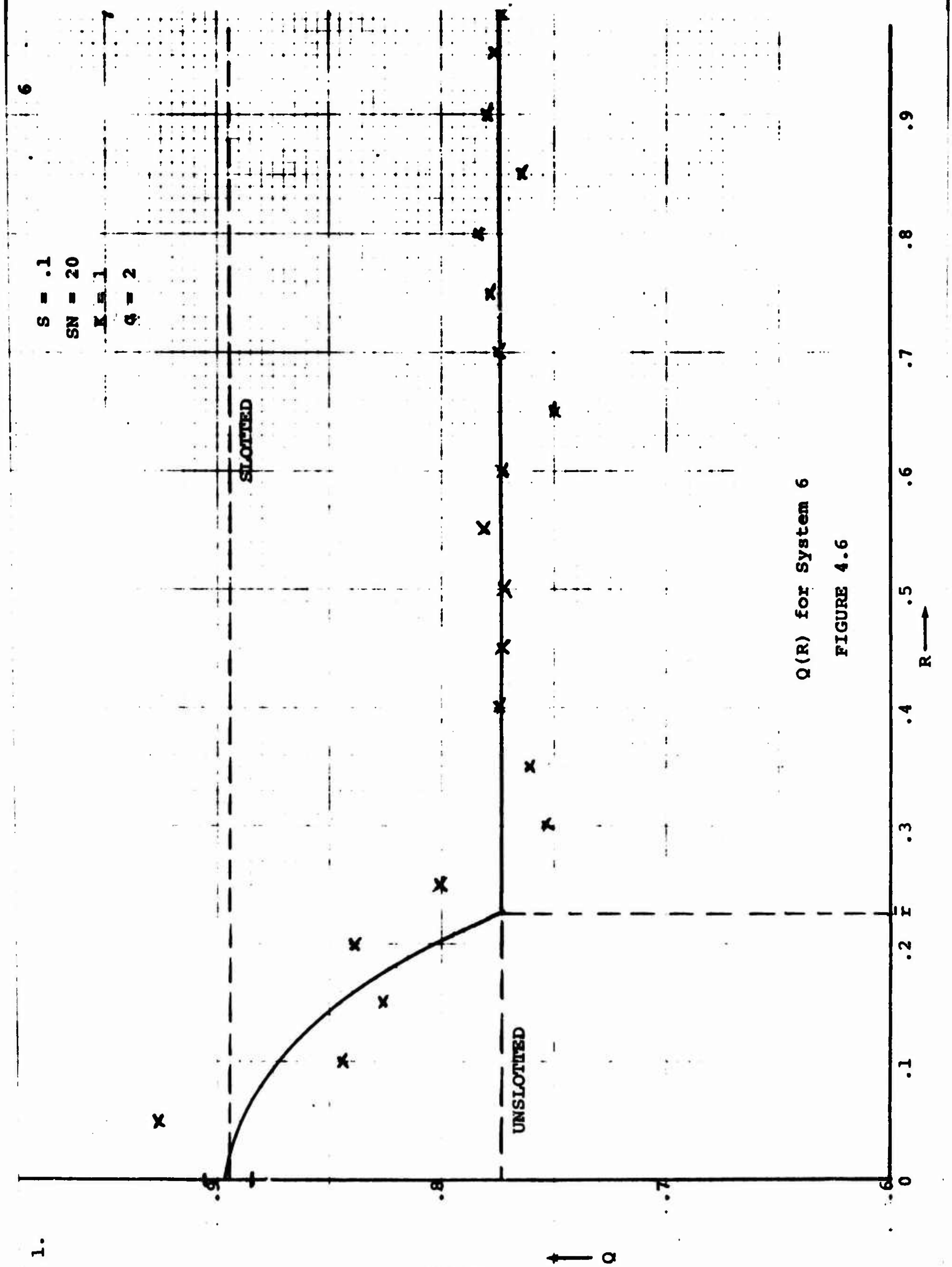
Q(R) for System 4

FIGURE 4.4



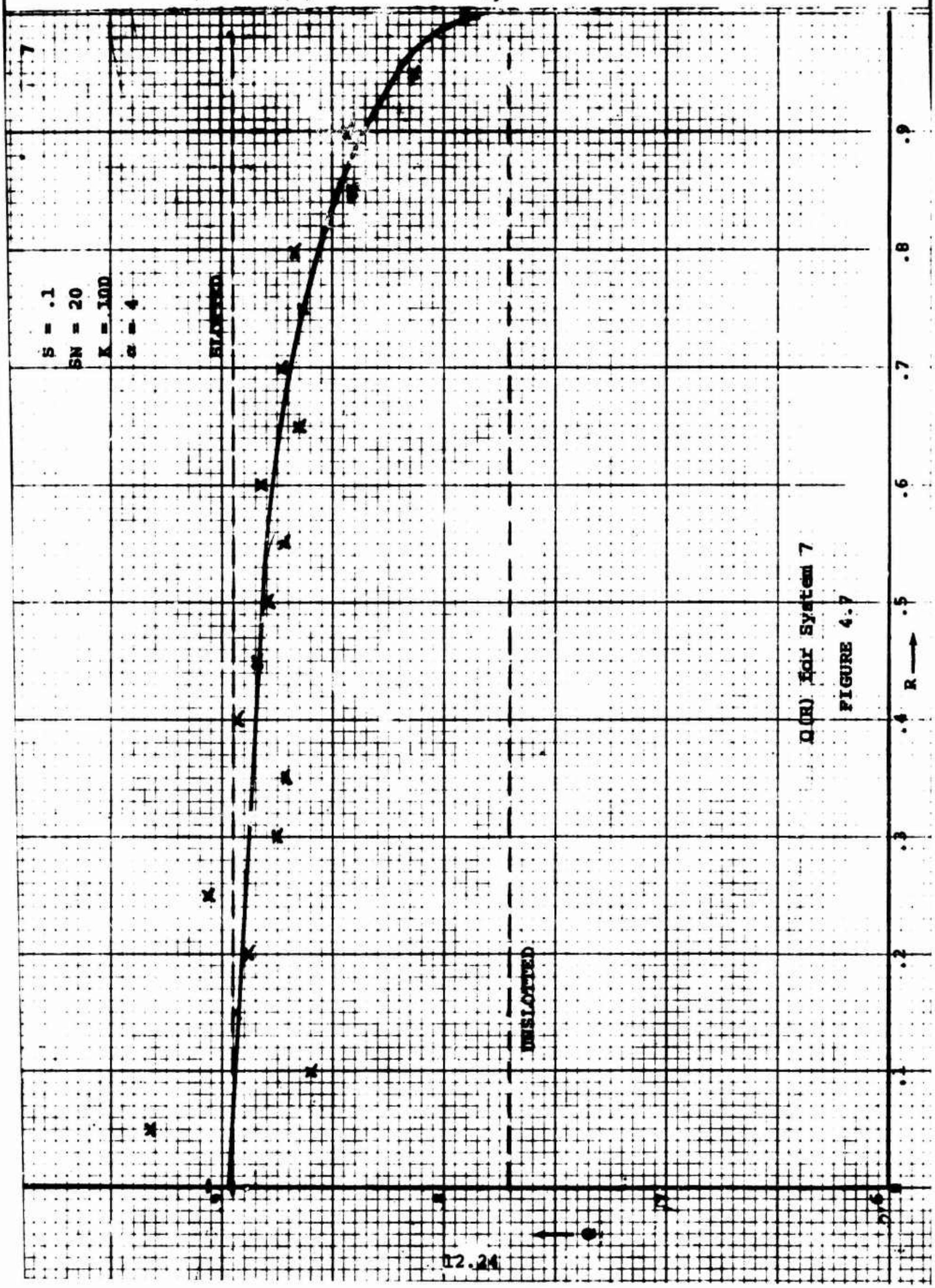
Q(R) for System 5

FIGURE 4.5



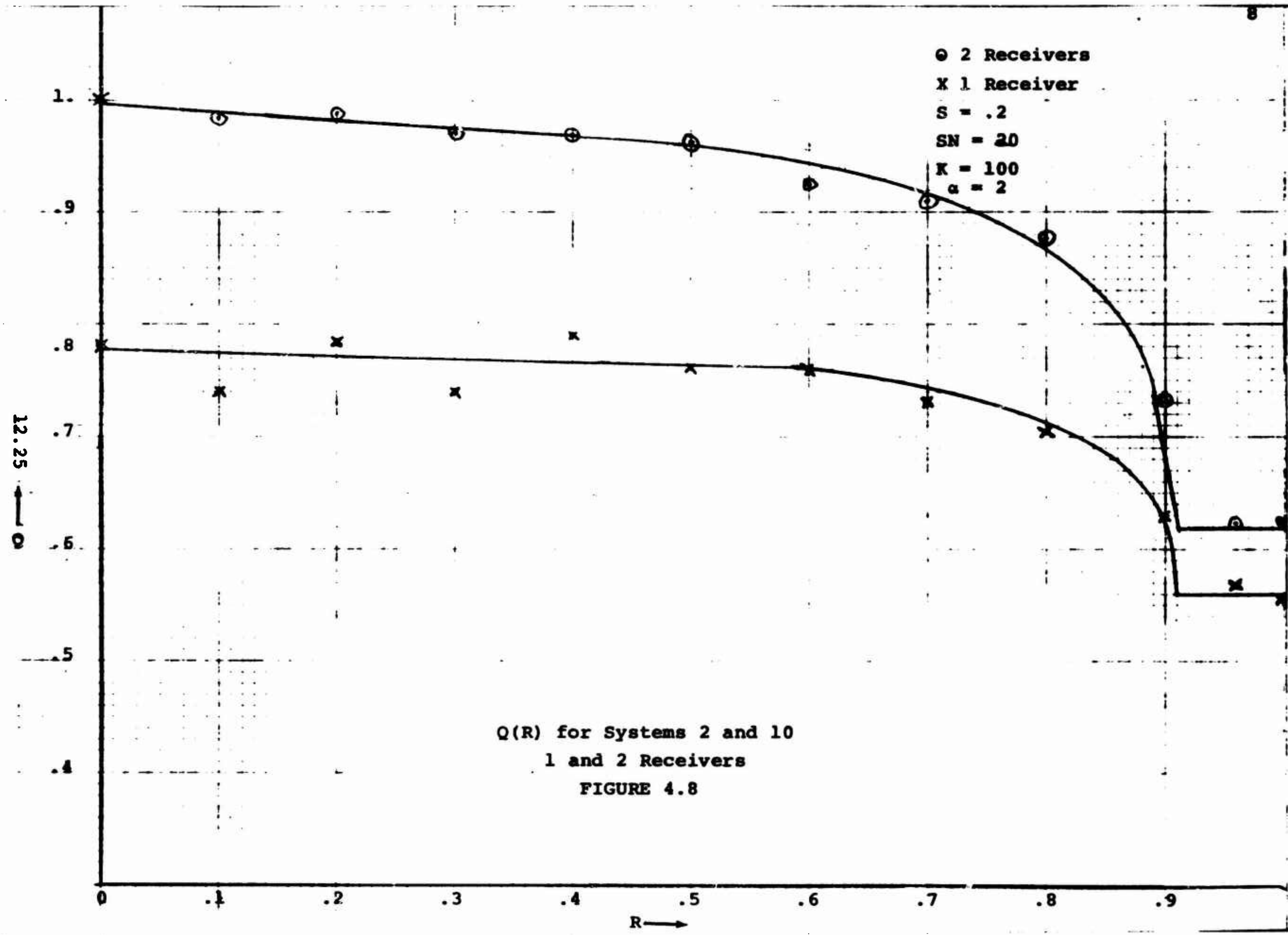
Q(R) for System 6

FIGURE 4.6



Q(R) for System 7

FIGURE 4.7



Q(R) for Systems 2 and 10
1 and 2 Receivers
FIGURE 4.8

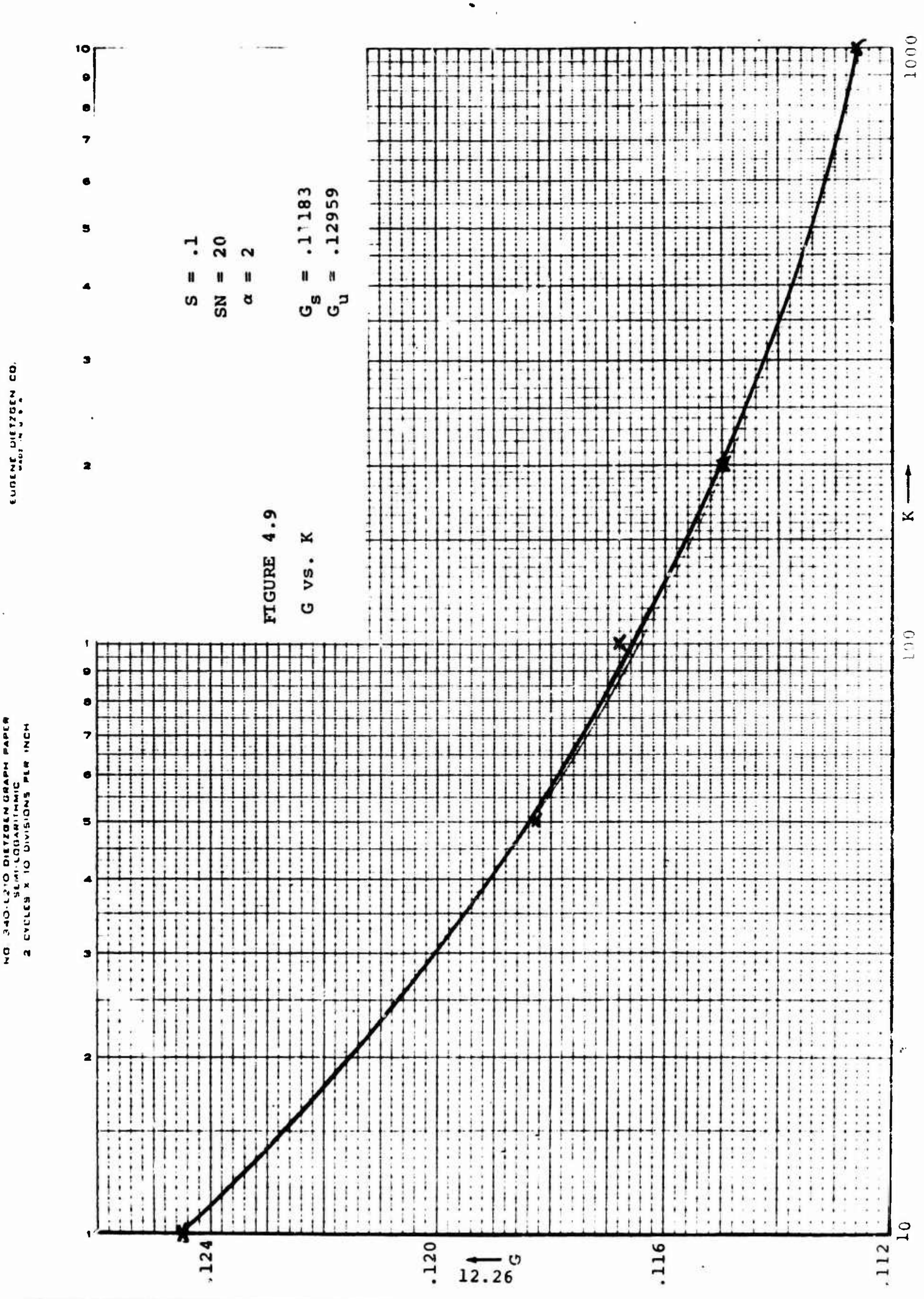


FIGURE 4.9

G vs. K

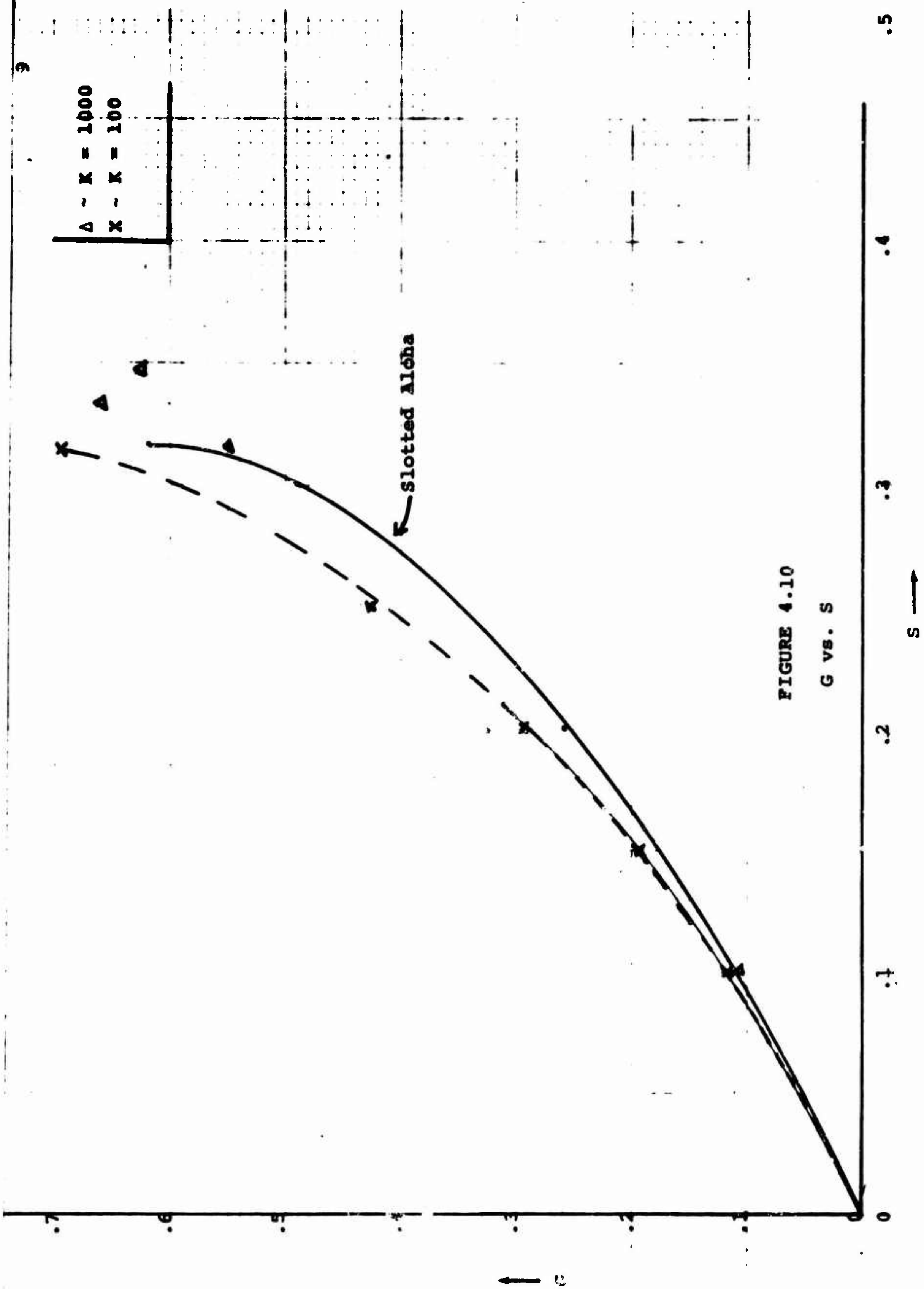


FIGURE 4.10

G vs. S

5. APPENDIX: METHOD OF SIMULATION

The radius from the station (R=o) to R=1 is divided into ND intervals. Associated with each interval, I, is a ring of width 1/ND and area 2 π I/ND. Thus, each ring is assigned a weight (proportional to radius) of arrivals in an array RDN, so that the arrival rate in the ring is RDN(I)* SIGMA; Q(I) is the current estimate of the probability a transmission from the I^{th} ring is successful. RDL(I) is defined proportional to RDN(I)/Q(I), so that the estimated arrival plus retransmission rate in the I^{th} interval is RDL(I) * GAMMA, where SIGMA~S and GAMMA~G.

Initially, we take GAMMA = SIGMA and RDL = RDN. The general step is to generate NS Poisson arrivals with arrival rate GAMMA. The transmissions are assigned radii according to the distribution RDL and the corresponding signal powers calculated in an array S. In an array ST, the instantaneous power is calculated for each time at which a transmission is initiated. Finally, in an array SMX the maximum power over the T seconds subsequent to each start of transmission is calculated.

To calculate the number of successful transmissions, the transmissions are considered in order. An indicator keeps track of how many receivers are busy. For a given transmission, the program first determines which interval from 1 to ND the transmission originates from; if it is in the I^{th} for example, then RBX(I) is bumped. Then the program makes the following tests to see if the transmission is successful:

1. Using the ST array, the program determines if the receiver can hear the beginning of the transmission. If so we go to Step 2. If not, we go on to the next transmission.

2. Is a receiver free? If one isn't, go on to the next transmission; if one is, mark the receiver busy for the next T seconds and go on to Step 3.

3. Can the entire transmission get through without being drowned out by subsequent competing transmissions? This is determined using the array SMX. If this test is passed, the transmission is assumed successful and a counter SBX(I) is bumped.

After all the transmissions are processed, and improved estimate for Q(I) given by

$$Q(I) = SBX(I)/RBX(I) \tag{A.1}$$

is obtained and this is repeated several times until the distribution settles down. It turns out that this process was very unstable, at least in the estimate for Q(I). The estimates for GAMMA were quite easy to obtain and reliable, but there were rather wide fluctuations in the Q(I) between iterations or when using different seeds for the random number generator. The reason is apparently because the power for originations near the origin become unboundedly large and hence have disproportionate influence, while the number of such events is quite small so the resulting variance is rather large. This was ameliorated to some extent by using exponential smoothing; that is, replacing (A.1) with

$$Q(I) := RLX * Q(I) + (1-RLX)\ SBX(I)/RBX(I) \text{ where}$$
$$0 \le RLX < 1.$$

CHAPTER 13

PACKET DATA COMMUNICATIONS ON MATV AND CATV SYSTEMS: A FEASIBILITY STUDY

1. INTRODUCTION

In 1970, the ARPA network was still written about in terms of goals to be reached.

> "For many years, small groups of computers have been interconnected in various ways. Only recently, however, has the interaction of computers and communications become an important topic in its own right. In 1968, after considerable preliminary investigation and discussion, the Advanced Research Projects Agency of the Department of Defense (ARPA) embarked on the implementation of a new kind of nationwide computer interconnection known as the ARPA Network. This network will initially interconnect many dissimilar computers at ten ARPA-supported research centers with 50-kilobit common-carrier circuits. The network may be extended to include many other locations and circuits of higher bandwidth.
>
> The primary goal of the ARPA project is to permit persons and programs at one research center to access data and use interactively programs that exist and run in other computers of the network. This goal may represent a major step down the path taken by computer time-sharing, in the sense that the computer resources of the various research centers are thus pooled and directly accessible to the entire community of network participants." [F.E. Heart, et.al., 1970]

Now these goals have been completely achieved with even further developments largely solidified. For example, TIPS (Terminal Interface Processors) to connect terminals to the ARPANET are a reality as are VDH (Very Distant Host) connections. In fact, in 1972 the result of the ARPA network development led to the conclusive statement that its performance is superior to other existing methods. Extensions to personal hand held radio terminals, are in the offing using a random access mode as begun in the University of Hawaii. The merits of the ALOHA packet mode have been thoroughly investigated with a conclusion:

> "...packet technology is far superior to circuit technology, even on the simplest radio transmission level, so long as the ratio of peak bandwidth to average bandwidth is large. Most likely, the only feasible way to design a useful and economically attractive personal terminal is through some type of packet communication technology. Otherwise one is restricted to uselessly small numbers of terminals on one channel. This result may also apply to many other important developments, only to be discovered as the technology of packet communication is further developed." [Roberts, 72]

Both as an adjunct to the interactive packet radio mode and as an extension of the ARPANET, this report describes the use of MATV and CATV coaxial cable systems for local distribution of packet data.

We first explain the need for a local distribution medium such as an MATV and CATV System, to augment a packet radio system in urban and suburban areas. We next investigate the properties of the coaxial cable systems in order to evaluate their data handling capability. To demonstrate the validity of our conclusions, actual designs are discussed for the existing CATV system in the Metropolitan Boston area. Specifications are given for all required digital equipment, and finally, test procedures are given. The overall conclusion of the study is that MATV and CATV interactive packet systems would form an excellent local distribution medium.

2. THE IN-BUILDING PROBLEM

In urban areas two difficulties impede the reception of radio signals in buildings:

1. The attenuation of signals in passing through building walls; and
2. The reception of multipath signals due to reflections off buildings.

In a study of the in-building reception problem, C. H. Vandament of Collins Radio concludes,

> "...signal strength environment on a city street will probably need to be 25 to 30 db higher than that previously considered if direct radiation into buildings is to be considered (i.e. no building distribution system employing amplification). This is quite unattractive since this implies either excessive transmit power and/or quite close repeater spacing." [Vandament, 1973]

Both of these problems can be avoided while still retaining the main idea of using ALOHA random access multiplexing. Instead of operating in an over-the-air broadcast transmission mode, data can be sent over the existing wideband coaxial cable facilities of master antenna (MATV) and cable television (CATV) systems. Recent FCC rulings require that all new CATV systems have two way capability. penetrations of 40-60% of U.S. homes is projected by the end of the decade [Sloan, 1971]. Moreover, most of the major new office buildings will have wide-band communication channels built in. For example, in the New York World Trade Center, there is a system of switched wideband communication channels. The user can select 60KHz audio channels or 15 MHz video channels and has an individual wideband cable connection to the central switch [Friedlander, 1972]. By 1980, "the wired city" will be close to reality.

Vandament, after considering radiation, telephone wires, power cables, and other special wiring schemes for the in-building problem, comes to a similar conclusion regarding the merits of coaxial cable transmission:

> "...Every building will require some analysis to determine which technique is required to deliver a useable signal to a terminal inside.

> If the building has windows and is relatively close to a system repeater, no special techniques will be required. At greater distances, a simple repeater with directional antennas focused on specific buildings will deliver the signals to interior users by radiation. Buildings which are effectively shielded must have a simple, dedicated repeater to receive a signal and pipe it into the building over conductors of one type or another...High grade coaxial cables solve that problem nicely, but this answer would probably be prohibitively expensive for the packet radio scenario of general distribution throughout every important building in the U.S. ...Where such cables exist, they do offer an attractive solution to the in-building distribution problem."
> [Vandament, 1973]

In the next five sections, we will show the technical merits of MATV systems for solution of the in-building problem and CATV systems for local distribution in high density suburban and urban areas. To do this, we first describe the properties of typical modem MATV and CATV systems in Sections 3 and 4 respectively.

3. A TYPICAL MATV SYSTEM

A master antenna television system as shown in Figure 3.1 serves a concentration of television sets such as in an apartment building, hotel, or motel. The main purpose of the MATV system, as

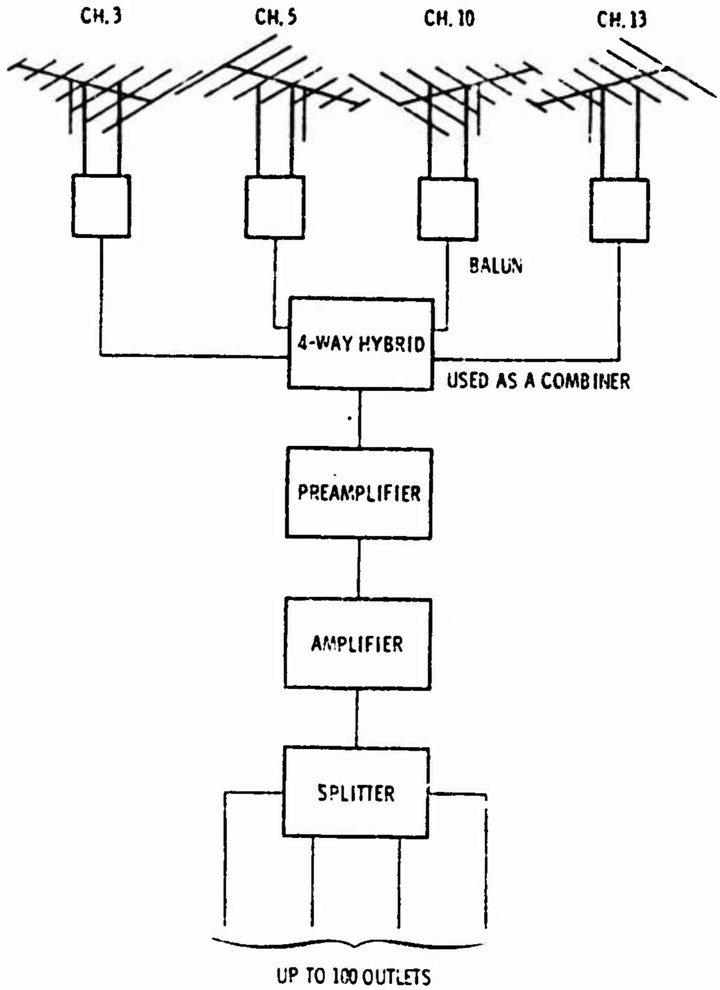


Figure 3.1 Typical Hotel or Apartment Building MATV System

shown in Figures 3.2, 3.3, 3.4, is to provide a usable signal to a large number of television sets fed by a local distribution network.

A number of television sets connected to the same antenna system without a signal amplifier would not provide any of the sets with a strong enough signal to produce good pictures. An all-channel

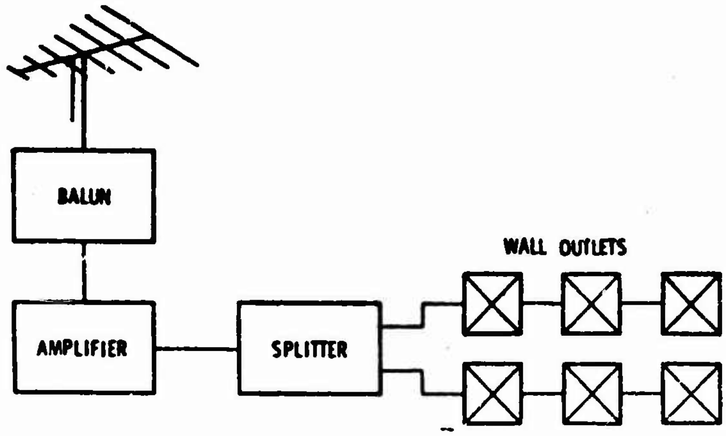


Figure 3.2 Typical Motel MATV System

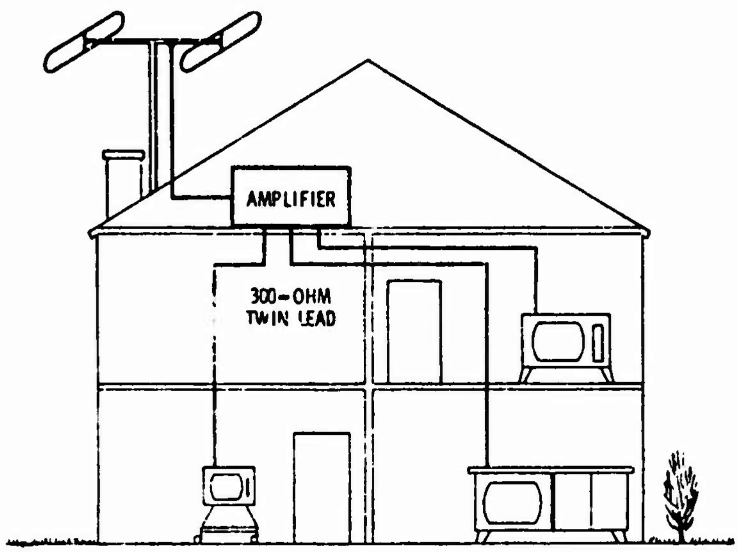


Figure 3.3 Typical Home MATV System

master antenna amplifier is connected to one antenna (sometimes more) which provides across-the-band amplification of all television signals in the VHF band and the f-m broadcast band.

Some MATV systems employ more than one amplifier. A separate single-channel amplifier may be used as shown in Figure 3.5, to provide greater amplification of a single channel. Generally, a separate antenna is used with a single-channel amplifier.

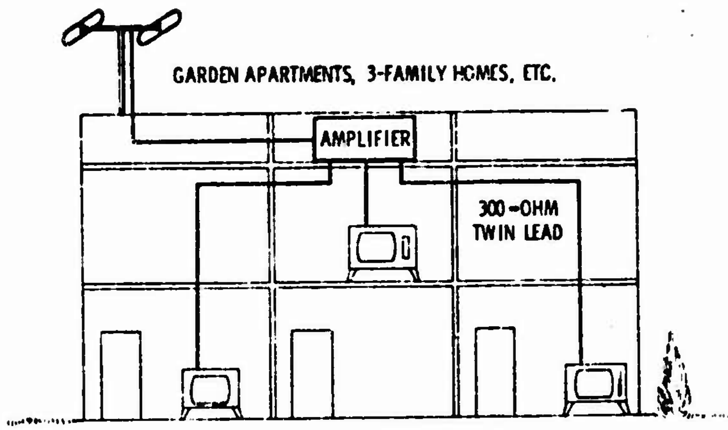


Figure 3.4 MATV System for Multiple Dwelling

For reception of UHF television stations, a UHF-to-VHF translator is required.

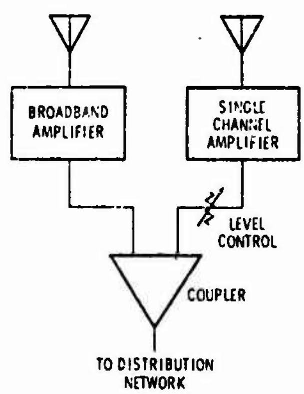


Figure 3.5

Use of Single Channel and Broadband Amplifiers for Receiving Distant Stations

For MATV systems with more than 100 outlets, further amplification may be required, and in a large office building cascades of two or three amplifiers might be expected. Nevertheless, compared to CATV systems - to be described in the next section - modern MATV systems are relatively uncomplicated media for data transmission. Signal to noise ratios of better than 43 db are reasonable; there are few environmental problems, since the system is indoors; there is minimal temperature variation; and amplifier cascades are low since the wide band capabilities of 0.5 inch coaxial cable are used. In a CATV system, the signal at the building is received from a cable distribution system rather than from antennas. However, the in-building part of the system is essentially unchanged from that used for an MATV system.

4. A TYPICAL CATV SYSTEM

CATV systems are an historical outgrowth of MATV and community antenna television systems. CATV systems perform roughly the same function for an entire town that a MATV system performs for a building, namely the distribution of TV singals to many terminals from a central reception area. Although the basic engineering strategies are the same for MATV and CATV systems, the CATV system is different in one crucial aspect; since it is larger, as many as thirty amplifiers may have to be cascaded to deliver a TV signal to the most distant terminal in the system. Therefore, the system design requirements are stringent; very high quality amplifiers and off the air reception equipment are essential. In order to motivate our proposal of data options and techniques for CATV systems, a detailed description of these systems is in order.

CATV systems in the U.S.A. are almost universally tree structured networks of coaxial cables installed for the distribution of broadcast type television signals from a central receiving station, called the "head end," to home type television receivers. Different television signals, which may be received at a central site or relayed over long distances by microwave systems, are processed at the head end and frequency division multiplexed onto coaxial cable for distribution. Coaxial cables now in use are universally 75 ohm inpedance types, usually of seamless aluminum sheathed construction, foam polyethylene dielectric, and solid copper or copper clad aluminum center conductor. The coaxial cables range in size from 0.75 inch outer diameter for "main trunk cables," through 0.5 inch size down to a 0.412 inch size for "local distribution." The service drop lines to the houses are usually flexible cables of about 0.25 inch diameter. The useful frequency range includes the VHF television band, 54-216 MHz, and broadband transistorized amplifiers are installed with equalizers to compensate for cable losses. Practical systems are aligned to be unity gain networks with amplifiers spaced about 20 db at the highest transmitted frequency.

Cable losses range from about 1 db/100' at 220 MHz for the 0.75 inch size cable to about 5 db/100' for the flexible service drop cables. Power division at multiple cable junctions and taps into subscribers' homes are accomplished through hybrid and directional couplers. All system components are carefully matched to 75 ohms to minimize internal signal reflections within the system.

System amplifiers are subject to rigorous linearity specifications. Amplifier overloads manifest themselves as cross-modulations between channels and as undesired second and third order intermodulation and harmonic products. Amplifier operating levels are bounded on the lower side by system signal to noise ratio objectives which are about 40 db over a 4 MHz band for reasonable acceptable performance as a television distribution system. A typical system will have amplifier inputs at about +10 dbmv and outputs at about +30 dbmv. System operating levels are controlled by automatic gain control circuits driven by pilot carriers and thermal compensation devices.

More recent cable systems have been built using a "hub" principal in which tree structured networks originate from a number of "hubs" throughout the community serviced (see Figure 4.1). The "hubs" may contain equipment for more elaborate control of signal levels and it may be possible to perform some special switching functions such as the interconnection of sub-trunks for special purposes.

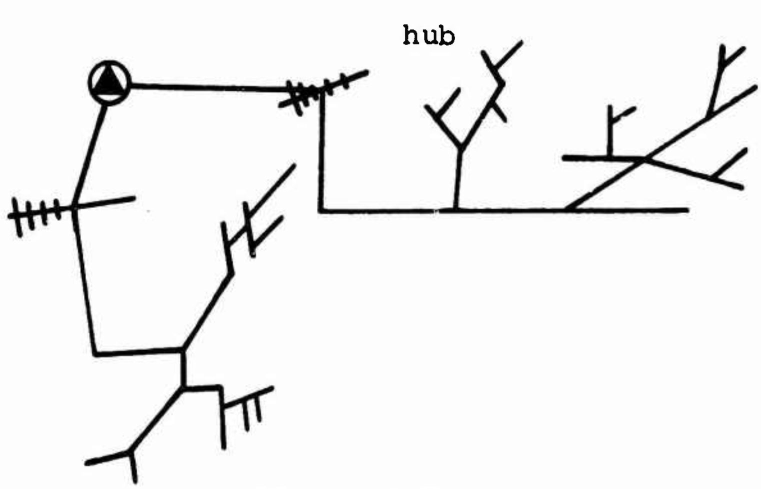


Figure 4.1 Hub System

Two-Way CATV Configurations:

FCC regulations now require that new CATV systems must have two-way capability. Practically speaking, this does not mean that all new systems are two-way systems, but rather that amplifier units are installed with forward amplifier modules in place and with distances between amplifiers constrained so that at some future date reverse amplifier modules can be installed for two-way operation. However, a number of actual fully two-way systems are presently being built and the number is increasing rapidly. Most present two-way systems use the configuration in Figure 4.2. Filters at each end of the station separate low (L) and high (H) frequencies and direct them to amplifiers usually referred to as "downstream" from the head end and "upstream" toward the head end.

A number of possible "two-way" configurations are shown in Figures 4.2, 4.3, and 4.4 [Jerrold, 1971]. The final choice between single cable/two-way, multi-cable two-way, and multi-cable without two-way filters will probably be made on the basis of marketing opportunities for special services.

There are no government regulations as to minimum specifications for a system. Hence, we will base our discussion on the characteristics of a representative two-way system, the Boston complex. This system is being built in about 10 stages, one of which is already installed and the final phase of which is to be completed within a year. At its completion, Boston will be one of the largest systems in existence in the U.S. The Boston system uses the "feederbacker" configuration shown in Figure 4.4 with the frequencies assigned to the upstream and downstream paths specifically indicated.

Data transmission on CATV systems will generally have to be fitted into space not being used to TV channels or for pilot frequencies. Hence, it is best to first describe the frequency allocation for video signals. TV channels are allocated six Megahertz

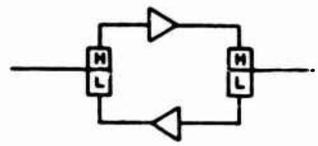


FIGURE 4.2

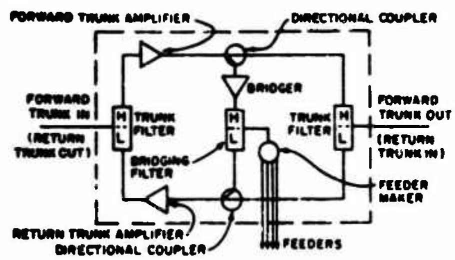


FIGURE 4.3 Two-way CATV Repeater (with feeders).

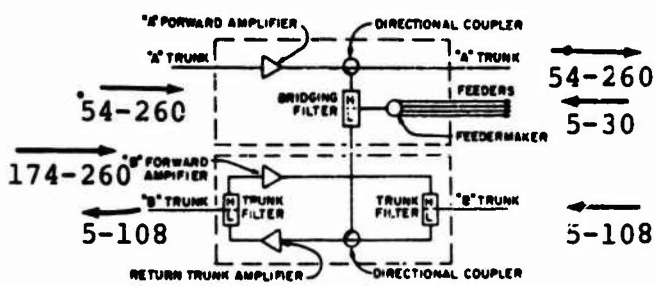


FIGURE 4.4 Dual Trunk/Single Feeder Station (Boston Configuration)

bandwiths. Broadcasted TV channels are in the Lo-VHF range 54 MHz-88MHz, the Hi-VHF range 174 MHz-216 MHz, and the UHF range 470 MHz-890MHz. The TV frequency allocations on cable are different. They partition the 54-300 MHzspectrum as follows:

Sub-VHF	5-54 MHz
Lo-VHF	54-88 MHz
Mid-band	88-174 MHz
Hi-VHF	174-216 MHz
Super-band	216-300 MHz

There is still discussion going on as to whether the mid-band should be used within a cable system because of danger of interference to aircraft navigation in case of signal leakage out of the cable. Data might be squeezed into bands not used by TV signals. Some space is available below Channel 2 at 48-54 MHz. The space between Channel 4 and Channel 5 (72-76 Mhz) might be used for low level data signals. High signal levels in this area could cause harmful picture interference on some TV sets.

A more likely situation is that two 6 MHz video channels - one upstream and one downstream - would be set aside for data transmission. The simple fact is that 21 to 30 channels are available on a single cable system and 42 to 60 channels on a dual cable system. These channels have not all yet been pre-empted by video transmission.

5. DATA OPTIONS ON MATV AND CATV SYSTEMS

Based on the details of MATV and CATV systems, we can demonstrate the merits of in-building and local distribution on MATV and CATV systems and describe detailed data options for experimentation and practical implementation.

Most two-way systems are being developed with an upstream channel designed to permit input from virtually any location in the network. The result is a large number of noise sources being fed upstream toward a common source. Individual cable television amplifiers usually have a noise figure of about 10db for a 6 MHz channel. Cascading amplifiers can increase effective system noise figure by 30db or more. Nevertheless, we shall see that system specifications on signal-to-noise ratio for CATV systems are stringent enough so that packets can be sent with existing analog repeaters, and no digital repeaters, such that bit rate error probabilities are negligible.

For example, in Boston the worst signal-to-thermal noise ratio is limited to 43db and the worst cross-modulation to signal ratio is limited to -47db. System operators may want to limit data channel carriers to a level of 10 to 20db below TV operating levels in order to minimize additional loading due to the data channel carriers [Switzer, 1972]. The cable operator, at least for the time being, is making his living by providing a maximum number of downstream television channels and will accept data channels only on a non-interfering basis.

Accepting these restrictions, in the worst case, we would be limited to 23db signal to thermal noise ratio and -27db cross-modulation to signal ratio. Let us consider both of these sets or restrictions to determine the resulting CATV system performance for random access packet transmission.

From the calculations in Appendix A, we have the error rates shown in Table 5.1. The calculations are performed for a FSK system with incoherent detection to determine a lower bound for system performance.

TABLE 5.1

ERROR RATES FOR FSK

System Label	Type of Specification	(N_C/S)	(S/N_r)	m	Pe
A	Boston Specs.	-47db	43db	2	$1/2\, e^{-5,148} \approx 1/2 \times 10^{-2,239}$
B	Boston Specs.	-47db	43db	4	$3/4\, e^{-571} \approx 3/4 \times 10^{-248}$
C	Boston Specs.	-47db	43db	8	$7/8\, e^{-104} \approx 7/8 \times 10^{-45}$
D	Boston Specs. - degraded by 20db	-27db	23db	2	$1/2\, e^{-51} \approx .5 \times 10^{-22}$

It is well known that for effecti signal-to-noise ratios above 20db there is a threshold effect for error probabilities. This is born out by the negligible error rates in Table 5.1. Even for the degraded specifications the error rate is low enough for the most stringent practical data requirements.

At a rate of 10^6 pulses/second the FSK signal will occupy the 6MHz bandwidth [Switzer, 1972] with negligible intermodulation into TV channels [Schwartz et al., 1966].

In Figure 5.1 we plot the maximum number of active terminals for the systems in Table 5.1 as calculated in Appendix B. The curves labeled A,B,C, and D correspond to the slotted systems in Table 5.1 labe[illegible] B,C, and D. The lines labeled A',B',C', and D', are for t[illegible]sponding unslotted systems.

We can now [illegible]ine Figure 5.1 to determine system performance under some typical data transmission requirements. For a data rate of 40 bits/second per terminals, a single trunk can handle 900 terminals with a slotted ALOHA system (1 Megabit/sec) with an error rate of 10^{-22} at a signal to noise ratio degraded by 20db. The average number of TV sets per trunk in Boston is approximately 27,000. Hence, the simplest modulation scheme will handle one third of all terminals in Boston as active terminals. At 100Kbits/second, the system will handle 900 active terminals.

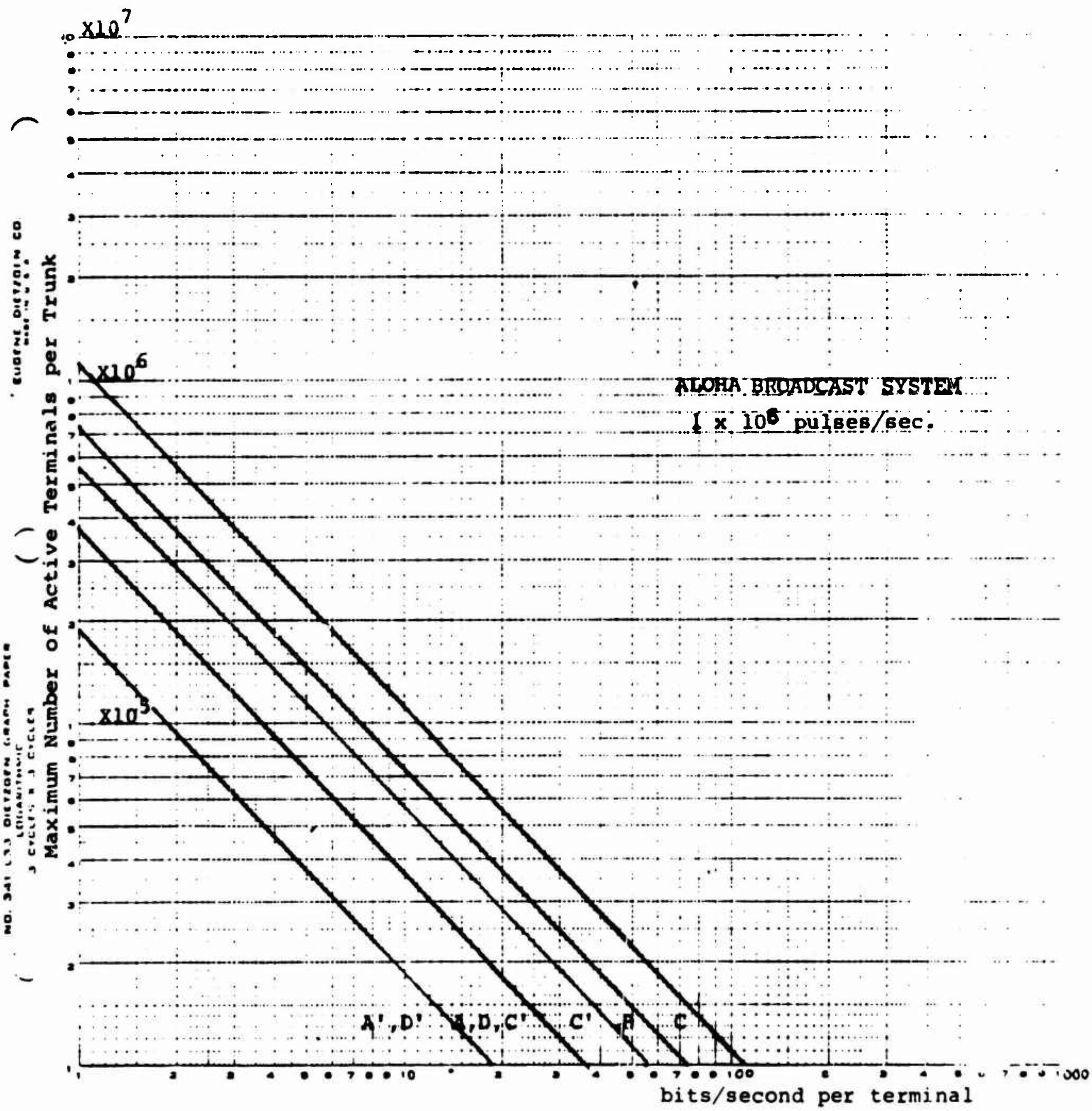


FIGURE 5.1
NUMBER OF TERMINALS PER TRUNK VS. BITS/SEC. PER TERMINAL

We will also consider the introduction of a number of basic data processing options into the CATV system in order to make our data transmission system adaptable to a variety of traffic requirements while satisfying the system constraints.

We will use the terminology of the cable TV industry in describing the direciton of signal flow. Signals traveling from the head end toward terminals will be said to be directed in the "forward" direction on a "forward" link and signals traveling from terminals toward the head end will be said to be directed in a "reverse" direction on a "reverse" link. A convenient synonym for "forward" will be "downstream", and for "reverse", will be "upstream". To install the device to be described in the forward and reverse channels simple diplex and triplex filters can be used. More detailed specifications for the filters and the data devices are given in Appendix E.

CARRIER FREQUENCY CONVERSION:

In the simplest version of a data system, two carrier frequencies are used; one for forward transmission from the head end to the terminals, and one for reverse transmission from the terminals to head end. Let us call these angular frequencies ω_f and ω_r respectively. The next simplest option is to use frequency converters at a small selected set of points in the system. In the forward direction, the converter converts from ω'_f to ω_f and in the reverse direction, it converts from ω_r to ω'_r. The net result is that the terminals still receive and transmit at the frequencies ω_f and ω_r. However, in the trunk between the converters and the head end, there are four frequencies in use, ω_r, ω'_r, ω_f and ω'_f so that in these trunks twice the traffic can be handled.

The converter and the other devices to be described in this section are shown schematically in Figure 5.2.

The advantages of this scheme are:

a) All terminals are identical.

b) The converters also act as digital repeaters to reshape signals.

c) The capacity of the system is increased since two channels are available in each direction.

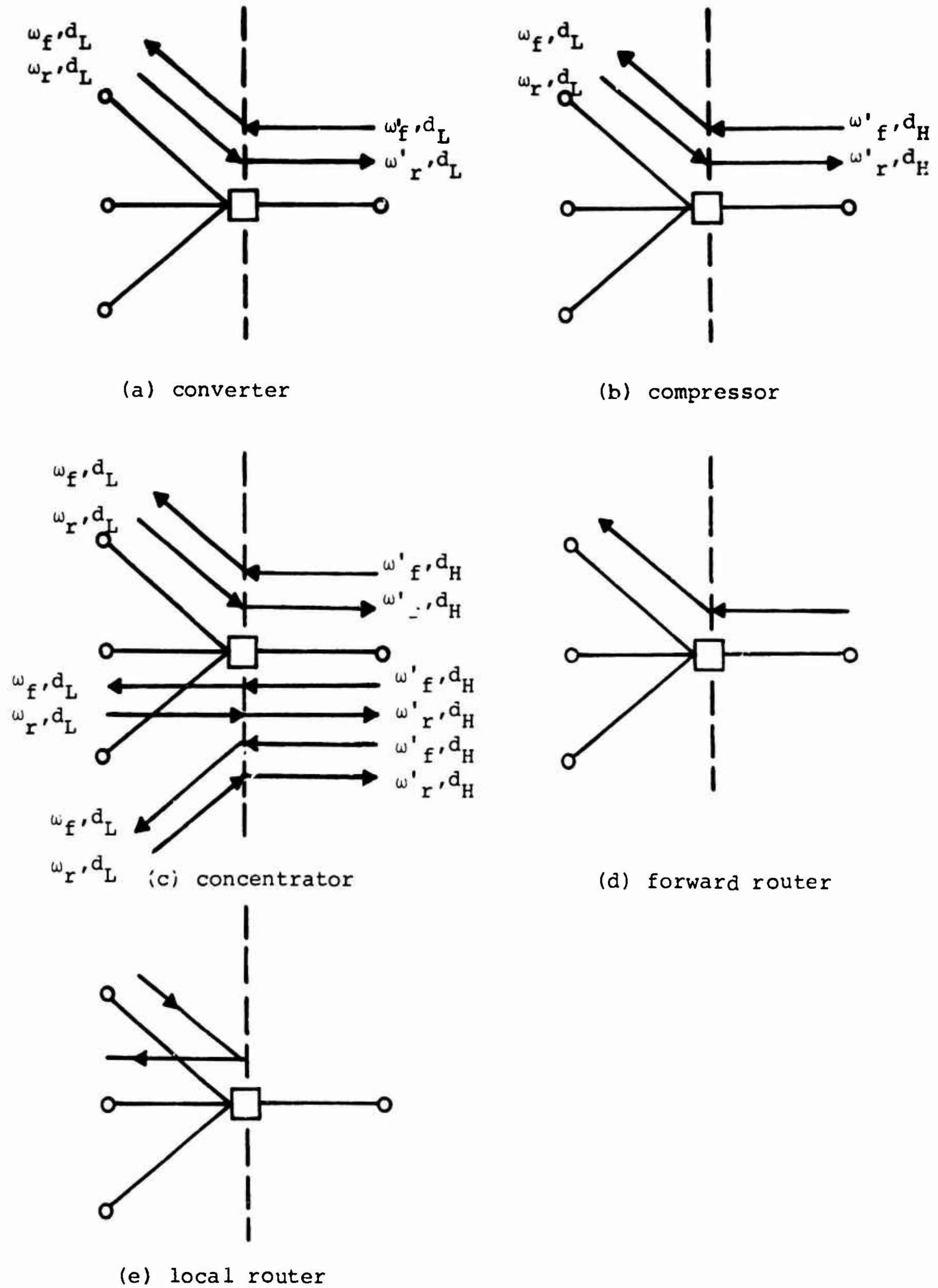


Figure 5.2

Device schematics; devices indicated by squares. Carrier frequencies ω_r, ω'_r, reverse; ω_f, ω'_f, forward. High data rate d_H; Low data rate, d_L.

ROUTING:

In the interactive packet system, there is no requirement for routing since the basic premise is that all receivers listen to all messages that reach them and merely select the ones addressed to them. Nevertheless, we will consider the addition of some primitive low cost routing schemes to study their effect on system capacity.

In the central transmission mode there is routing needed in the reverse direction since all messages reach the head end along the unique paths from the originating terminal. In the forward direction the signals at frequency ω'_f are blocked by filters at the converters and yields a simple form of routing.

In fact, at any section of trunk not requiring signals at ω'_f filters can be added to block ω'_f. Adding these filters does not increase system capacity but may be useful if the frequency ω'_f can be used for local signaling when not being used for data transmission.

At any junction containing a converter, digital routers must be added to send messages at ω_f down the trunk to which they are addressed rather than all trunks. Such a router may be added at any other point in the system as well. This is called "forward routing" and can increase system capacity. Forward routing requires a digital router which can read and interpret message addresses.

Let us now consider these system options in the presence of local traffic. The option of frequency conversion is unaffected and performs in exactly the same manner as in the central transmission mode. However, an extra routing option is available for local transmission. In particular, if two terminals are on the same trunk, then the message between them can be intercepted and routed at a routing station rather than travel all the way to the head end. Such routing is called "local routing". Local routing reduces the traffic on the main trunk.

COMPRESSION:

As the next more complicated option, the data rate as well as the carrier frequency is changed at a converter, i.e., a compressor can be used. The advantages of this arrangement are:

A. All terminals operate at low data rate.

B. On heavily used lines near the head end a higher data rate, say one megabit/second, can be used to increase the number of potential active users or decrease the delay.

C. Even though a section of the trunk can carry high data rate traffic at carrier frequencies ω'_r anand ω'_f other users can still use the system at the low data rates at ω_r and ω_f. Thus, the number of compressors required is small.

D. At the low data rate, signal distortion is kept to a minimum.

CONCENTRATION:

Finally, the compressor at junctions may be replaced by a concentrator. That is, messages arriving simultaneously on two or more links in the reverse direciton are buffered and sent out sequentially at the higher data rate. This essentially makes the system downstream from the concentrator appear to operate at the higher data rate and hence increases the system capacity even further.

With the use of converters, compressors , concentrators and routers, we have a highly flexible interactive packet data system which obviously meets most of the system requirements of capatibility with the CATV system, the packet radio system and the population. In particular, the system has the following characteristics:

A. The number of active users that may have access to the system can be readily controlled by varying the number of converters, compressors , concentrators and routers.

B. The transmission rates at every terminal are at the low data rate of 100 kilobits/second.

C. Terminal equipment is inexpensive because modulation takes place at the low data rate.

D. All terminals receive and transmit at the same frequencies and data rates.

FREQUENCY DIVISION MULTIPLEXING:

In case the data rate is limited by the head end mini-computer, an available option is to frequency division multiplex several 100Kbits/sec. channels, each of which is processed by a separate head end mini-computer.

The assignment of these options in an optimal fashion requires detailed expressions for the traffic in the links. Formulae for the traffic are given in Appendix C and are used to give an augmented design for the Boston system in Appendix D.

6. FUNCTION OF EXPERIMENTAL DIGITAL SYSTEM DESIGN

The previous considerations indicate the desirability of developing a system of MATV and/or CATV lines for inter- and intrabuilding communication to and from packet data terminals. A picture of the system is shown in Figure 6.1.

In order to communicate through the cable system, each terminal will be connected to an interface and modem. A minicomputer will be connected to the cable system at its head end by means of a similar modem and modem-to-computer interface. The minicomputer is used as a test device to:

A. Receive and generate basic traffic to establish the viability of the system configuration.

B. Provide message loading to determine limits of system performance.

C. Demonstrate the functional capabilities required for the head end processor.

D. Determine the performance required to communicate with external facilities.

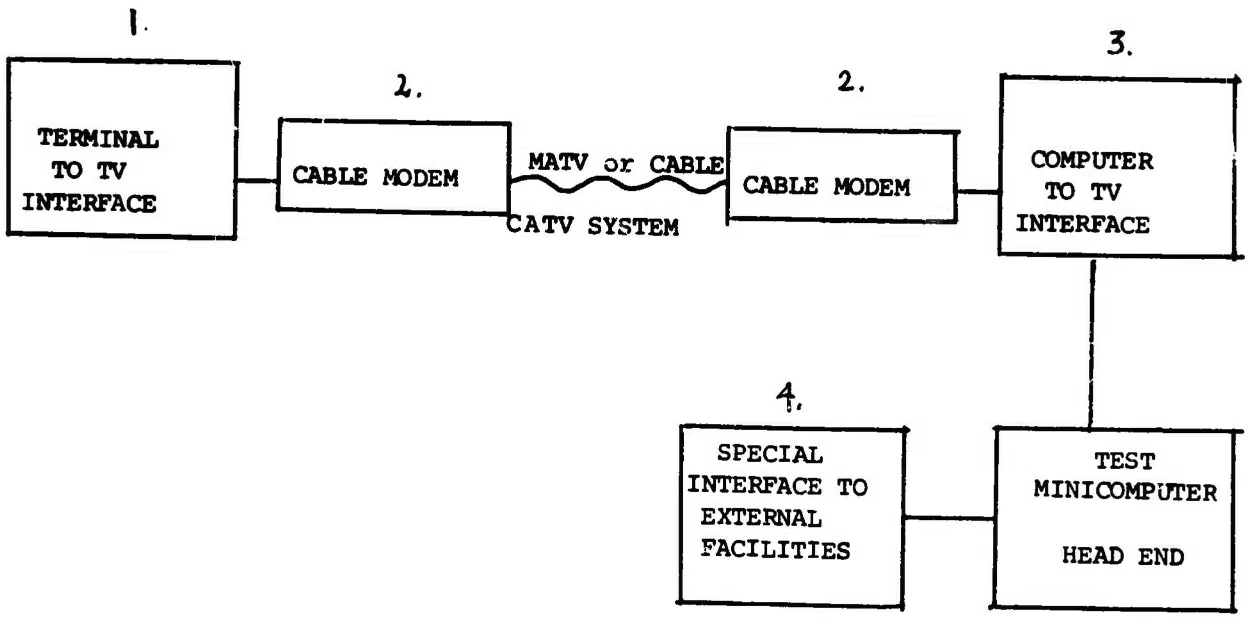


FIGURE 6.1 SYSTEM DIAGRAM

7. SYSTEM CONSTRAINTS

In specifying hardware, a dominant consideration is the fact that the interactive packet cable transmission system must interface with a CATV system and a packet radio transmission system, among others, and an existing population of unsophisticated users. Thus, any hardware innovation must satisfy the following three classes of system constraints.

Interface With CATV System:

Two Way Options:

The data transmission system must be readily adaptable to a wide variety of existing CATV system designs and two-way options.

Data Rates:

The data signals must not cause visible interference with video signals.

Installation:

If auxiliary data equipment is to be added to the CATV system, it must satisfy the following requirements:

- It can be installed with only minor changes in the CATV system.
- It need be installed in only a small number of locations.
- It can be installed rapidly in early hours of the morning to prevent interference with TV service.

Low Cost:

To maximize the marginal utility of data distribution over the CATV system, any equipment introduced must be inexpensive. Depending upon the data rates and bandwidths involved, the radio and CATV systems can be arranged to share some modules, at the baseband or IF frequency range.

Interface With Population:

Population Density Variations:

Standard transmission configuration options must be available for systems of various sizes, population densities and percent of active users. Because of the huge number of potential users, all terminal equipment must be simple and inexpensive.

Unsophisticated Users:

To minimize user interaction with the system operating mode, all terminal equipment must be the same for each location; it must use the same frequencies and data rates; and it must have no options for equipment modification by the user.

8. COMPONENTS OF EXPERIMENTAL SYSTEM

With the overall system plan described in Section 6 and the specifications described in Section 7, we can now develop a detailed description of each system component. These components are examined in depth since the system under consideration must be compatible with both CATV and data transmission technologies. Systems similar to the one proposed have never been designed or built. Hence, major system components must be designed to establish the difficulty of meeting technical requirements and to ultimately establish a cost for each component.

A future report will study in detail the cost/performance tradeoffs of this system versus other local transmission methods involving other technologies such as packet radio, polled multidrop lines, dial up, and other communication techniques.

8.1 MATV AND CATV SYSTEMS

The digital MATV and CATV study should be carried out in two phases. Phase one will develop data transmission techniques for a locally built MATV system. In the second phase, this technology will be transferred to an existing CATV system. The advantages of this mode of operation are the following:

- The MATV system will be local and hence there would be no initial problems of equipment transportation.
- There are no initial restrictions on TV interference or picture quality.
- There are no initial restrictions on carrier frequency or bandwidth.
- The initial system can be varied in structure and performance to approximate the degradation in any CATV system.
- A final advantage of performing the tests on high quality, well built MATV system is that it would completely prove out the viability of using MATV systems to solve the in-building distribution problem. Although MATV equipment is generally poorer quality than CATV systems, there should be only minor technical problems with the local MATV system since cascades are low and environmental conditions are good. The local system will also provide a standard to which older, poorly built MATV systems must be raised for data transmission capabilities.

The MATV system should meet the following specifications:

- The system will have a separate antenna and pre-amplifier for each channel, so the levels for each channel can be controlled independently.

- All available off-the-air channels will be received.
- The system will cover the full VHF and UHF range.
- The head end amplifiers will be driven at full output capability of about 55 dbmV in order to test significant noise and cross modulation figures.
- The system will be two-way so that at any tap a signal modulated by digital data can be inserted and received at any other tap. This will be done by feeding the signal back to the head end and redistributing it through an amplifier after conversion.
- The system will contain at least one extender amplifier on a leg of at least 1500 feet to simulate paths in a large building.
- The system will contain converters so that signals can be converted from a sub-VHF channel to a mid band channel.

A block diagram for the system is shown in Figure 8.1.1 and a bill of materials using Jerrold equipment is given in Table 8.1.1.

Once the equipment and techniques are perfected for the MATV data system, they must be tested and modified under environmental operating conditions on an actual CATV system. CATV systems use much higher quality equipment than do MATV systems, but have much longer cascades. Since the test CATV system should be an operating system, there will be certain restrictions on test operations.

The requirements on the data transmission are as follows:

- The data signal bandwidth be limited to an available 6 MHZ TV channel. The placement of the signal is still being investigated. If operation were to be at $\sim$70 MHZ, the 4 MHZ guard band between 70 and 74 MHZ could be used.

- The data signal must not yield visible interference with TV service. A reasonable guarantee is achieved if the data signal is kept 20 db below the video transmission.

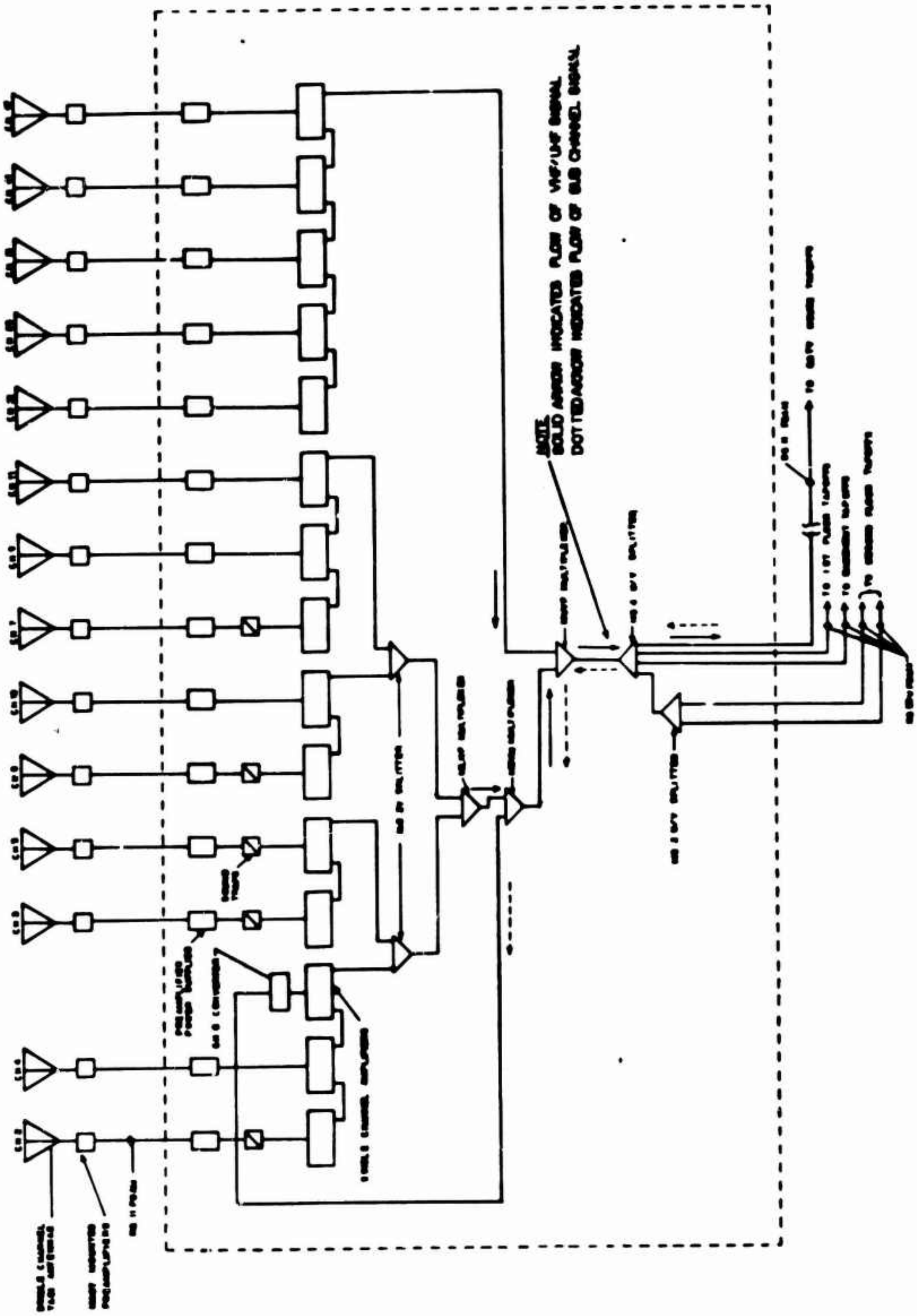


FIGURE 8.1.1
BLOCK DIAGRAM OF MATV SYSTEM

MATV SYSTEM BILL OF MATERIALS

QUANTITY	JERROLD CATALOG NUMBER	DESCRIPTION
4	J-Series	Lo band VHF antenna
5	J-Series	Hi band VHF antenna
5	J-275	UHF antenna
9	TPR	VHF preamplifiers
5	TPR (1443, 3968)	UHF preamplifier
2	PPS-8A	preamplifier power supply
10	THPM	VHF amplifier
5	UCA	UHF amplifier
3	TLB-2	trap filter
2	THB-2	trap filter
1	SCON-Sub-V	subchannel to VHF converter
2	1597	four-way splitter, UHF/VHF
2	1592B	two-way splitter, VHF
1	LHS-76	VHF multiplexer
1	FCO-320	UHF/VHF multiplexer
1	.'CO47	subchannel - VHF, multiplexer
1	1596A	two-way splitter, VHF/UHF
38	UT-82	Subscriber taps
1	SPS-30/60A	power supply
1	--	rack and miscellaneous hardware
1	SLE-300-2W	CATV extender amplifier
4000 ft.	JA-500-cc-J	5 inch coaxial cable

TABLE 8.1.1

8.2 TERMINAL INTERFACE

A. The interface can use a non-slotted ALOHA random access mode. The first transmission, as well as subsequent retransmissions, will be randomized over time.

B. The interface will transmit and receive data to a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem or utilize modem provided clock signals along with modem data.

C. The interface will provide and receive data for the interactive terminal at the rate required by the particular terminal. The exact rate chosen will be selected from the rates given below by means of simple plug changes or terminal block jumper wiring at the interface unit. The data rates possible are:

110	bits per second
134.5	bits per second
150	bits per second
300	bits per second
600	bits per second
1200	bits per second
2400	bits per second

The terminal input and output rates shall be independently selectable. In addition, it should be possible to control the transfer of data to and from the terminal by means of an externally supplied clock signal(s) which is below 2400 bits per second.

D. The interface shall be capable of receiving or transmitting 5, 6, 7, and 8 bit characters. The number of bits per character shall be selectable independently for transmitter and receiver by means of plugs or jumper straps within the interface.

E. The interface shall be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

F. The following formats will be used for the TV/Terminal system:

1. Data Packets

HEADER	TEXT	CHECKSUM

2. Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D.	SOURCE I.D.	CONTROL CODES	MESSAGE NUMBER	TERMINAL RCVR RATE
40 bits	40 bits	8 bits	4 bits	4 bits

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation.

There are three possible means of identifying source and destination in the TV/Terminal Interface.

A. "Log in" to the interface via the terminal and establish the ID codes which the interface then uses in each header.

B. Use short, hard wired (or switch selectable) interface ID codes for header information. The user of the terminal "logs in" to the head end to identify fully.

C. Use full switch selectable ID codes on the interface into which the user sets his ID number. A likely choice is a Social Security number of 9 4-bit digits. No further identification of the user is required to the head end.

The first alternative is unattractive because the implementation of a dialogue capability between the terminal and the interface will be more costly and complicated than is warranted. A hardware design which allows (C), also allows (B) as an alternate later on with only software changes while the reverse is not true. For this reason, the initial test interfaces will be built to allow insertion of a 36 bit (9 4-bit digits) ID code for source and destination. If at a later date it is desired to use a log in procedure at the head end, the switches will be used to set in a smaller interface ID code.

Since there are boundaries at 8 and 16 bits due to most mini-computers which might be used at the head end of the CATV/MATV systems, initial choice of 40 bit fields for both source and destination ID's is recommended. An 8 bit control code field should also be adequate. The control code field will contain information such as message ID and acknowledge bits. The total header length is then 96 bits or six 16 bit words.

G. The following description of the proposed operation of the transmitter section of the TV/Terminal interface assumes that the terminal has not been operated with the interface prior to this operation.

1. The switches, jumpers, and plugs on or in the interface should be set to conform to the parameters of the terminals. This includes input and output data rates, character size, and interfacing conventions such as EIA, current loop, etc.

2. The initial parameters which are required for operation with a given head end computer are set in via switches, plugs, or jumpers. These parameters include source and destination I.D. codes and special escape characters which, upon receipt by the interface, cause initiation of various functions (if any). As an example, the character sequence which, when received from the terminal, causes the interface to initiate message transmission is one such parameter.

3. The terminal should be connected to the interface and modem, and A.C. power should be applied to all units.

4. The terminal operator should compose the first message and enter it on the terminal keyboard. The interface will accept and buffer the message up to a maximum number of characters. For interactive terminals, the maximum number of characters will be 125, the maximum line length. Depending on the terminal type and mode of operation, the interface might provide data character echoing to indicate correct message receipt to the terminal operator.

5. Upon receipt of the escape character sequence from the terminal (line terminator such as carriage return, line feed, or a similar sequence), or upon filling of the interface buffer, a transmission would be initiated from the interface, and the message would be transmitted at 100K or 1M bits per second. Upon transmission of the message, a timer would be started to time out the reception of an acknowledgement from the message destination.

The timer will serve two functions: (a) time out the reception of an acknowledgement (of the transmitted message) from the head end for retransmission by the terminal interface, and (b) randomize the starting time for the retransmission. The fixed minimum delay before acknowledgement should be slightly greater than 14 milliseconds. The exact number will be chosen later and will reflect the ease of generation of the hardware,and be based on a multiple of an available clock rate.

The random interval of delay with respect to the starting time for the retransmission will vary from 0 to ≈ 5 full packet intervals (0 to ≈ 64 milliseconds) with an exact choice determined in the same manner as for the fixed interval. The maximum random interval will be divided into 1/2 millisecond sub-intervals as the various transmission starting times, corresponding to 128 possible starting delays. If the timer runs out without the reception of an acknowledgement, the message is retransmitted. The transmitter retransmits a message N times, where N is selected between 0 and 15 by means of jumpers or a switch in the interface. If, after N retransmissions, no acknowledgement has been received, an indicator will be activated to notify the terminal operator of failure to communicate with the head end computer.

There will be a "Packet Acknowledged" light on the interface which allows the terminal operator to know if the packet he last sent has been acknowledged and tells him when he can start to send the next packet. In the case of an unacknowledged packet, the interface will refuse any new data from the terminal, and that state may be cleared by pressing a "reset" button on the interface.

6. As soon as an acknowledgement is received, the local copy of the transmitted message may be released and a new message accepted from the terminal for transmission. With this type of operation, only one message may be handed at a given time by the interface. Once a given message has been accepted for transmission by an interface, the terminal user must wait until either the message is sent and acknowledged or the message is cleared by the interface reset button.

7. A preliminary flow chart for operation of the transmitter is shown in Figure 8.2.1.

H. The description of the proposed operation of the receiver section of the TV/Terminal interface assumes that the terminal has not been operated with the interface prior to this operation.

1. The parameters of the terminal should be set into the interface as in steps G1, G2, and G3 above.

2. Upon detection of modulation on the TV channel, the raw digital data will be passed through the receiver and bit and character synchronization will be obtained.

3. The header will be inspected to see for which terminal the message is destined. If the message is not for this terminal, it will be ignored.

4. If the message is destined for this terminal, the checksum will be checked to insure that the message is error free. If there are errors, the message will be ignored.

5. If the checksum is valid, the header will be inspected to see if the message indicates an acknowledgement or other control function. If so, appropriate action will be taken. For example, if the message is an acknowledgement, the transmitter will be notified to drop the copy of the last message sent and will be able to accept a new message.

6. The message text will be received and stored in a buffer in the interface, and if an acknowledgement is required, the interface will transmit one to the message

TV/TERMINAL INTERFACE

TRANSMITTER FLOW CHART

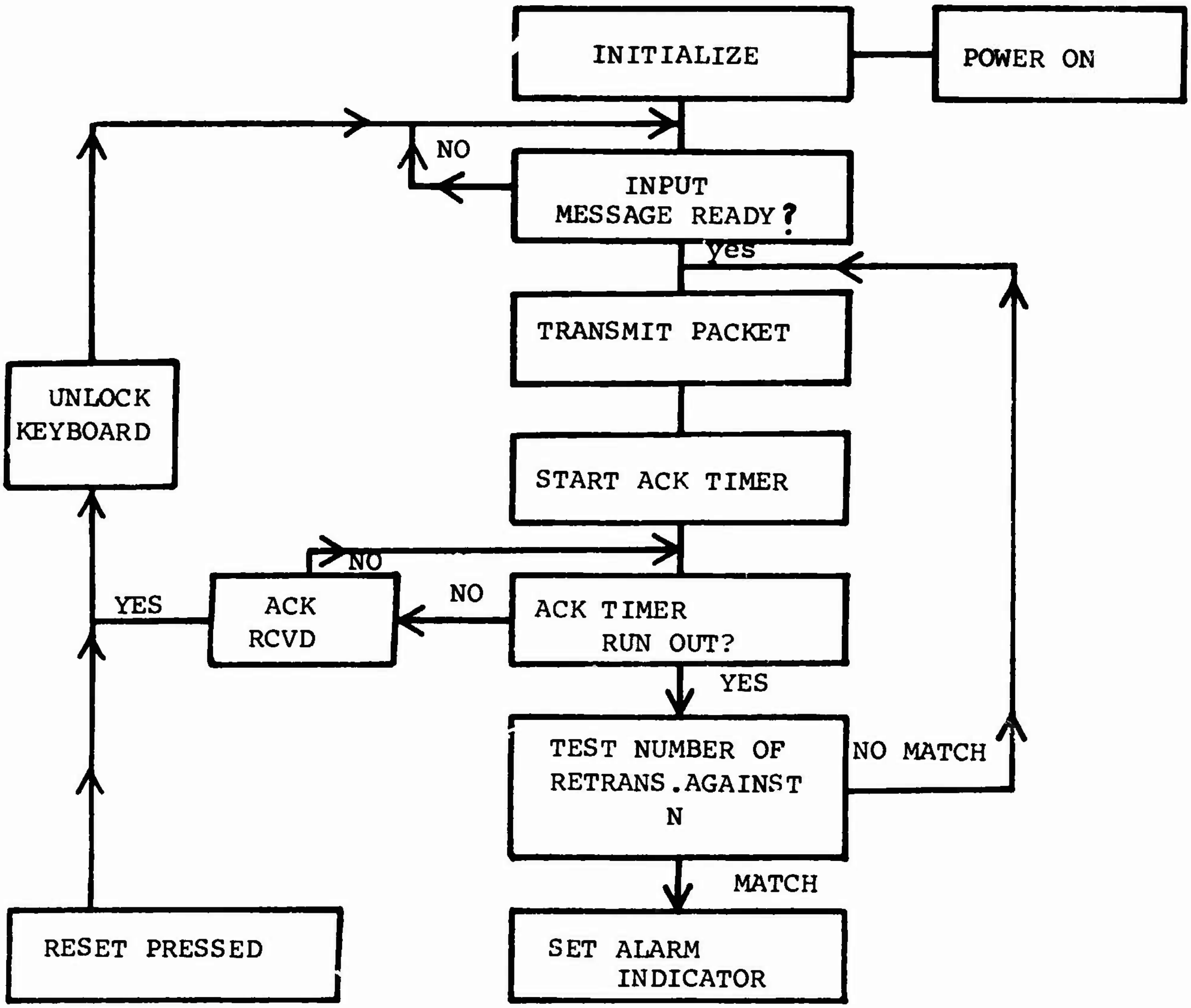


FIGURE 8.2.1

source. The interface will sense when the terminal is turned off, using the Data Terminal Ready (DTR) signal and will reject all packets received for that terminal during terminal power off conditions. The head end will not send messages to the same terminal interface without waiting a fixed "safety" period. The duration of this safety period will be determined at the head end by the specific terminal receiver data rate information which is a part of the header of all packets which originate from that terminal interface. The acknowledgement from the terminal interface will be sent to the head end simultaneously with the sending of the message from the interface to the terminal.

7. The number of the most recently received message will be stored in the interface logic to allow detection of duplicate messages.

8. A preliminary flow chart of the operation of the receiver is shown in Figure 8.2.2.

I. A Cyclic Redundant Code (CRC) checksum will be employed on the data for error control in the TV/Terminal system. The checksum for the message text will be chosen to provide adequate undetected error probability under channel conditions which are expected to be encountered in the TV system. The analysis and experience of the Packet Radio and ALOHA systems will be used to pick the best checksum. For hardware estimates, assume that the ARPANET type of 24 bit CRC will be used in the TV/Terminal interface.

TV/TERMINAL INTERFACE

RECEIVER FLOW CHART

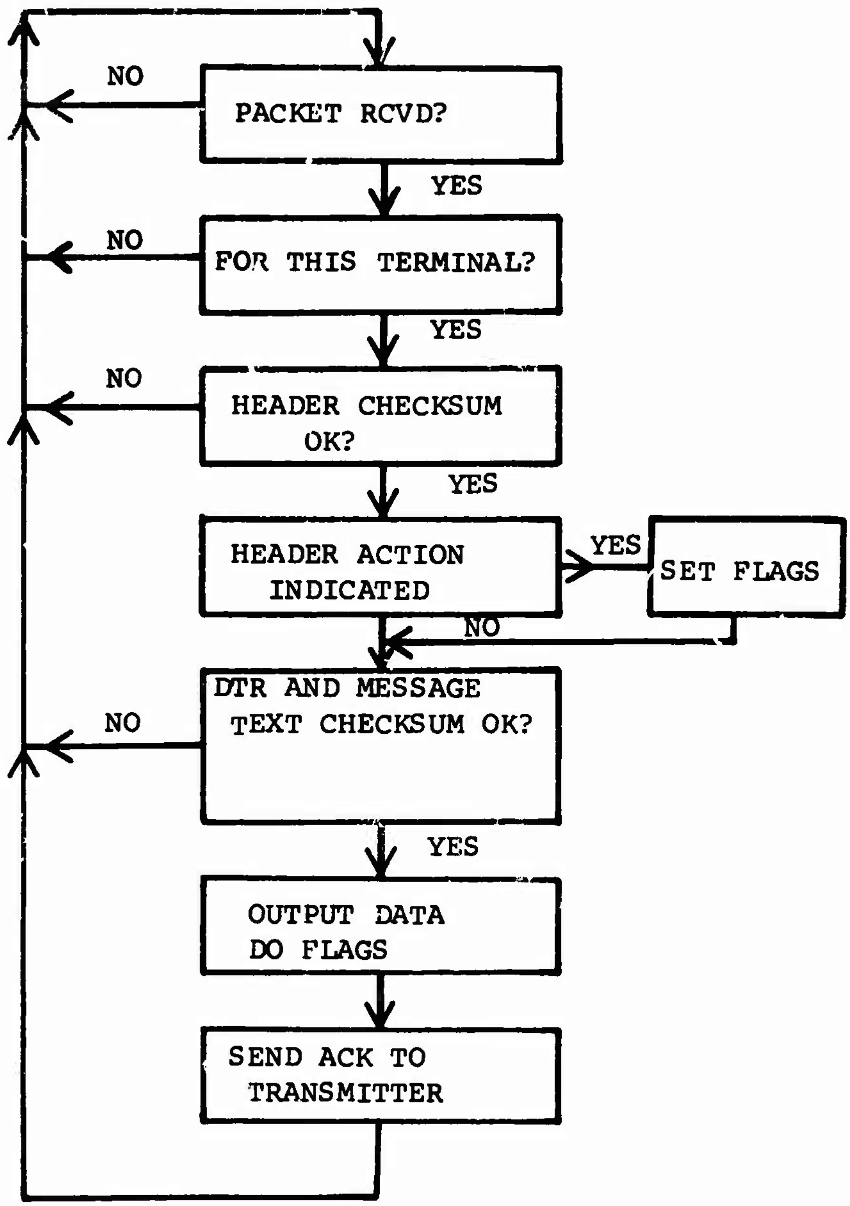


FIGURE 8.2.2

Electrical Specifications

A. The interface shall be capable of being configured to match a wide variety of terminal requirements. This flexibility shall be provided either by having a set of the various possible circuits required in each interface and selecting the proper form by jumpers of plugs, by having a set of sub-modules which can be interchanged in a given interface to adopt to a particular terminal's requirements, or, ideally (but possibly quite difficult), by having a single "universal" type of interface circuit which can be easily adapted to each terminal requirement. The types of requirements which can be expected are:

1. TTY (Current loop @ 20ma or 60ma)
2. EIA-RS-232 (Either full or partial depending on the terminal)
3. CCITT (Either full or partial depending on the terminal)
4. MIL STD 188B
5. Standard low level Signaling Interface
6. Specials

In all cases, the interface circuits should prevent or minimize damage to either the interface or to the terminal under conditions of short circuit, open circuit, or voltage or current transients. State-of-the-art isolation techniques will be used, including optically-coupled interface circuits and protective diodes on sensitive elements.

B. The interface will be designed to match the choice of modem. The same isolation and protective features used in the connection of the interface to the terminal will be included in the circuits which connect the interface to the modem.

C. The power supply and interface circuits will be housed together. Input power requirements are:

1. 110 volts A.C.
2. 60 Hz, single phase
3. Estimated input power required less than 100 watts

The power supplies will provide suitable voltages and currents for the operation of all circuits and modules within the interface. Standard modular off-the-shelf supplies will be used whenever possible. The interface will have a master power switch, power-on indicator, adequate protective fusing, transmission and reception status indicators, and a reset button.

D. The logical design of the interface will be realized using standard integrated circuits and components. The possible use of Large Scale Integration (LSI) logic modules, such as a microcomputers and Read Only Memory (ROM), will be investigated. The characteristics of available microcomputers will be carefully studied and a choice will be made based on the TV/Terminal interface requirements. At a later date, for reasons of improved microcomputer components or desired compatibility with other packet systems, a different microcomputer can be selected and programmed to perform the same functions without major hardware changes. The project will result in a reliable, cost effective interface with sufficient flexibility to allow reasonable system modifications during testing.

Mechanical Specifications

A. The interface will be packaged in an enclosure which is capable of standing alone. The estimated size of the interface is approximately 19" wide, 20" deep, and 5" high. The interface will have a 6' power cord and suitable connectors or terminal strips to match the connections to the terminals and modem with which the interface will operate. Sound construction practice will be used throughout with an emphasis on ease of assembly and maintenance along with low cost. The estimated weight of the interface is less than 30 pounds.

B. The front panel of the TV/Terminal interface will have a power switch and whatever other switches and visual indicators are required for operator assurance, such as power-on and incomplete transmission. All cther switches, connectors, and seldom used controls will be mounted in a conveniently accessible, yet protected, fashion. Some connectors will be mounted on the rear panel of the interface, while other controls and terminals will be accessible only with a set of tools such as a screwdriver or key to open a service access panel or to remove the protective cover(s).

BLOCK DIAGRAM OF TERMINAL INTERFACE (FIGURE 8.2.3)

The cable modem data and control signals pass through appropriate level-conversion logic on entering the terminal interface. Modem control signals and status signals are set or sensed by the micro computer program via paths in the micro computer I/O multiplexer and controller. All incoming data are passed through a data examination register where special high speed character detection logic looks for synchronization characters and enables the checksum verification logic upon synchronization. All synchronized data are stored automatically in a high speed received packet buffer (First In, First Out - FIFO). Upon completion of reception of a packet, the micro computer is interrupted and begins to process the packet. The checksum is verified, and, if false, the buffer is cleared, and the sync logic is reinitialized. If the checksum is verified as true, the packet header is examined for destination, and if no match is detected, the receiver section is reinitialized and the buffer is cleared.

If a destination match is detected, the appropriate acknowledge and control information is stored in The Random Access Memor , (RAM), and the data is enabled to pass on a byte basis to the Universal Asynchronous Receiver - Transmitter, (UAR-T), which converts the parallel data bytes to serial asynchronous data at the terminal speed. The data passes through appropriate terminal level conversion circuits before leaving the interface. The terminal control and status sense circuits are also connected to the micro computer and the I/O multiplexer. The micro computer can insert or delete characters in the actual data stream between the received packet buffer and the UAR-T, as for example in the case of an acknowledge for a previously transmitted message where no data would be passed to the terminal.

Data from the terminal is automatically stripped of start-stop bits by another UAR-T and stored in byte format in the transmit packet buffer. Upon detection of the terminal special end-of-line character or on filling up a packet, the micro computer is interrupted and attaches the header information to the data in the transmit packet buffer. At the appropriate randomized time, the data is gated to the output register and the checksum generation logic is enabled. The information about the message for retransmission purposes is stored in RAM and the message is recirculated in the transmit buffer at the same time it is sent to the modem. The micro computer controls the number and timing of retransmissions based on acknowledge information from received packets. The crystal clock provides timing pulses to all circuits as required.

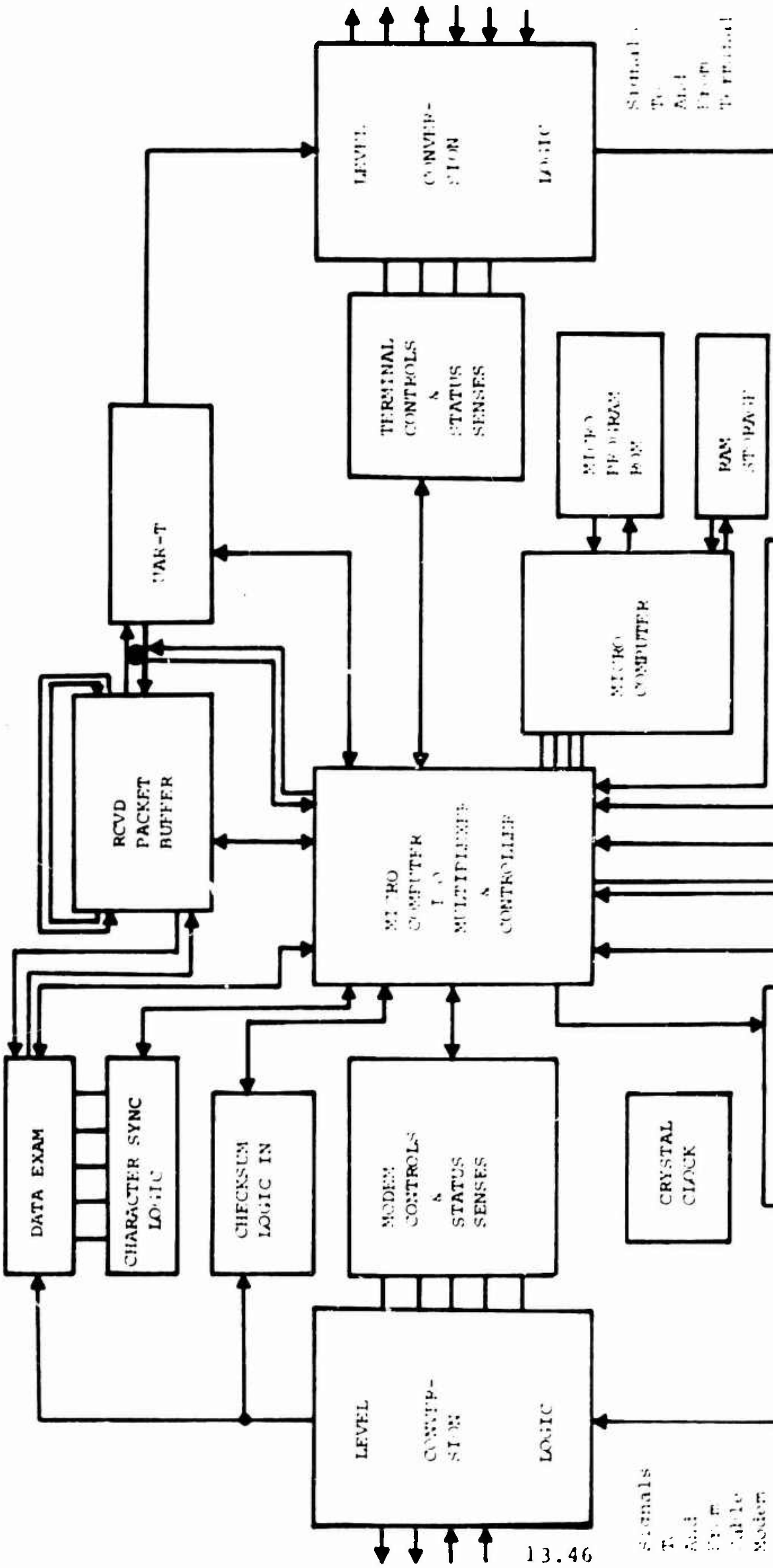


Figure 8.2.3 BLOCK DIAGRAM OF TERMINAL INTERFACE

8.3 MODEMS

For initial experiments a 2-phase differential phase shift keyed (DPSK), coherently detected signal with a 100 kilobit data rate is adequate. This selection of parameters enables the use of much off-the-shelf equipment while still giving the properties of a system meeting practical data requirements. For example, modems are commercially available to produce a baseband PSK binary data stream which can then be modulated onto a video carrier. These modems can produce a 0 dbmv signal on a 75 ohm line with good capture properties and excellent phase stability. The off-the-shelf modem will probably have poor acquisition time, on order of 100 milliseconds, which will limit the system to less than two hundred terminals in initial experiments.

Later experiments can be performed with 4 level DPSK and at a Megabit/sec data rate. However, the short acquisition times required for 1 Megabit/sec data rate would require special development of a Surface Acoustic Wave Device type detector [Matthaei,1973]. The final selection of parameters for a practical data transmission system depends upon several variables whose values are uncertain and subject to change with improvements in technology:

1. The bandwidth of the head end minicomputer
2. The amount of core at the head end minicomputer
3. The bandwidth of the links to external test facilities.
4. The throughput at the terminals
5. The subscriber saturation level of CATV system

The specifications to be met by the final modems are given in Table 8.3.1.

MODEM SPECIFICATIONS

type of modulation	DPSK
data throughput	100 KB/sec. or 1MB/sec.
carrier frequency	5-240 MHZ
nominal signal level	input 10 dbmv,output 32 dbmv
impedance	75Ω
reflection coefficient	-23 db
noise figure	7.5 db
power hum	60 db below signal level

Table 8.3.1

Modems meeting these specifications are available with performance parameters and resulting system degradation indicated in Table 8.3.2, [Cuccia, 1973]. The signal degradation is within acceptable limits for error rates better than 10^{-20}.

Carrier Instability in Carrier Source	1/2° RMS in a PLL with Bandwidth = 0.03% Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Modulator	2°	0.10 db
Rise Time in Each Phase Change	1/4 Bit Period	0.40 db
Amplitude Unbalance in QPSK Modulator	0.2 db, 10°/db	0.10 db
Data Asymmetry from Data Source	±2%	0.05 db
Group Delay Distortion (Modulator)	25°	0.20 db
Clock Instability	1° RMS in the Bit Synchronizer PLL	0.10 db
	SOURCE TOTAL =	1.00 db

DEMODULATOR

Incidental FM from all Oscillators in RF Channel and Carrier Reconstruction	1/2° RMS in PLL with Bandwidth = 0.03% of Bit Rate Bandwidth	0.05 db
Static Phase Error in QPSK Demodulator	2°	0.10 db
Reference Phase Noise in Reference PLO	1°	0.10 db
Total Group Depay Distortion (Total Channel from Modulator)	20°	0.35 db
Timing Jitter in Matched Filter Sampler	4%	0.25 db
Timing Bias Error in Matched Filter Sampler	1.5%	0.15 db
DC Offsets-Total Receive	4%	0.10 db
Data Waveform/Matched Filter Detector Mismatch	--	0.60 db
	SOURCE TOTAL =	1.70 db

Table 8.3.2

8.4 TEST MINICOMPUTER

A. General

The minicomputer used as the test head end controller in the Terminal/MATV-CATV system will have the following characteristics:

A. 16 bit word length
B. ∿1 μ sec cycle time core memory
C. Memory size expandable to at least 32k words

In order to adequately test the Terminal/TV System, a variety of peripherals and interfaces provided by the mini computer vendor may be required. In addition, several custom interfaces may have to be developed for cases where standard interfaces are not available.

B. Data Rate Constraints

The data rates expected in the head end computer system are shown in Figure 8.4. The data to and from the cable modem will flow at the rates of either 10^5 or 10^6 bits per second, although the flow will not be continuous in general. At the rate of 10^6 bits/second, a 16 bit computer word will be assembled in the interface every 16 μ seconds. The transfers of these words into and out of the computer will require the availability of a high speed data channel, or alternately a Direct Memory Access (DMA) channel as a required option on the computer. Transfers to and from memory will be accomplished automatically without requiring program intervention.

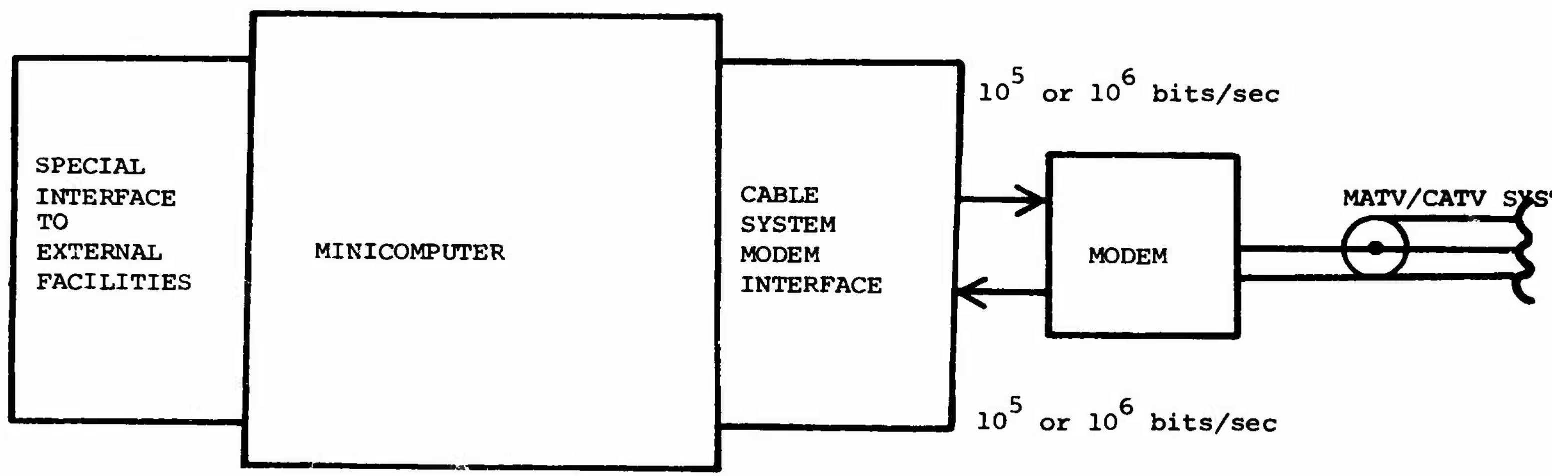


Figure 8.4.1 Head End Computer System Data Rates

C. Functional Description

The computer at the head end must implement the programs to test the following functions for the system:

- Buffering of packets to and from terminals.
- Flow control of cable system
- Implementation of network protocols.

The headend computer will provide sufficient buffer storage to match the difference between the peak to average data rates seen on the cable system and any external sources of data. Initially, storage for a total of the order of 100 packets will be provided.

The head end computer will provide the required cable system flow control by means of acknowledges for received packets. When traffic from the cable terminals begins to congest the system, the head end will effect the flow control by refusing to acknowledge packets which cannot be accepted. This will cause the terminal to retransmit messages, producing the same effect as errors on the cable system. The head end computer will provide the implementation of all protocols required to communicate on the system.

The following is a list of some of the major program pieces which must be implemented:

1. Cable modem interface handler
2. Special interface to external facilities handler
3. Monitor or supervisor for system
4. Cable flow control routine
5. Buffer allocation and management
6. Miscellaneous background tasks such as garbage collect, timeouts, accounting, etc.

8.5 MINICOMPUTER TO CATV INTERFACE

This specification describes the interface between the head end computer and the MATV and/or CATV system. A diagram of the system is shown in Figure 8.5.1.

FUNCTIONAL SPECIFICATIONS

A. The MATV-CATV computer interface will use a non-slotted ALOHA random access mode. The first transmission as well as subsequent retransmission of messages will be randomized over time.

B. The interface will be able to simultaneously and independently transmit to and receive data from a suitable modem at a rate of 100K or 1M bits per second. Data will be supplied to and received from the modem in bit serial form. The interface will be able to provide crystal controlled clock signals to the modem, and receive modem clock signals along with modem data.

C. The interface will be capable of receiving or transmitting 8 bit characters. The computer program shall be able to control the actual number of valid character bits in each received or transmitted character on a message by message basis.

D. The interface will be capable of transparently receiving or transmitting all bits included in each character, including a parity bit, independently for transmitter and receiver.

E. The following formats will be used for the TV/Terminal system:

(1) Data Packets

HEADER	TEXT	CHECKSUM

(2) Acknowledge Packets

HEADER	CHECKSUM

The format within the header will be as follows:

DEST. I.D. 40 bits	SOURCE I.D. 40 bits	CONTROL CODES 8 bits	MESSAGE NUMBER 4 bits	TERMINAL RCVR RATE 4 bits

COMPUTER TO TV/TERMINAL CABLE SYSTEM INTERFACE

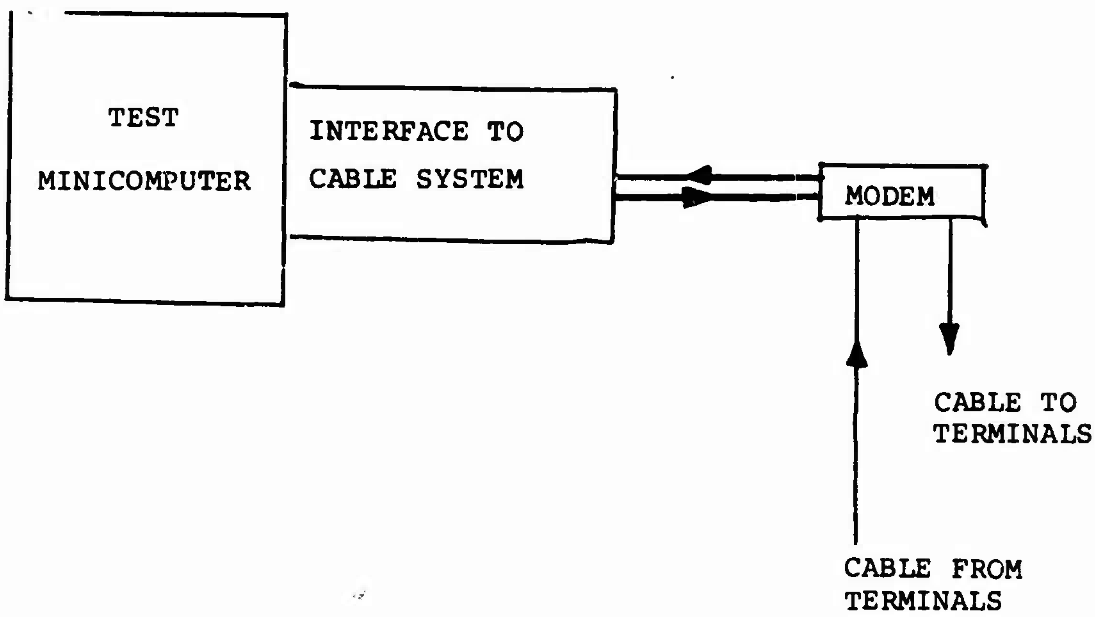


FIGURE 8.5.1

The field sizes are chosen to allow expansion to a reasonable number of terminal interfaces on a given TV system, and to provide sufficient control codes for system operation. The destination and source I.D. fields will each be 40 bits long. This length is the lowest 16 bit word boundary which will contain the temporary 36 bit source and destination identifier.

F. The transmitter section will operate in the following manner:

1. Messages within the computer which are to be transmitted will be placed in one of two queues, a high priority acknowledgement queue or a regular message queue.

2. All messages in the high priority queue will be transmitted before any regular messages are sent. The high priority queue will be serviced before each regular message transmission.

3. Copies of transmitted acknowledgements will not be kept, since acknowledgements are never retransmitted.

4. Copies of all transmitted regular messages will be kept in the computer until either receipt of an acknowledgement from the destination or the occurrence of a program-determined number of unsuccessful retransmissions.

5. The transmitter program will set up a parameter table for each terminal active on the cable system. This table will contain terminal-specific information, such as terminal output rate and character set; and will be used to format each transmitted message to match the characteristics of the destination terminal and to allow the spacing of messages for each specific terminal in time to match terminal output speed.

6. The transmitter program will operate in the computer in conjunction with other programs which will deal with the various functions required of the head end test mini computer system. There will be a monitor or executive program which will control the activities of

the cable system and external interface programs. This monitor will also control or provide various required console related activities and accounting functions.

7. Transfer of messages from the computer to the interface logic will be by Direct Memory Access (DMA) of the head end computer.

8. The transmitter interface hardware will calculate the message checksum for each message and append it to the message. In addition, the hardware will automatically perform such routine tasks as synchronization character generation, special escape character generation (for transparency), and others as required. When the interface hardware completes the transmission of each message, a program interrupt will be generated.

G. The receiver section will operate in the following manner:

1. Message transfers between the cable system interface and the computer memory will be by means of DMA.

2. The interface logic will compute the checksum of the received message and notify the computer by interrupt if an error has occurred. In this case, the computer program will discard the data for that message.

3. The interface logic will automatically acquire character synchronization and will strip synchronization and control characters from the input data stream.

4. The interface will allow the transparent reception of full binary text.

5. As each message is received, its checksum will be verified and a computer interrupt will be generated.

6. The receiver section of the computer program will, upon notification by a receiver interrupt, examine the new message and dispatch the message to the appropriate queue for further action. Sample actions are given below:

a. New message - determine destination queue and forward message, send acknowledgement to terminal, and check for retransmission.

b. Acknowledge - remove copy of acknowledged message from transmitter retransmission storage.

c. Duplicate - discard

7. The receiver section of the program will operate under the control of the computer monitor program and will interface to the transmitter and protocol programs.

ELECTRICAL SPECIFICATIONS

A. Modem Connection

The interface will be designed to match the choice of modem. Interface circuits should prevent or minimize damage to either the computer or the modem under conditions of short circuit, open circuit, or voltage or current transients. State-of-the-art isolation techniques will be used, including optically-coupled circuits and protective diodes on sensitive elements.

B. Computer Connection

The interface will be designed to connect to the Input/Output bus of the head end computer. The manufacturer's recommendations and specifications will be met in all cases to insure proper computer and interface operation.

C. Power

Power for the interface logic will be obtained from the computer power supply. Power for circuits beyond the isolation devices will be obtained by small modular power supplies which will be incorporated into the interface assembly.

D. Logical Design

The logical design of the interface will be realized using standard integrated circuits and components either of the types recommended by the minicomputer manufacturer or their equivalent. The project will result in a reliable cost effective interface with sufficient flexibility to allow reasonable system modifications during testing.

MECHANICAL SPECIFICATION

A. The interface will be packaged on circuit boards of the type used by the minicomputer for interface circuits. The circuit boards will plug into the I/O bus of the minicomputer in the enclosure provided by the minicomputer and will connect to the modem with a cable and connector. Estimated weight of the interface is less than 10 lbs. The prototype version of the interface will most likely use wire-wrapped sockets for the integrated circuits to facilitate design changes during testing.

B. There will be no operational controls associated with the interface other than program control. Test modes and switches for their control will be provided.

BLOCK DIAGRAM
OF MINICOMPUTER TO CATV INTERFACE

Figure 8.5.2

Assume that any frequency or phase acquisition is accomplished by the modem. The received bit stream is examined by the interface hardware for sync characters and synchronization is automatically accomplished without computer intervention. The interface hardware also handles DLE doubling and other special signaling conventions such as STX detection and ETX detection on the cable system automatically. As each character is received, it is packed into a minicomputer 16 bit word and when the 16 bits are ready, the word is transferred to core memory on a DMA cycle stealing basis. A computer interrupt is generated at the end of a packet or upon an error condition.

The transmit section of the interface is also automatic in that after initialization of the interface for a new packet, all communication discipline is accomplished automatically by the interface hardware, and words of data are simply requested by the interface via DMA from core, unpacked and serialized. The hardware also calculates and appends the checksum.

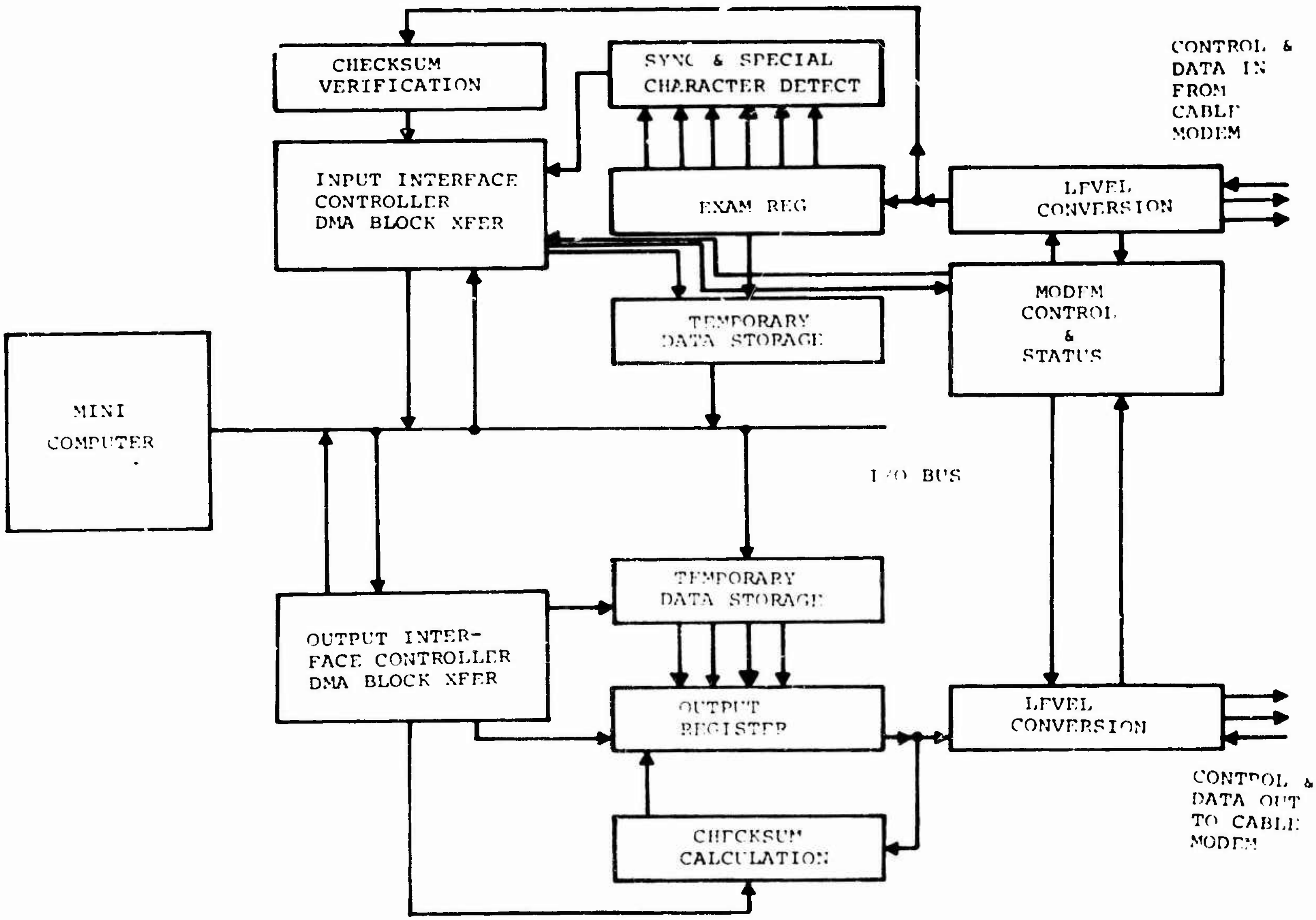


Figure 8.52 BLOCK DIAGRAM OF MINICOMPUTER TO CABLE SYSTEM INTERFACE

8.6 MINICOMPUTER TO EXTERNAL TEST FACILITIES INTERFACE

This specification describes the interface between the head end test minicomputer and a communications link to external test facilities.

Functional Specifications

A. The interface will communicate with the external test facilities over leased lines using commercially available modems, and will provide all of the control signals required to operate these modems.

B. The interface will connect to the I/O system of the selected minicomputer and will comply with all specifications set forth for such connections.

C. The interface will operate in full duplex mode at a rate of up to at least 4800 bits per second.

Electrical Specifications

A. The interface will be able to connect to a suitable data modem operating at 4800 bps full duplex. The connections will conform to EIA RS-232-C. The interface will incorporate adequate isolation features to prevent damage to itself or to the modem under conditions of short circuit or open circuit signal leads. In addition, sensitive interface elements will be protected from damage by voltage or current transients from external sources.

B. Power Supplies

The interface will be powered from the minicomputer logic power supplies, except for circuits which are to be isolated from the computer system. Any isolated sections of interface logic circuits will be powered by isolated power supplies with characteristics to match the degree of isolation desired. It is estimated that less than 50 watts of DC power will be required for the interface circuits.

The repeaters at d=5 do not have messages arriving from other stations. They only receive their own traffic at Poisson rate. Repeaters at $d < 5$ which are on the "axes" (denoted by circles) have messages arriving from the three neighbors at d+1, as well as their own Poisson traffic.

Repeaters off the axes at distance $d < 5$ have input at each time point from the two neighbors at d+1 as well as their own Poisson traffic.

The network is activated at t=0 by having random Poisson arrivals with mean λ at each of the 61 repeaters. This input traffic at each repeater is converted to received messages in each of the two possible modes for different values of m by use of the "transfer functions".

$$(1) \quad P_{kj} = \frac{\binom{m}{j}}{m^k} \sum_{\nu=0}^{\min(k-j,m-j)} (-1)^{\nu} \cdot \binom{m-j}{\nu} \frac{k!}{(k-j-\nu)!} (m-j-\nu)^{k-j-\nu} \; ;$$

$$(2) \quad P^{*}_{kj} = \binom{m}{j} \sum_{\nu=0}^{j} (-1)^{\nu} \binom{j}{\nu} \left(\frac{j-\nu}{m}\right)^{k} . \qquad j=0,\ 1,\ 2,\ldots,\min(k,m)$$

These calculations give us $P^R_{(d,j)}(0)$ for all repeaters with coordinates (d,j), d=1, 2, 3, 4, 5, j=1,...,4d, an (0,0) the station at the origin.

We can now determine message traffic at each repeater by using equations which describe message transmission in the direction of of the "origin".

<u>For Time t=1</u>

<u>When d=5; j=1, 2,...,20</u> , the repeaters at d=5 receive only their generated Poisson traffic. Thus, for time 1 we generate 61 Poisson traffic numbers which describe direct (i.e., at the source) message input. When $d \le 4$, the repeater at coordinates (d,j) also receive traffic from its neighbors at further distance

by one unit. The following equations describe messages arriving at each repeater for arbitrary time $t > 1$.

<u>On the Axis</u>:

$$P_{(0,0)}(t) = P^R_{(1,1)}(t-1)+P^R_{(1,2)}(t-1)+P^R_{(1,3)}(t-1)+P^R_{(1,4)}(t-1)+\text{Poisson}$$

<u>At d=1</u> $(P_{1,1}, P_{1,2}, P_{1,3}, P_{1,4})$

$$P_{(1,1)}(t) = P^R_{(2,1)}(t-1)+P^R_{(2,2)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

$$P_{(1,2)}(t) = P^R_{(2,2)}(t-1)+P^R_{(2,3)}(t-1)+P^R_{(2,4)}(t-1)+\text{Poisson}$$

$$P_{(1,3)}(t) = P^R_{(2,4)}(t-1)+P^R_{(2,5)}(t-1)+P^R_{(2,6)}(t-1)+\text{Poisson}$$

$$P_{(1,4)}(t) = P^R_{(2,6)}(t-1)+P^R_{(2,7)}(t-1)+P^R_{(2,8)}(t-1)+\text{Poisson}$$

<u>At d=2</u> $P_{2,1}, P_{2,3}, P_{2,5}, P_{2,7}$

$$P_{(2,1)}(t) = P^R_{(3,1)}(t-1)+P^R_{(3,2)}(t-1)+P^R_{(3,12)}(t-1)+\text{Poisson}$$

$$P_{(2,3)}(t) = P_{(3,3)}(t-1)+P^R_{(3,4)}(t-1)+P^R_{(3,5)}(t-1)+\text{Poisson}$$

$$P_{(2,5)}(t) = P^R_{(3,6)}(t-1)+P^R_{(3,7)}(t-1+P^R_{(3,8)}(t-1)+\text{Poisson}$$

$$P_{(2,7)}(t) = P^R_{(3,9)}(t-1)+P^R_{(3,10)}(t-1)+P^R_{(3,11)}(t-1)+\text{Poisson}$$

<u>At d=3</u> $P_{3,1}, P_{3,4}, P_{3,7}, P_{3,10}$

$$P_{(3,1)}(t) = P^R_{(4,1)}(t-1)+P^R_{(4,2)}(t-1)+P^R_{(4,16)}(t-1)+\text{Poisson}$$

$$P_{(3,4)}(t) = P^R_{(4,4)}(t-1)+P^R_{(4,5)}(t-1)+P^R_{(4,6)}(t-1)+\text{Poisson}$$

$$P_{(3,7)}(t) = P^R_{(4,8)}(t-1)+P^R_{(4,9)}(t-1)+P^R_{(4,10)}(t-1)+\text{Poisson}$$

$$P_{(3,10)}(t) = P^R_{(4,12)}(t-1)+P^R_{(4,13)}(t-1)+P^R_{(4,14)}(t-1)+\text{Poisson}$$

At d=4 $P_{(4,1)}(t)$, $P_{(4,5)}(t)$, $P_{(4,9)}(t)$, $P_{(4,13)}(t)$;

$$P_{(4,1)}(t) = P^R_{(5,1)}(t-1)+P^R_{(5,2)}(t-1)+P^R_{(5,20)}(t-1)+\text{Poisson}$$

$$P_{(4,5)}(t) = P^R_{(5,6)}(t-1)+P^R_{(5,5)}(t-1)+P^R_{(5,7)}(t-1)+\text{Poisson}$$

$$P_{(4,9)}(t) = P^R_{(5,10)}(t-1)+P^R_{(5,11)}(t-1)+P^R_{(5,12)}(t-1)+\text{Poisson}$$

$$P_{(4,13)}(t) = P^R_{(5,15)}(t-1)+P^R_{(5,16)}(t-1)+P^R_{(5,17)}(t-1)+\text{Poisson}$$

Off the Axes:

$$P_{(d,j)}(t) = P^R_{(d+1,j)}(t-1+P^R_{(d+1,j+1)}(t-1)+\text{Poisson};\quad j=2,3,\ldots,d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+1)}(t-1)+P^R_{(d+1,j+2)}(t-1)+\text{Poisson};\quad j=d+2,d+3,..2d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+2)}(t-1)+P^R_{(d+1,j+3)}(t-1)+\text{Poisson};\quad j=2d+2,..3d;\ d=2,3,4.$$

$$P_{(d,j)}(t) = P^R_{(d+1,j+3)}(t-1)+P^R_{(d+1,j+4)}(t-1)+\text{Poisson};\quad j=3d+2,..4d;\ d=2,3,4.$$

These equations relate arriving and received messages over neighboring time points and repeaters. Thus, the arriving number of messages can be computed in the grid at each point in time and each repeater.

In terms of a flow diagram, the procedure for analyzing this and all finite grids follows:

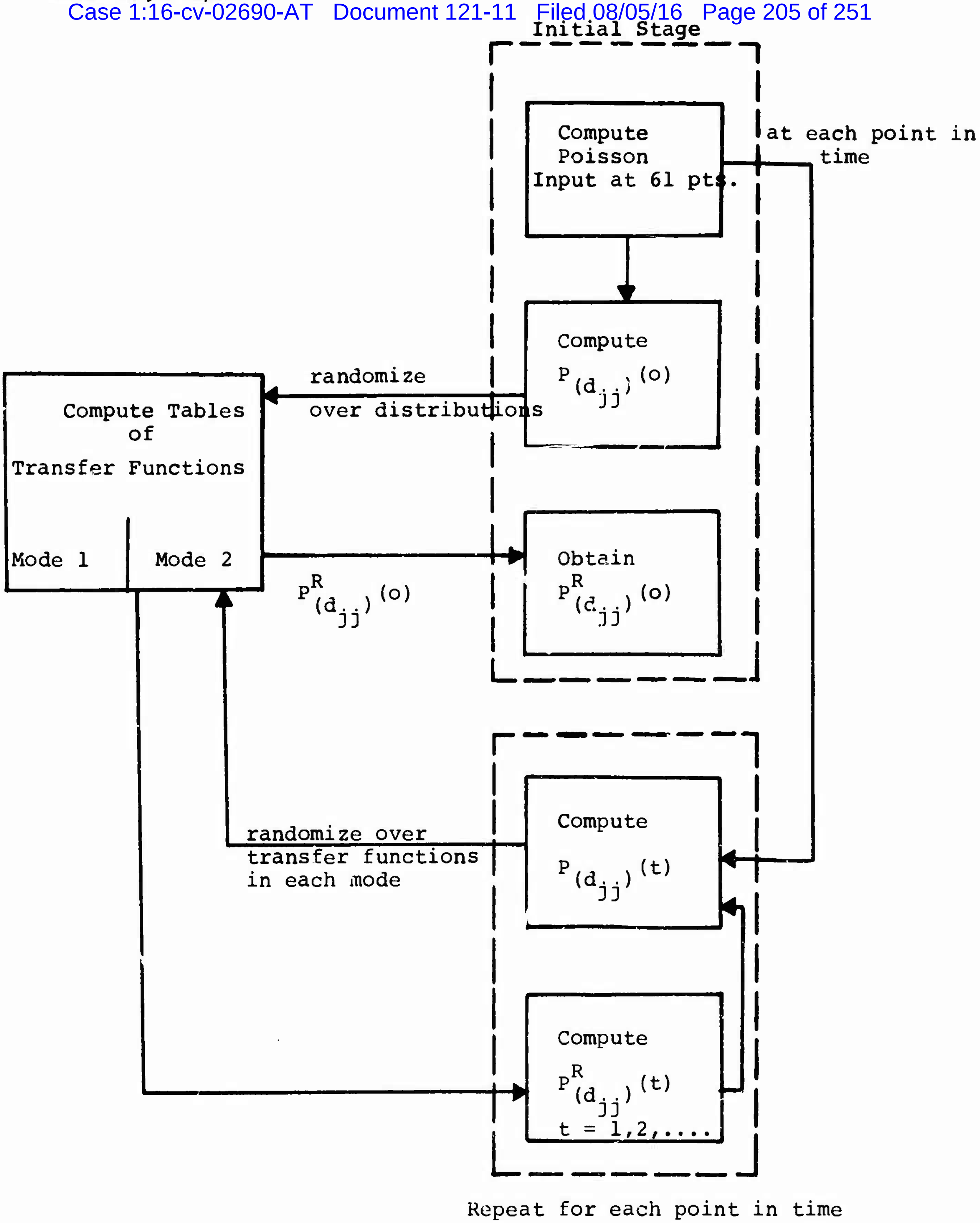

Initial Stage
Compute Poisson Input at 61 pts.
at each point in time
Compute
P(djj)(o)
randomize over distributions
Compute Tables of Transfer Functions
Mode 1
Mode 2
P^R(djj)(o)
Obtain
P^R(djj)(o)
randomize over transfer functions in each mode
Compute
P(djj)(t)
Compute
P^R(djj)(t)
t = 1,2,....
Repeat for each point in time

FIGURE 2

The parameters are, λ = mean Poisson arrival at each time point, at each repeater, m the number of slots in each mode one and two.

The output of the computer analysis is processed and presented in two forms, tabular and graphical. The tabular format is for each m and λ mode,

t	0	1	2	3	4	5	6	7	. . .
$P_{(0,0)}(t)$									. . .
$P^R_{(0,0)}(t)$									. . .
$P_{(1,1)}(t)$									. . .
$P^R_{(1,1)}(t)$									
$P_{(1,2)}(t)$									
$P^R_{(1,2)}(t)$									
⋮	⋮	⋮	⋮	⋮					

Various graphic analyses are also obtained.

A. A graph of arrived and received messages at the origin as a function of time for various values of m and λ.

B. A frequency histogram of arrivals off the axis. There are 24 points of the axis at distance 2, 3, 4,...

We take for each time t;

$$f(x) = \frac{\text{number of stations with x arrivals at time t}}{24}.$$

This is plotted for each time point.

C. The same histograms as in b except on the axis. There are 16 points on the axes at distances 1, 2, 3, 4.

D. The mean number of arrived and received messages $\hat{\lambda}(t)$ and $\hat{\lambda}^R(t)$ as a function of time on and off the axis. These are given by,

$$\hat{\lambda}(t) = \sum_{x=1} xf(x), \quad \hat{\lambda}^R(t) = \sum_{y=1} xf^R(x)$$

where f(x) if the frequency of arrivals and $f^R(x)$ is the frequency of received messages.

$$\hat{\lambda}_A(t) = \sum_{x=1} xf_A(x), \quad \hat{\lambda}_A^R(t) = \sum_{x=1} xf_A^R(x),$$

where $f_A(x)$ and $f_A^R(x)$ are frequencies on the axis of arriving and received messages. Some numerical results follow.

8.1 Summary of Initial Computer Analysis

Attached, are two curves which represent a summary of data compiled from a preliminary computer investigation of a closed grid network. The grid selected for initial analysis is the closed boundary grid at distance five. We combined computer runs with the closed form theoretical analyses of sections 4 and 5 of this report to obtain some observations of network behaviour.

The first six curves represent a study of messages arriving and being received at the origin (fixed ground station) as a function of time. We used 20 computer runs for each of the first fifty time units. In this initial study the number of slots was kept fixed at 100, but λ(the mean number of messages originating at a given repeater) was set at 10, 20 and 30. All calculations were carried out for mode 1 and mode 2.

The message flow and reception at the origin settle down at about t=4 and remained relatively constant. For λ=10 the number of arriving messages seemed to have a mean at about 155 and the number of received messages averaged to about 31. Since the system behaviour for λ=10, m=100 settled down so quickly it seems reasonable to combine all time point data past t=10 to estimate the probability density function of arrivals and receptions at the origin in each of modes 1 and 2 when λ=10. The curves would seem to indicate asymptotic Poisson behaviour with means about 31, 155 in mode 1 and about 100, 300, in mode 2 respectively. Saturation occurs quickly in mode 2 for λ=10 or more. These results are summarized in the last four curves of probability density functions.

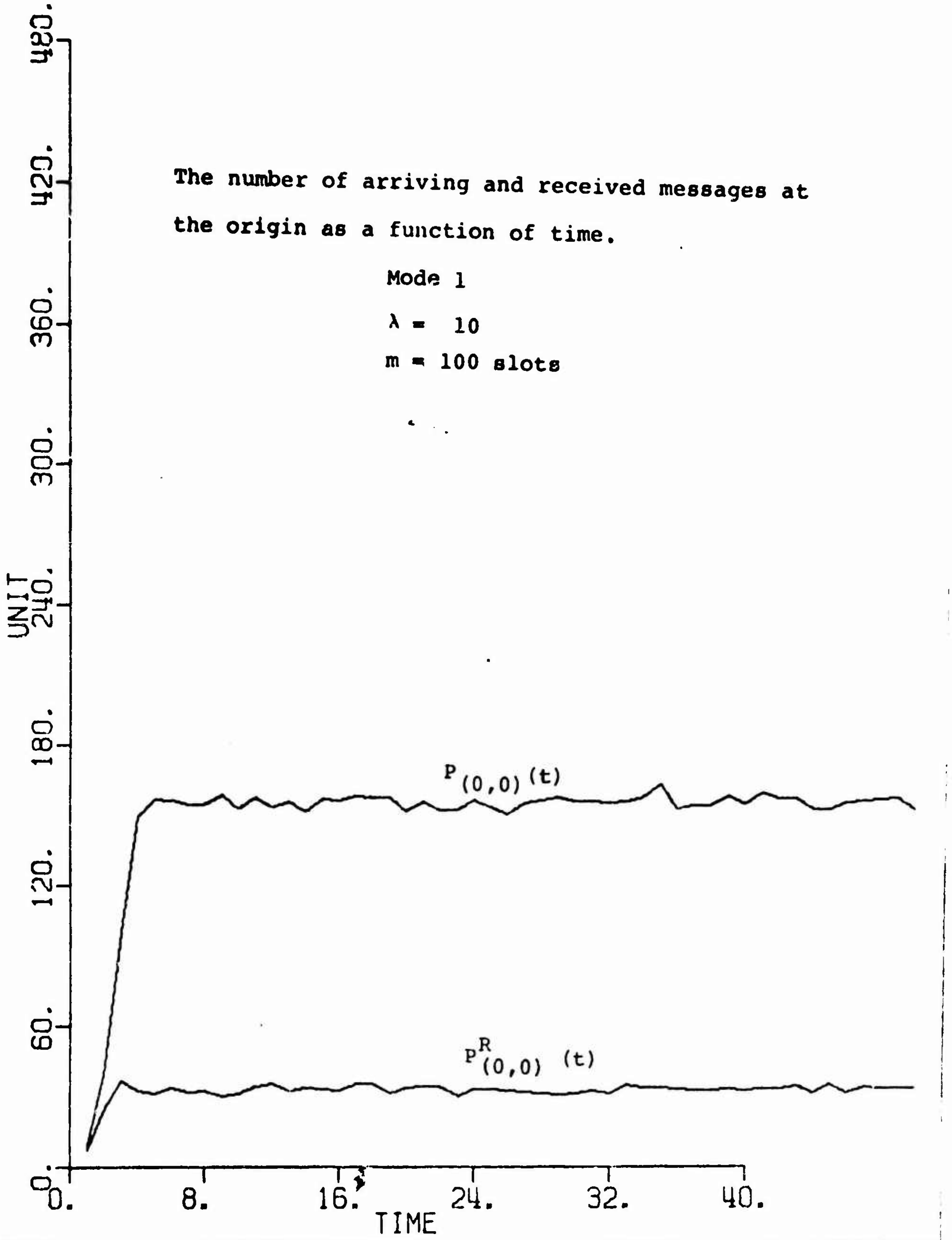

The number of arriving and received messages at the origin as a function of time.
Mode 1
λ = 10
m = 100 slots
$P_{(0,0)}(t)$
$P^R_{(0,0)}(t)$
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 3

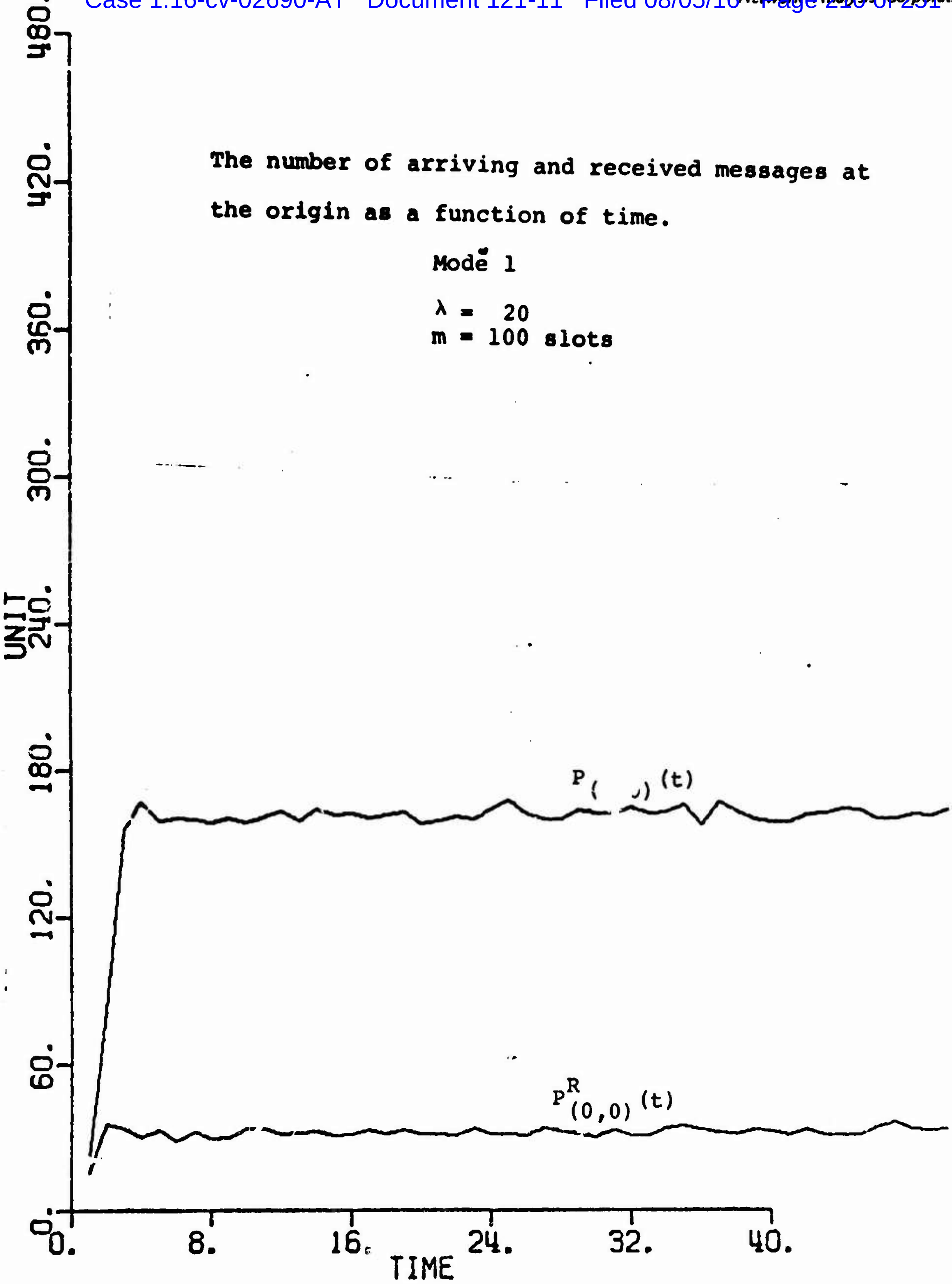

The number of arriving and received messages at the origin as a function of time.
Mode 1
λ = 20
m = 100 slots
P( ) (t)
P^R (0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 4

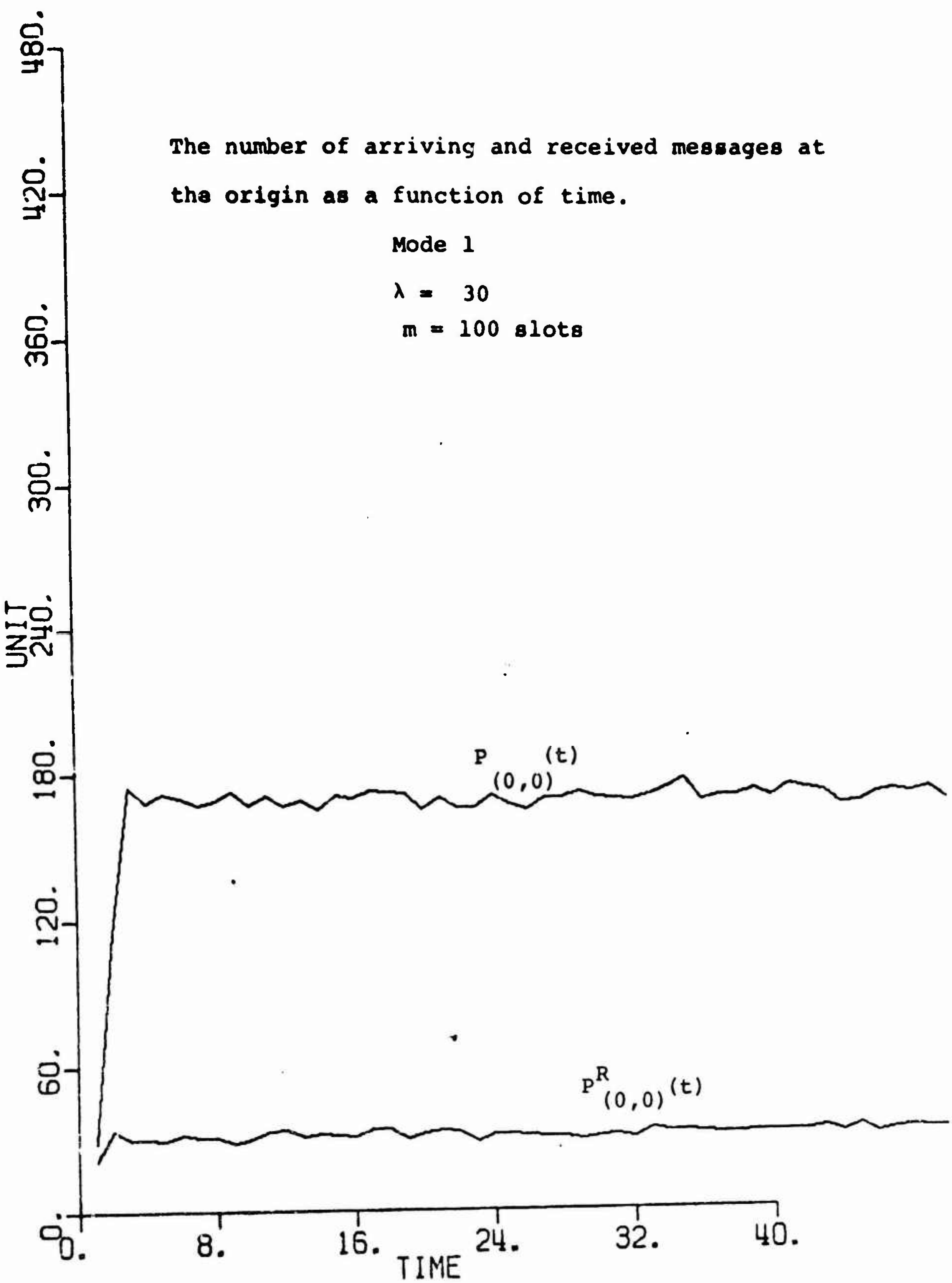

The number of arriving and received messages at
the origin as a function of time.
Mode 1
λ = 30
m = 100 slots
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
$P_{(0,0)}(t)$
$P^R_{(0,0)}(t)$
0.
8.
16.
24.
32.
40.
TIME

FIGURE 5

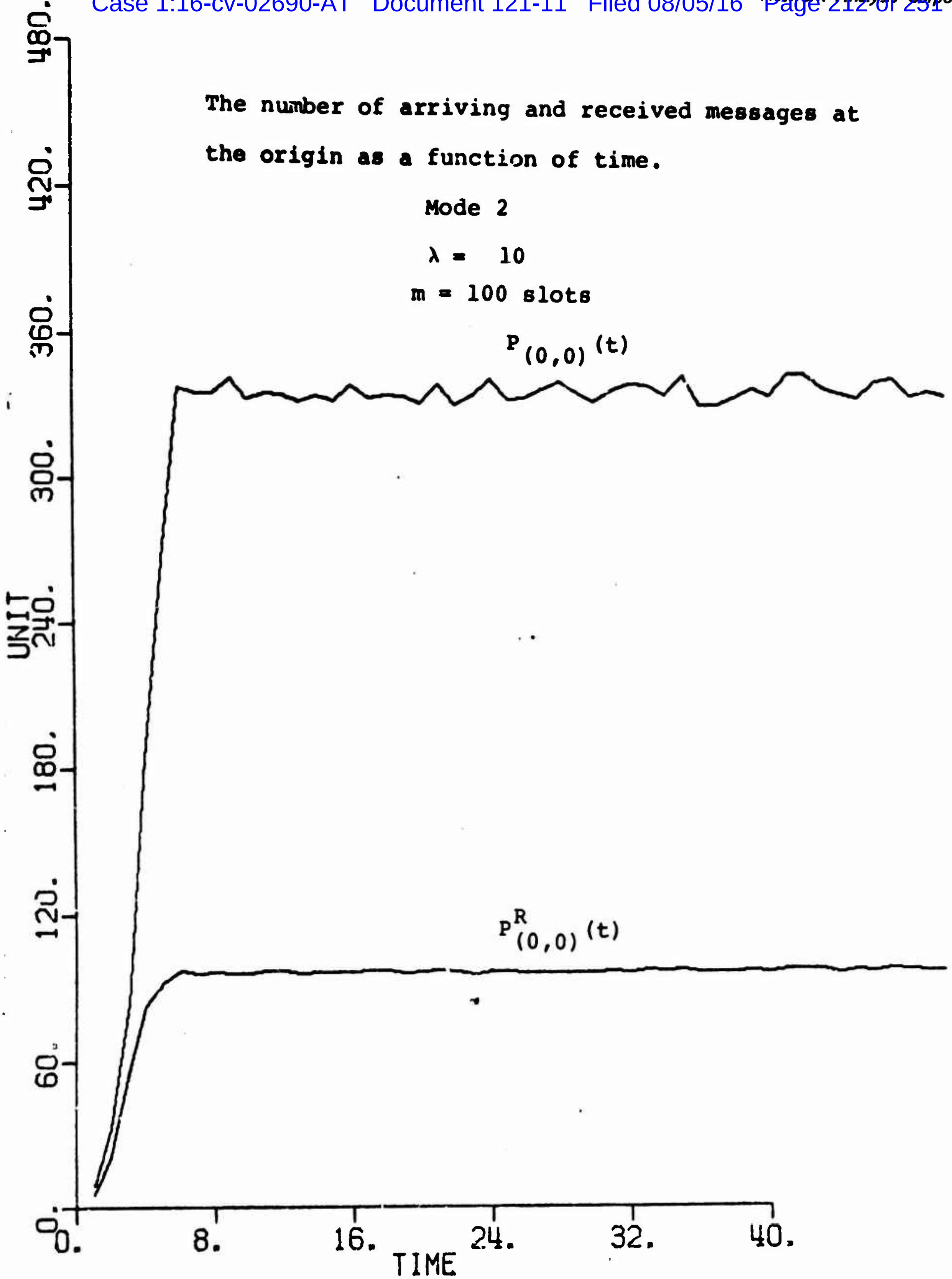

The number of arriving and received messages at the origin as a function of time.
Mode 2
λ = 10
m = 100 slots
P (0,0) (t)
P R (0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 6

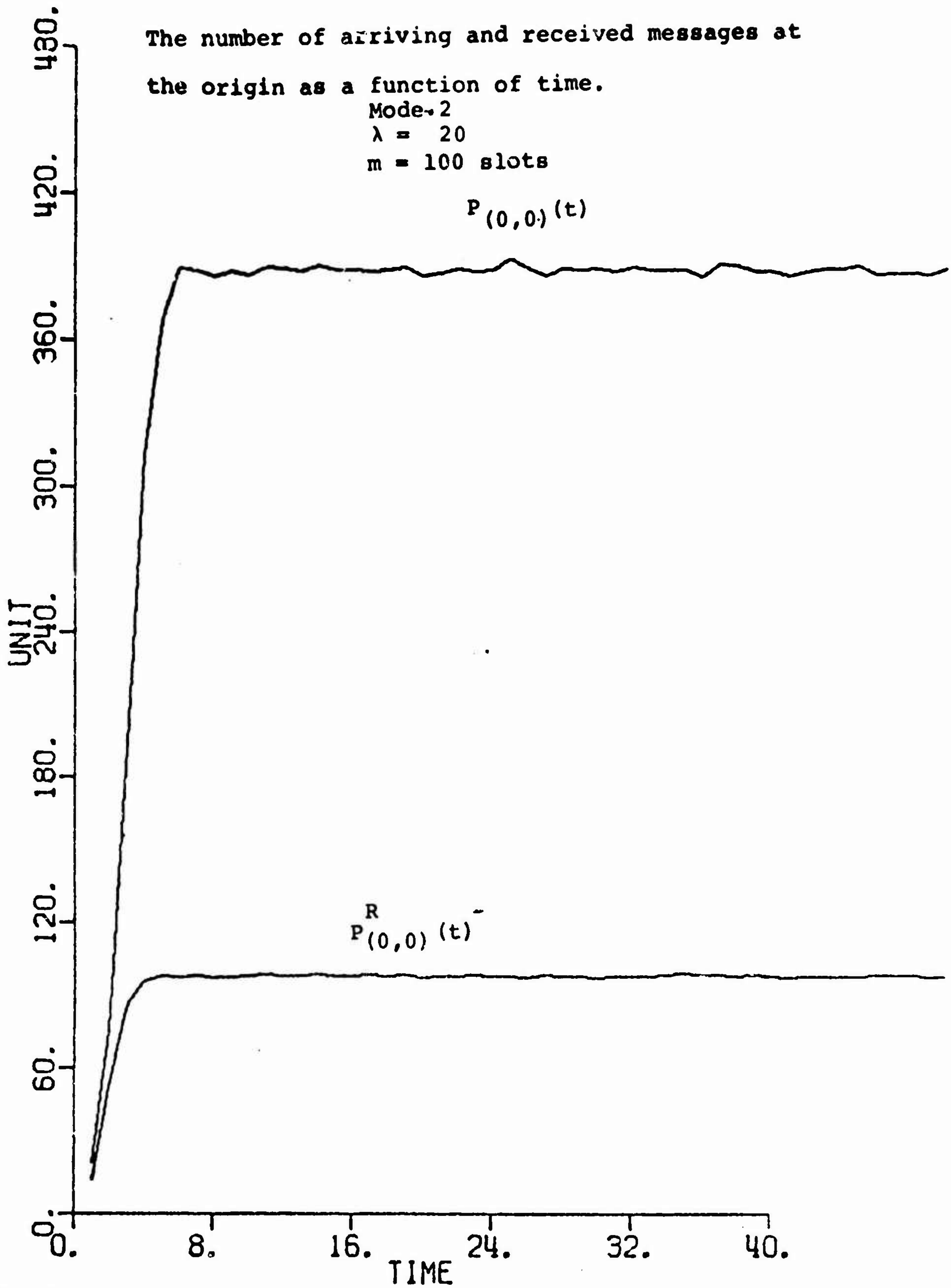

The number of arriving and received messages at
the origin as a function of time.
Mode- 2
λ = 20
m = 100 slots
P (0,0) (t)
R
P (0,0) (t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
430.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 7

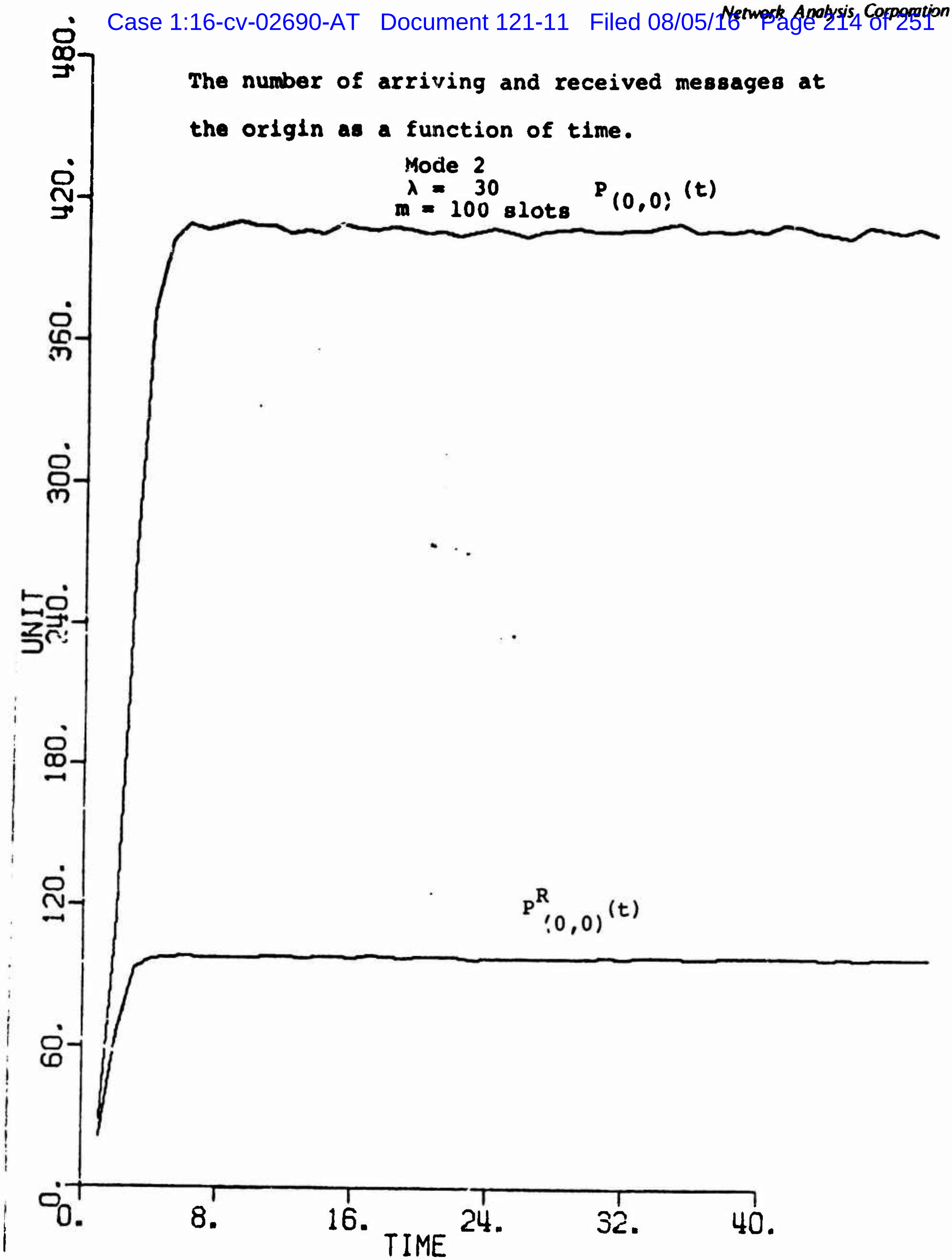

The number of arriving and received messages at
the origin as a function of time.
Mode 2
λ = 30
m = 100 slots
P(0,0)(t)
PR(0,0)(t)
UNIT
0.
60.
120.
180.
240.
300.
360.
420.
480.
0.
8.
16.
24.
32.
40.
TIME

FIGURE 8

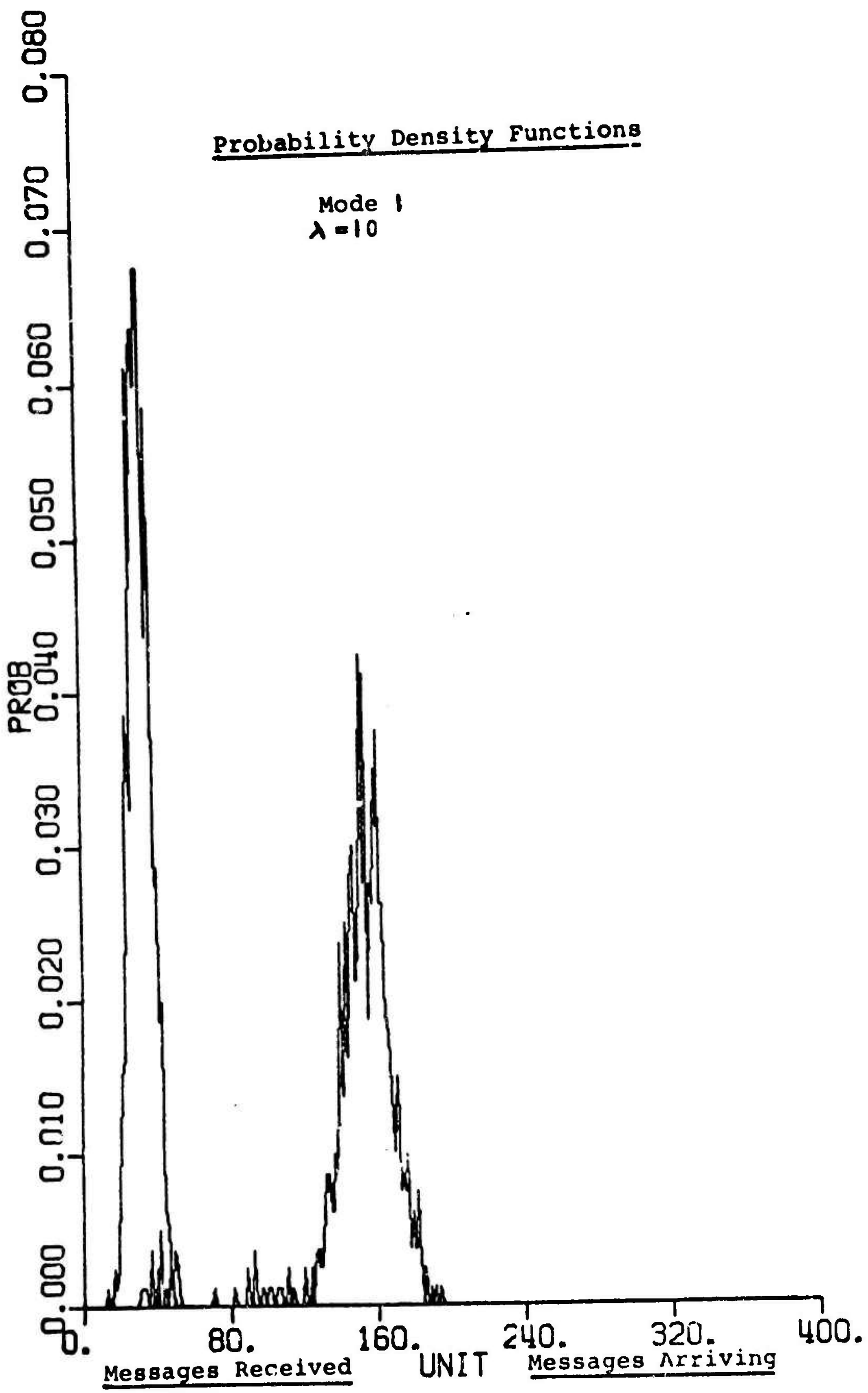

Probability Density Functions
Mode 1
λ =10
PROB
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
0.
80.
160.
240.
320.
400.
Messages Received
UNIT
Messages Arriving

FIGURE 9

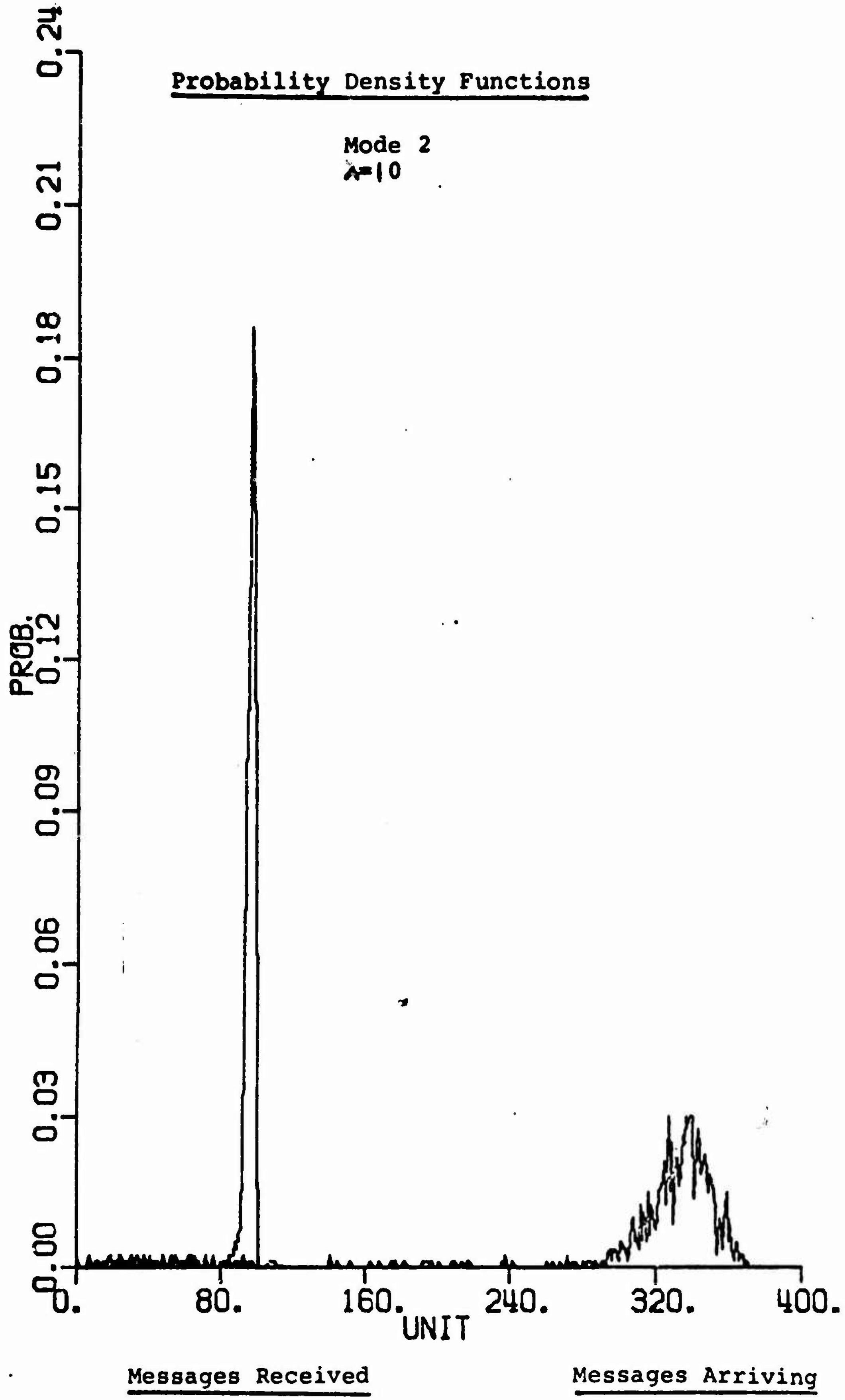

Probability Density Functions
Mode 2
λ=10
PROB.
0.24
0.21
0.18
0.15
0.12
0.09
0.06
0.03
0.00
0.
80.
160.
240.
320.
400.
UNIT
Messages Received
Messages Arriving

FIGURE 10

9. DYNAMICS OF A SINGLE MESSAGE ON ROUTE

In this section, we will develop the theoretical basis for a computer analysis of the dynamics of a single message originating at a repeater in the net and attempting to reach the ground station at the origin. The equations derived are directed towards a computer analysis. Let us assume that the given message originates at a repeater with coordinates (i,j) at time t. If the incoming and acceptance numbers at (k,j) at time t are respectively $X_{(i,j)}(t)$ and $X^A_{(i,j)}(t)$, we assume the given message is one of the $X_{(i,j)}(t)$ messages. Furthermore, we assume that each of the $X_{(i,j)}(t)$ messages is equally likely to be one of the accepted messages. Under these assumptions, it follows that at (i,j), there are two types of messages which have arrived. The first type is one message (the given one), the second type are $X_{(i,j)}(t)-1$ messages. The probability of acceptance at (i,j) is given by the hypergeometric probability density function:

$$\frac{\begin{pmatrix} X_{(ij)}(t)-1 \\ X^A_{(ij)}(t)-1 \end{pmatrix}}{\begin{pmatrix} X_{(ij)}(t) \\ X^A_{(ij)}(t) \end{pmatrix}} \qquad (22)$$

At each repeater on every path to the ground station the same analysis applies. At any given repeater, on the path, say with coordinates (k,e) there may be several copies of the original message which arrives.

Suppose (k,e) is on a path from (i,j) to (0,0) and the number of paths from (i,j) to (k,e) is w. Then at (k,e), at time t plus the distance from (i,j) to (k,e), either 0, 1, 2,..., up to w copies of the message may arrive. If d is the distance from (i,j) to (k,e) and at time (t + d), $X_{(k,e)}(t+d)$ and $X^{A}_{(k,e)}(t+d)$ messages respectively arrive and are accepted then we can compute the probability that exactly Z copies of the original messages are accepted. The computation of the required probabilities is a direct extension of

$$P\{\text{exactly } Z \text{ copies of original message is accepted at } (k,e) \text{ at time } t + d/\nu \text{ copies are amongst the arrivals}\}$$

$$= \frac{\binom{\nu}{Z}\binom{X_{(k,e)}(t+d)-\nu}{X^{A}_{(k,e)}(t+d)-Z}}{\binom{X_{(k,e)}(t+d)}{X^{A}_{(k,e)}(t+d)}} \quad ; \quad Z = 0, 1, 2, \ldots, \nu.$$

Equation(25) is valid at every repeater along every path from (i,j) to (0,0), and in particular at the origin. The only ingredient needed to apply the equations to a computer analysis and generate numerical values is a formula for the probability that exactly ν copies of the message arrive at each repeater. This formula can be obtained recursively using the idea of isodesic line and wedge joint density functions as developed in Section 7.

If a single copy of the given message is accepted at its origination repeater, it is:

a) repeated to each of two repeaters one unit closer to the origin if it is not on an axis;

b) repeated to the one repeater one unit closer to the origin if it is on an axis.

We will focus only on (a) since (b) is essentially identical as far as the analysis is concerned. The message, when accepted at its origination, is then repeated to repeaters at (i-1,j-1) and (i-1,j). Acceptances at (i-1,j-1) and (i-1,j) are determined according to Equation (23) The isodesic line joint density of receptions and acceptances are computed at (i-1,j-1) and (i-1,j). This joint density then determines arrivals and acceptances at (i-2,j-2), (i-2,j-1) and (i-2,j). The process then continues recursively until all computations are carried out at the origin.

9.1 Outline of Computer Analysis

In our computer analysis we used the above results to compute the probability distributions and mean value of the number of copies accepted at the origin of a single message which originates at distance of 5, 4, 3, 2, 1, 0 units from the ground station. For convenience and realism of the numerical results, we selected each originating repeater to have the maximum number of paths to the origin. The coordinate system we used for these calculations is given in Figure11, below:

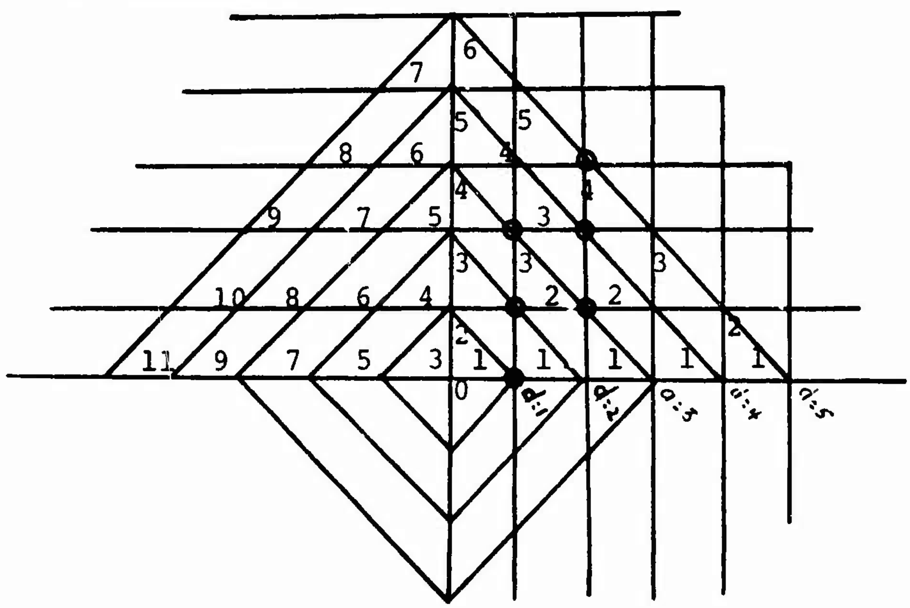


FIGURE 11

The repeaters selected for originating messages at distances 5,4,3,2,1,0 are respectively at (5,4), (4,3), (3,3), (2,2), (1,1), (0,0). The routes are designated in Figure 12, and the maximum number of copies of an originating message which can be received along each repeater on the route is given in Table 1 below. Note that the maximum number of possible copies is given by the number of paths from an originating repeater to the receiving repeater. Note in Table 1 that no copies can be received at a repeater further from the origin than the originator.

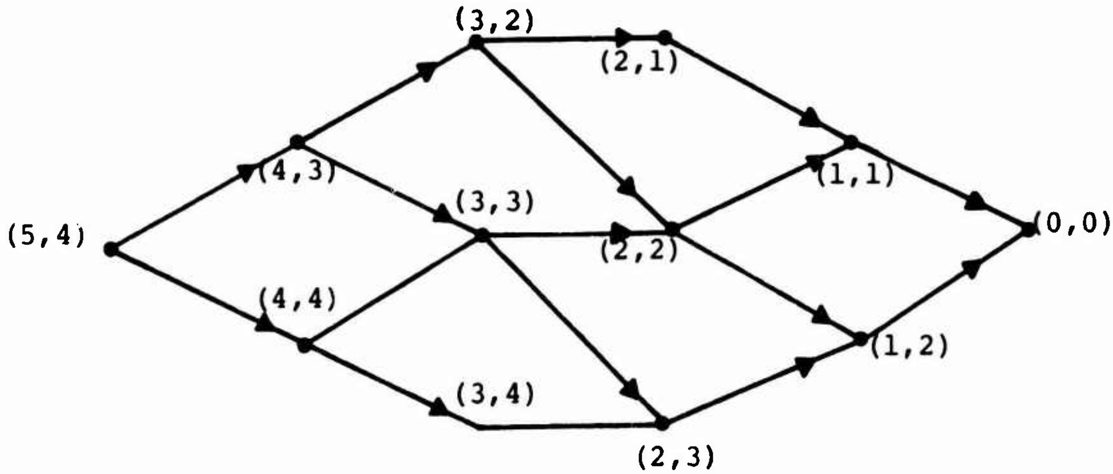


Fig. 12 Routing From (5,4) to (0,0)

	(0,0)	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(3,2)	(3,3)	(3,4)	(4,3)	(4,4)	(5,4)
(0,0)	1	1	1	1	2	1	3	3	1	6	4	10
(1,1)		1	0	1	1	0	2	1	0	3	1	4
(1,2)			1	0	1	1	1	2	1	3	3	6
(2,1)				1	0	0	1	0	0	1	0	1
(2,2)					1	0	1	1	0	2	1	3
(2,3)						1	0	1	1	1	2	3
(3,2)							1	0	0	1	0	1
(3,3)								1	0	1	1	2
(3,4)									1	0	1	1
(4,3)										1	0	1
(4,4)											1	1
(5,4)												1

Table 1 Maximum Number of Copies - Between Two Repeaters

A. The Equation for $Z_0(j;t)$

Clearly $Z_0(j;t)$ is simply given by

$$Z_0(j;t) = \frac{A_{(0,0)}(1,j;t)}{A_{(0,0)}(0,0;t)} \; ; \; j = 0, 1; \quad t = 0, 1, 2, \ldots, 40. \tag{24}$$

B. The Equation for $Z_1(j;t)$

$$Z_1(j;t) = \sum_{\mu=0}^{1} \binom{\mu}{j} \cdot \frac{A_{(0,0)}(\mu,j;t)}{A_{(0,0)}(0,0;t)} \cdot f_1^1(\mu;t-1) \tag{25}$$

for $j = 0, 1; \quad t = 1, 2, \ldots, 40$; where

$$f_1^1(j;t) = \frac{A_{(1,1)}(1,j;t)}{A_{(1,1)}(0,0;t)} \; ; \; j = 0, 1; \quad t = 0, 1, 2, \ldots, 39.$$

C. The Equation for $Z_2(j;t)$

$$Z_2(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^2(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{26}$$

for $j = 0, 1, 2; \quad t = 2, 3, \ldots, 40$; where

$$f_2^2(i,j;t) = \sum_{\mu=0}^{1} f_1^2(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for $t = 1, 2, \ldots, 39; \quad i = 0, 1; \quad j = 0, 1$; where

$$f_1^2(j;t) = \frac{A_{(2,2)}(1,j;t)}{A_{(2,2)}(0,0;t)} \; ; \; t = 0, 1, 2, \ldots, 38; \quad j = 0, 1.$$

D. The Equation for $Z_3(j;t)$

$$Z_3(j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{2} f_3^3(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} \; ; \tag{27}$$

for $t = 3, \ldots, 40; \quad j = 0, 1, 2, 3$; where

$$f_3^3(i,j;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^3(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \cdot \frac{A_{(1,1)}(\mu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\nu,j;t)}{A_{(1,2)}(0,0;t)} \; ;$$

for t = 2, ..., 39; i = 0, 1; j = 0, 1, 2; where

$$f_2^3(i,j;t) = \sum_{\mu=0}^{1} f_1^3(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(2,2)}(\mu,i;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu,j;t)}{A_{(2,3)}(0,0;t)} ;$$

for t = 1, 2, ..., 38; i = 0, 1; j = 0, 1; where

$$f_1^3(j;t) = \frac{A_{(3,3)}(i,j;t)}{A_{(3,3)}(0,0;t)} ; \; t = 0, 1, 2, ..., 37; \; j = 0, 1.$$

E. The Equation for $Z_4(j;t)$

$$Z_4(j;t) = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} f_4^4(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for t = 4, 5, ..., 40; j = 0, 1, 2, ..., 16; where

$$f_4^4(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^4(\nu,\mu,\rho;t-1) \binom{\nu+\mu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for t = 3, 4, ..., 39; i = 0, 1, 2, 3; j = 0, 1, 2, 3; where

$$f_3^4(i,j,k;t) = \sum_{\mu=0}^{1} \sum_{\nu=0}^{1} f_2^4(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(2,1)}(\mu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\mu+\nu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\nu,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for t = 2, 3, ..., 38; i = 0, 1; j = 0, 1, 2; k = 0, 1; where

$$f_2^4(i,j;t) = \sum_{\mu=0}^{1} f_1^4(\mu;t-1) \binom{\mu}{i} \binom{\mu}{j} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu,j;t)}{A_{(3,3)}(0,0;t)} ;$$

for t = 1, 2, ..., 37; i = 0, 1; j = 0, 1; where

$$f_1^4(j;t) = \frac{A_{(4,3)}(1,j;t)}{A_{(4,3)}(0,0;t)} ; \; t = 0, 1, ..., 36; \; j = 0, 1. \quad (28)$$

The numbers in Table 1 give the upper limits of the summation for the possible copies of messages which can be received at each repeater of a single message originating at a repeater further from the origin but within the net of Figure 12. With the selected net and the numbers of Table 1, we can use the results of section 12 to obtain numerical data.

At time zero a random number of messages has arrived at each repeater. To compute the distribution of copies arriving at (0,0) from (5,4) we assume one of the messages arriving at (5,4) is singled out and followed along the route using the hypergeometric analysis of section 12. The procedure was used for $t = 0, 1, 2, \ldots, 40$ in conjuction with the random Poisson number generator developed and discussed earlier.

Specifically we seek to compute the five numbers:

$Z_0(j;t)$; $j = 0, 1$; $t = 0, 1, 2, \ldots, 40$;

$Z_1(j;t)$; $j = 0, 1$; $t = 1, 2, \ldots, 40$;

$Z_2(j;t)$; $j = 0, 1, 2$; $t = 2, 3, \ldots, 40$;

$Z_3(j;t)$; $j = 0, 1, 2, 3$; $t = 3, 4, \ldots, 40$;

$Z_4(j;t)$; $j = 0, 1, 2, 3, 4, 5, 6$; $t = 4, 5, 6, \ldots, 40$;

$Z_5(j;t)$; $j = 0, 1, 2, 3, \ldots, 10$; $t = 5, 6, \ldots, 40$;

where $Z_k(j;t)$ is the probability that exactly j copies of a message originating at a repeater at distance k at time t-k, are accepted at the origin at time t. For the computer analysis we considered one repeater at each of the distances, as in Figure 11. The maximum j values are given by the first row of Table 1. Using the hypergeometric analyses the following equation can be used to compute each of the $Z_k(j;t)$; as a function of:

1) λ = mean number of originations at each repeater.
2) m = number of slots fixed at 100.
3) Each of two capture modes 1 and 2.

For ease of notation we denote:

$$A_{(ij)}(w,X;t) = \binom{X_{(ij)}(t)-w}{X^A_{(ij)}(t)-X}.$$

F. The Equation for $Z_5(j;t)$

(6.6) $$Z_5(j;t) = \sum_{\mu=0}^{4} \sum_{\nu=0}^{6} f_5^5(\mu,\nu;t-1) \binom{\mu+\nu}{j} \cdot \frac{A_{(0,0)}(\mu+\nu,j;t)}{A_{(0,0)}(0,0;t)} ;$$

for $t = 5, 6, \ldots, 40;\ j = 0, 1, 2, \ldots, 10;$ where

$$f_5^5(i,j;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{3} \sum_{\rho=0}^{3} f_4^5(\nu,\mu,\rho;t-1) \binom{\mu+\nu}{i} \binom{\mu+\rho}{j} \cdot \frac{A_{(1,1)}(\mu+\nu,i;t)}{A_{(1,1)}(0,0;t)} \cdot \frac{A_{(1,2)}(\mu+\rho,j;t)}{A_{(1,2)}(0,0;t)} ;$$

for $t = 4, 5, 6, \ldots, 39;\ i = 0, 1, 2, 3, 4;\ j = 0, 1, 2, \ldots, 6;$

where

$$f_4^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{2} \sum_{\rho=0}^{1} f_3^5(\nu,\mu,\rho;t-1) \binom{\nu}{i} \binom{\nu+\mu}{j} \binom{\mu+\rho}{k} \cdot \frac{A_{(2,1)}(\nu,i;t)}{A_{(2,1)}(0,0;t)} \cdot \frac{A_{(2,2)}(\nu+\mu,j;t)}{A_{(2,2)}(0,0;t)} \cdot \frac{A_{(2,3)}(\mu+\rho,k;t)}{A_{(2,3)}(0,0;t)} ;$$

for $t = 3, \ldots, 38;\ i = 0, 1;\ j = 0, 1, 2, 3;\ k = 0, 1, 2, 3;$

where

$$f_3^5(i,j,k;t) = \sum_{\nu=0}^{1} \sum_{\mu=0}^{1} f_2^5(\mu,\nu;t-1) \binom{\mu}{i} \binom{\mu+\nu}{j} \binom{\nu}{k} \cdot \frac{A_{(3,2)}(\mu,i;t)}{A_{(3,2)}(0,0;t)} \cdot \frac{A_{(3,3)}(\mu+\nu,j;t)}{A_{(3,3)}(0,0;t)} \cdot \frac{A_{(3,4)}(\nu,k;t)}{A_{(3,4)}(0,0;t)} ;$$

for $t = 2, 3, \ldots, 37;\ i = 0, 1;\ j = 0, 1, 2;\ k = 0, 1;$ where

$$f_2^5(i,j;t) = \sum_{\mu=0}^{1} f_1^5(\mu,t-1) \binom{\mu}{i} \binom{\nu}{j} \cdot \frac{A_{(4,3)}(\mu,i;t)}{A_{(4,3)}(0,0;t)} \cdot \frac{A_{(4,4)}(\mu,j;t)}{A_{(4,4)}(0,0;t)} ;$$

for t = 1, 2, 3, ..., 36; i = 0, 1; j = 0, 1; where

$$f_1^5(j;t) = \frac{A_{(5,4)}(1,j;t)}{A_{(5,4)}(0,0;t)}; \quad t = 0, 1, 2, \ldots, 35; \quad j = 0, 1.$$

9.2 Probability of at Least One Message Getting Through

The first set of curves, Figures13-18, plot the probability of at least one message getting through as a function of the mean number of originations at each repeater. There is one set of curves for each unit of distance d ranging from 0 to 5. Each figure contains one curve for mode 1 and one curve for mode 2. The number of slots was fixed at 100. The data for the curves is summarized in Table 2 below.

	λ=1		λ=3		λ=5	
distance	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.398	.589	.285	.421	.264	.431
1	.243	.434	.192	.273	.117	.321
2	.355	.614	.166	.341	.129	.326
3	.428	.695	.164	.358	.119	.285
4	.613	.874	.250	.534	.159	.271
5	.740	.967	.341	.692	.157	.198

Table 2: Probability That at Least One Message Gets Through

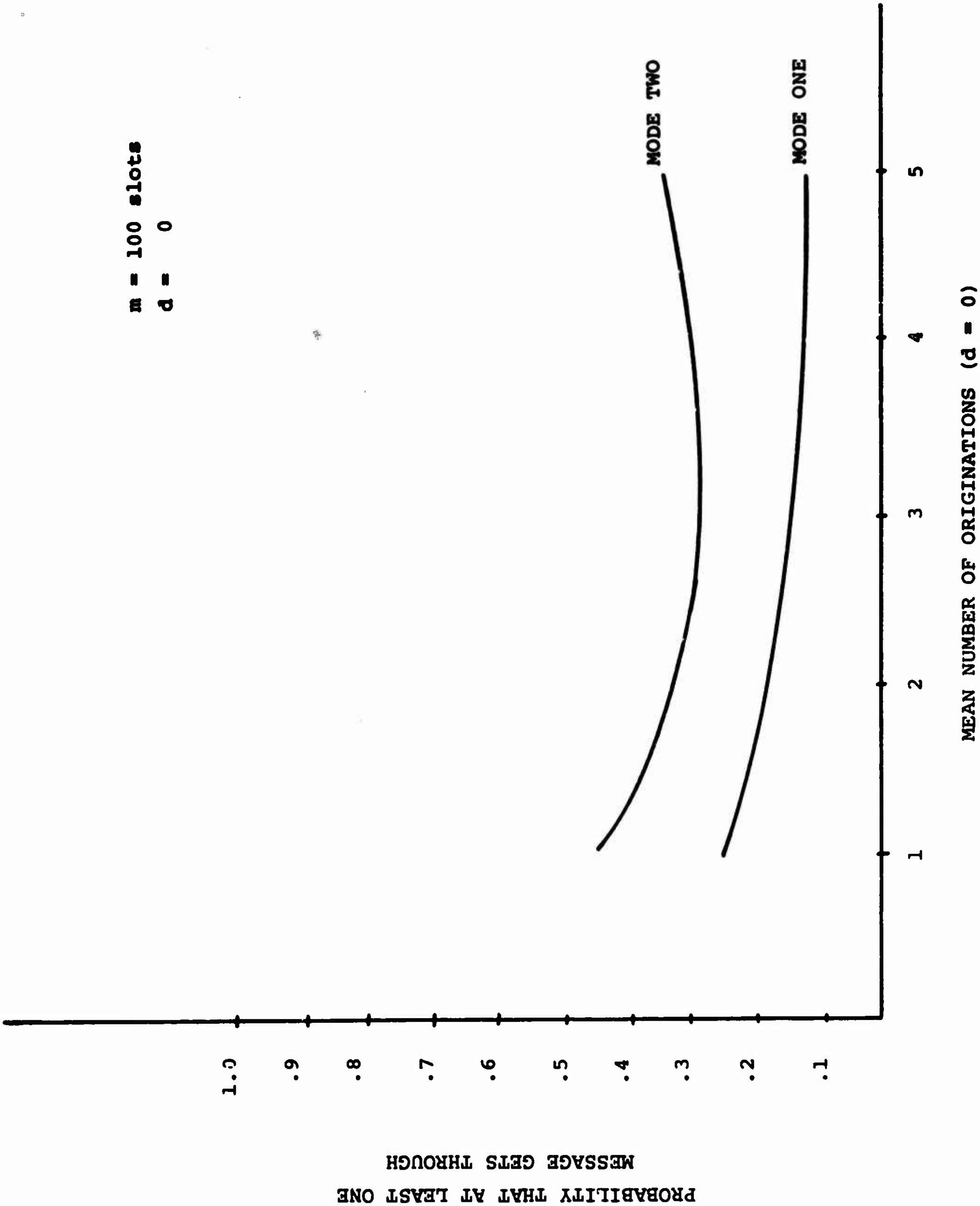

m = 100 slots
d = 0
MODE TWO
MODE ONE
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 0)
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 13

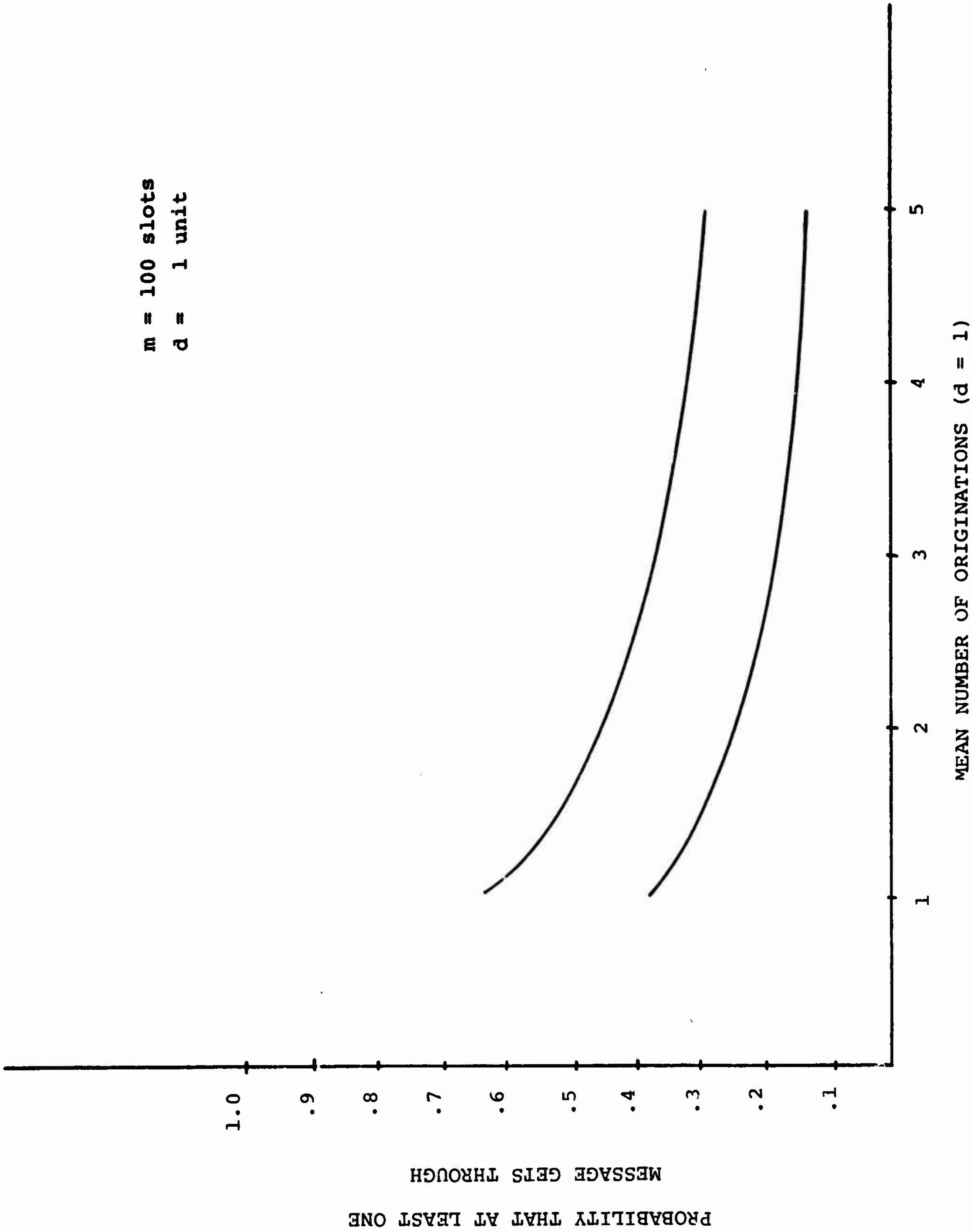

m = 100 slots
d = 1 unit
MEAN NUMBER OF ORIGINATIONS (d = 1)
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1

FIGURE 14

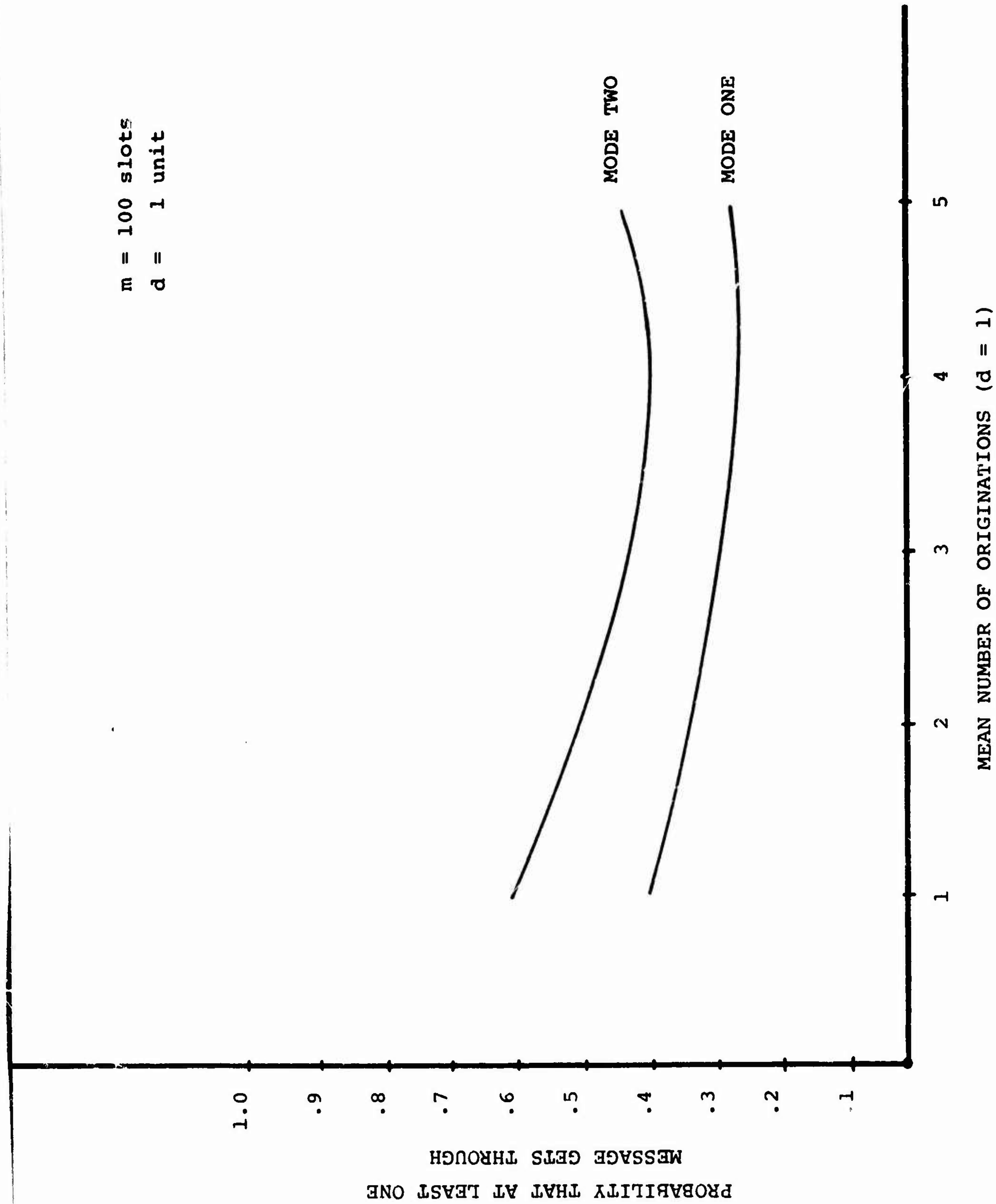

m = 100 slots
d = 1 unit
MODE TWO
MODE ONE
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
MEAN NUMBER OF ORIGINATIONS (d = 1)

FIGURE 15

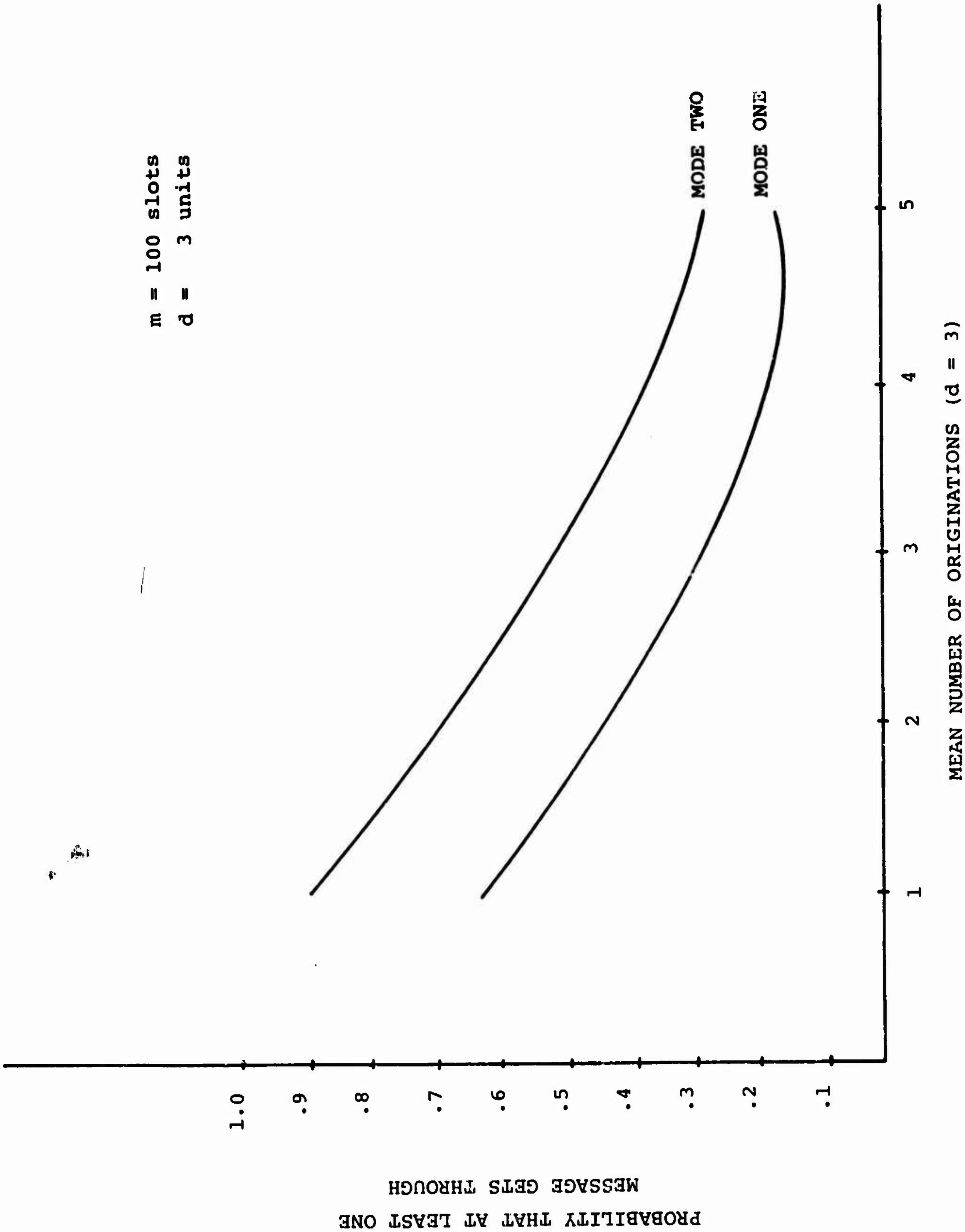

m = 100 slots
d = 3 units
MODE TWO
MODE ONE
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 3)

FIGURE 16

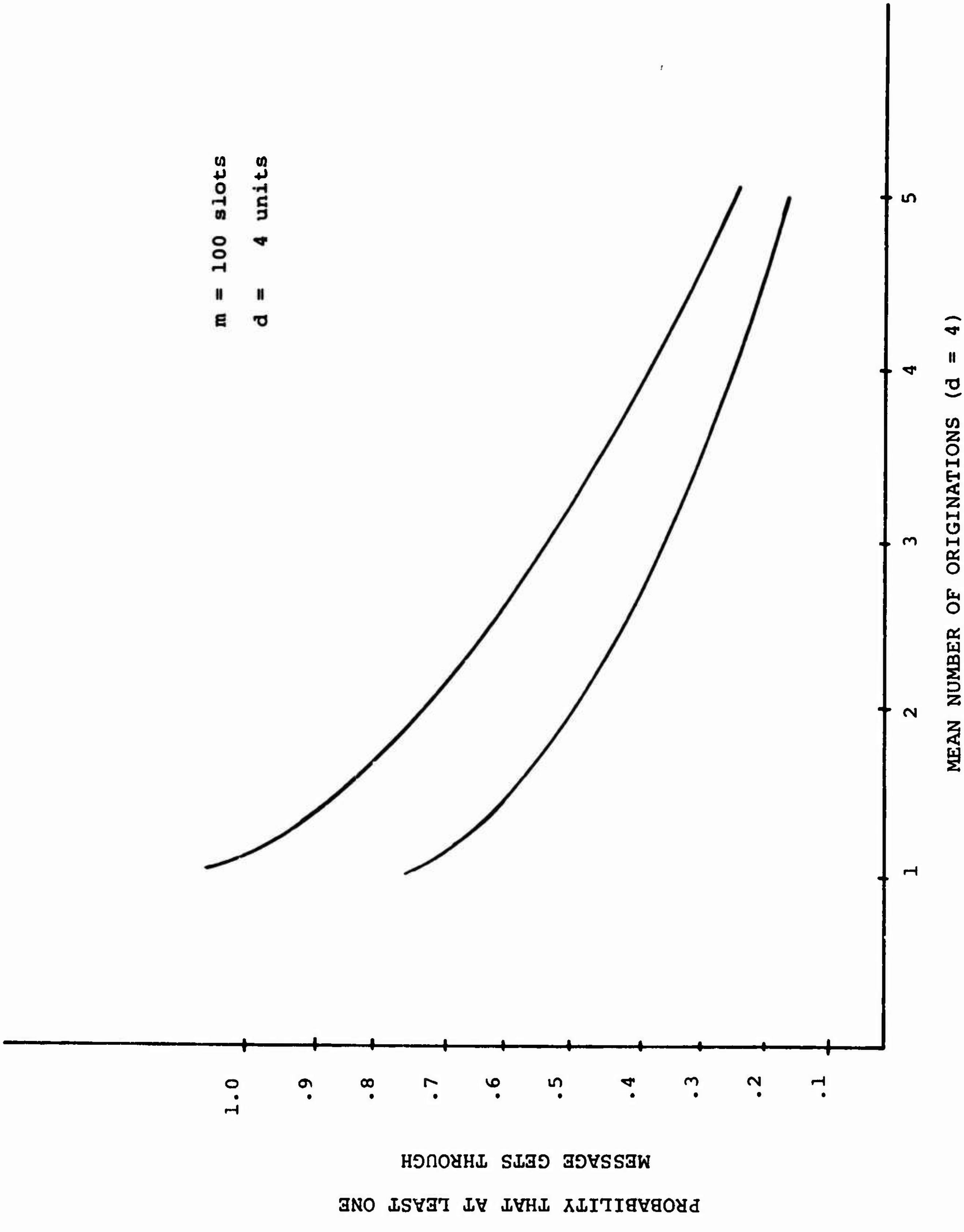

m = 100 slots
d = 4 units
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 4)
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 17

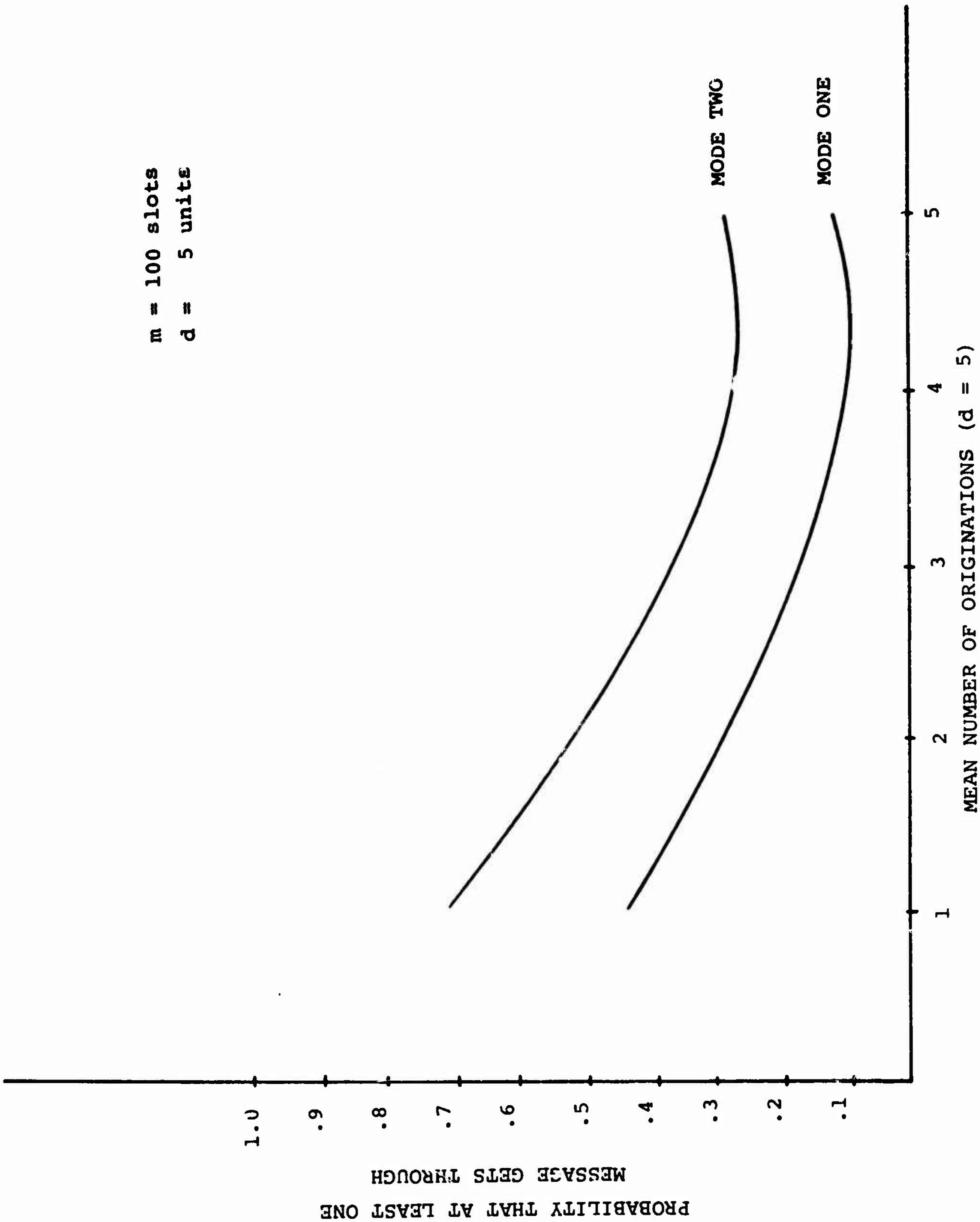

m = 100 slots
d = 5 units
MODE TWO
MODE ONE
1
2
3
4
5
MEAN NUMBER OF ORIGINATIONS (d = 5)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH

FIGURE 18

9.3 Distribution of Message Explosion as a Function of Slot Size and Mean Number of Originations

The equations for message explosion derived earlier in the report were used to obtain numerical data for message explosion. The results of the numerical analysis follow in Tables 3 through 26 and Figures 19,20, and 21.

9.4 Distributions of Copies Getting Through as a Function of Slot Size and Mean Originations

Tables 3-26 contain the probability distributions for the number of copies of a single message which are received at the origin (ground stations) for each distance (d=0,2,3,4,5,) of origination of the message. The tables vary according to mode (each of two modes), mean number of messages originating at each repeater (λ=1,3,5), and each of four slot sizes (m=25, 50, 75, 100). This produces a total of 4 x 3 x 2 = 24 tables.

In table 27, we summarize the results of the twenty-four tables by considering only the probability that at least one copy of the message gets through as a function of distance and the three parameters; mode, mean, and slot size.

The results of table 27 are presented pictorially in figures 19, 20, and 21 for distances of zero, two and four respectively of origination of the message.

Mode 1, λ= 1, m = 25

distance \ #	0	1	2	3
0	.786	.214		
1	.921	.079		
2	.900	.096	.004	
3	.897	.097	.005	
4	.831	.153	.015	.001
5				

Table 3

Mode 2, λ= 1, m = 25

distance \ #	0	1	2	3	4
0	.689	.311			
1	.834	.166			
2	.759	.219	.021		
3	.716	.245	.037	.002	
4	.554	.324	.103	.018	.002
5					

Table 4

Mode 1, $\lambda = 3$, $m = 25$

distance \ #	0	1	2
0	.805	.195	
1	.942	.058	
2	.952	.047	.001
3	.966	.033	.001
4	.953	.045	.002
5			

Table 5

Mode 2, $\lambda = 3$, $m = 25$

distance \ #	0	1	2	3
0	.739	.261		
1	.890	.102		
2	.887	.107	.006	
3	.892	.101	.007	
4	.826	.153	.019	.001
5				

Table 6

Mode 1, λ = 5, m = 25

distance \ #	0	1	2
0	.813	.187	
1	.951	.049	
2	.963	.036	.001
3	.979	.021	.001
4	.977	.022	.001
5			

Table 7

Mode 2, λ = 5, m = 25

distance \ #	0	1	2
0	.753	.247	
1	.914	.086	
2	.916	.080	.004
3	.930	.066	.004
4	.900	.091	.008
5			

Table 8

Mode 1, $\lambda = 1$, $m = 50$

distance \ #	0	1	2	3
0	.755	.245		
1	.878	.122		
2	.823	.166	.001	
3	.796	.185	.019	.001
4	.661	.272	.059	.007
5				

Table 9

Mode 2, $\lambda = 1$, $m = 50$

distance \ #	0	1	2	3	4	5
0	.605	.395				
1	.736	.264				
2	.602	.338	.060			
3	.531	.356	.103	.010		
4	.306	.360	.232	.083	.017	.002
5						

Table 10

Mode 1, $\lambda = 3$, $m = 50$

distance \ #	0	1	2
0	.788	.212	
1	.926	.074	
2	.921	.076	.003
3	.926	.070	.006
4	.882	.108	.009
5			

Table 11

Mode 2, $\lambda = 3$, $m = 50$

distance \ #	0	1	2	3	4
0	.709	.291			
1	.860	.140			
2	.816	.170	.014		
3	.800	.178	.021	.001	
4	.875	.258	.058	.008	.001
5					

Table 12

Mode 1, λ = 5, m = 50

distance \ #	0	1	2
0	.794	.206	
1	.937	.063	
2	.441	.057	.002
3	.954	.044	.002
4	.935	.062	.004
5			

Table 13

Mode 2, λ = 5, m = 50

distance \ #	0	1	2	3
0	.733	.267		
1	.890	.110		
2	.868	.124	.008	
3	.870	.120	.010	
4	.791	.180	.027	.002
5				

Table 14

Mode 1, λ = 1, m = 75

distance \ #	0	1	2	3	4
0	.711	.289			
1	.831	.169			
2	.742	.235	.023		
3	.868	.269	.043	.002	
4	.512	.342	.121	.023	.003
5					

Table 15

Mode 2, λ = 1, m = 75

distance \ #	0	1	2	3	4	5	6
0	.526	.474					
1	.655	.345					
2	.486	.408	.106				
3	.392	.408	.175	.025			
4	.184	.304	.295	.158	.049	.008	.001
5							

Table 16

Mode 1, λ = 3, m = 75

distance \ #	0	1	2	3
0	.782	.218		
1	.913	.087		
2	.895	.100	.005	
3	.891	.103	.006	
4	.818	.161	.019	.001
5				

Table 17

Mode 2, λ = 3, m = 75

distance \ #	0	1	2	3	4
0	.674	.326			
1	.815	.185			
2	.750	.225	.025		
3	.725	.233	.040	.002	
4	.561	.313	.103	.020	.002
5					

Table 18

Mode 1, $\lambda = 5$, $m = 75$

distance \ #	0	1	2
0	.788	.212	
1	.929	.071	
2	.924	.074	.003
3	.932	.065	.003
4	.894	.098	.008
5			

Table 19

Mode 2, $\lambda = 5$, $m = 75$

distance \ #	0	1	2	3
0	.711	.289		
1	.863	.137		
2	.819	.168	.013	
3	.818	.163	.019	.001
4	.703	.239	.051	.006
5				

Table 20

Mode 1, $\lambda = 1$, $m = 100$

distance \ #	0	1	2	3	4	5	6
0	.602	.398					
1	.757	.242					
2	.645	.302	.053				
3	.572	.325	.093	.013			
4	.387	.369	.184	.050	.008		
5	.260	.318	.250	.122	.039	.008	.003

Table 21

Mode 2, $\lambda = 1$, $m = 100$

distance \ #	0	1	2	3	4	5	6	7	8
0	.411	.589							
1	.566	.434							
2	.386	.433	.181						
3	.305	.399	.240	.056					
4	.126	.244	.308	.215	.087	.019	.002		
5	.003	.110	.200	.248	.211	.126	.054	.014	.001

Table 22

Mode 1, $\lambda = 3$, $m = 100$

distance \ #	0	1	2	3	4
0	.715	.285			
1	.808	.192			
2	.834	.143	.023		
3	.836	.147	.016		
4	.750	.211	.035	.004	
5	.659	.267	.063	.010	.001

Table 23

Mode 2, $\lambda = 3$, $m = 100$

distance \ #	0	1	2	3	4	5	6
0	.579	.421					
1	.727	.273					
2	.659	.279	.062				
3	.642	.276	.073	.008			
4	.466	.338	.149	.040	.007	.001	
5	.308	.333	.223	.098	.030	.007	.001

Table 24

Mode 1, $\lambda = 5$, $m = 100$

distance \ #	0	1	2	3
0	.736	.264		
1	.803	.117		
2	.871	.119	.010	
3	.881	.110	.009	
4	.841	.140	.018	.001
5	.843	.133	.022	.002

Table 25

Mode 2, $\lambda = 5$, $m = 100$

distance \ #	0	1	2	3	4	5
0	.569	.431				
1	.679	.321				
2	.674	.283	.043			
3	.715	.214	.064	.007		
4	.729	.169	.076	.021	.004	
5	.802	.115	.056	.020	.005	.001

Table 26

	λ = 1								λ = 3								λ = 5							
	M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100		M = 25		M = 50		M = 75		M = 100	
d	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0	.214	.311	.245	.395	.289	.474	.398	.587	.195	.261	.212	.291	.218	.326	.285	.421	.187	.247	.206	.267	.212	.289	.264	.431
1	.079	.166	.122	.264	.269	.345	.243	.434	.058	.110	.074	.140	.087	.185	.192	.273	.049	.086	.063	.110	.071	.137	.197	.321
2	.100	.241	.177	.398	.258	.514	.355	.614	.048	.113	.079	.184	.105	.250	.166	.341	.037	.084	.059	.132	.076	.181	.129	.326
3	.102	.284	.204	.469	.304	.608	.428	.695	.034	.108	.074	.200	.109	.275	.164	.358	.021	.070	.046	.130	.068	.182	.119	.285
4	.169	.446	.339	.694	.488	.816	.613	.874	.047	.174	.118	.325	.182	.437	.250	.534	.023	.100	.065	.207	.106	.297	.159	.271
5							.740	.997							.341	.692							.157	.198

TABLE 27

THE PROBABILITY OF AT LEAST ONE MESSAGE GETTING THROUGH

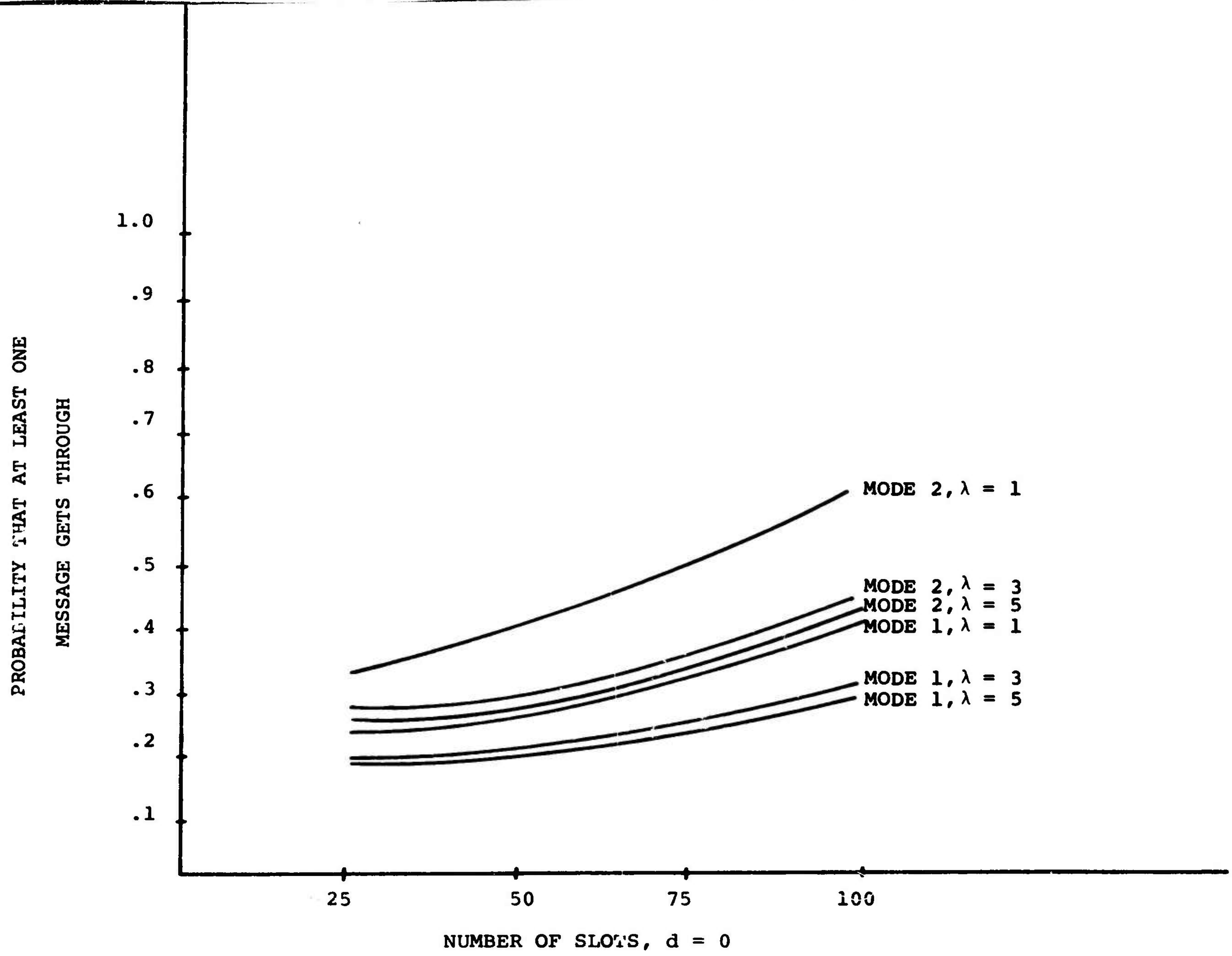

PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
MODE 2, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 1
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
NUMBER OF SLOTS, d = 0

FIGURE 19

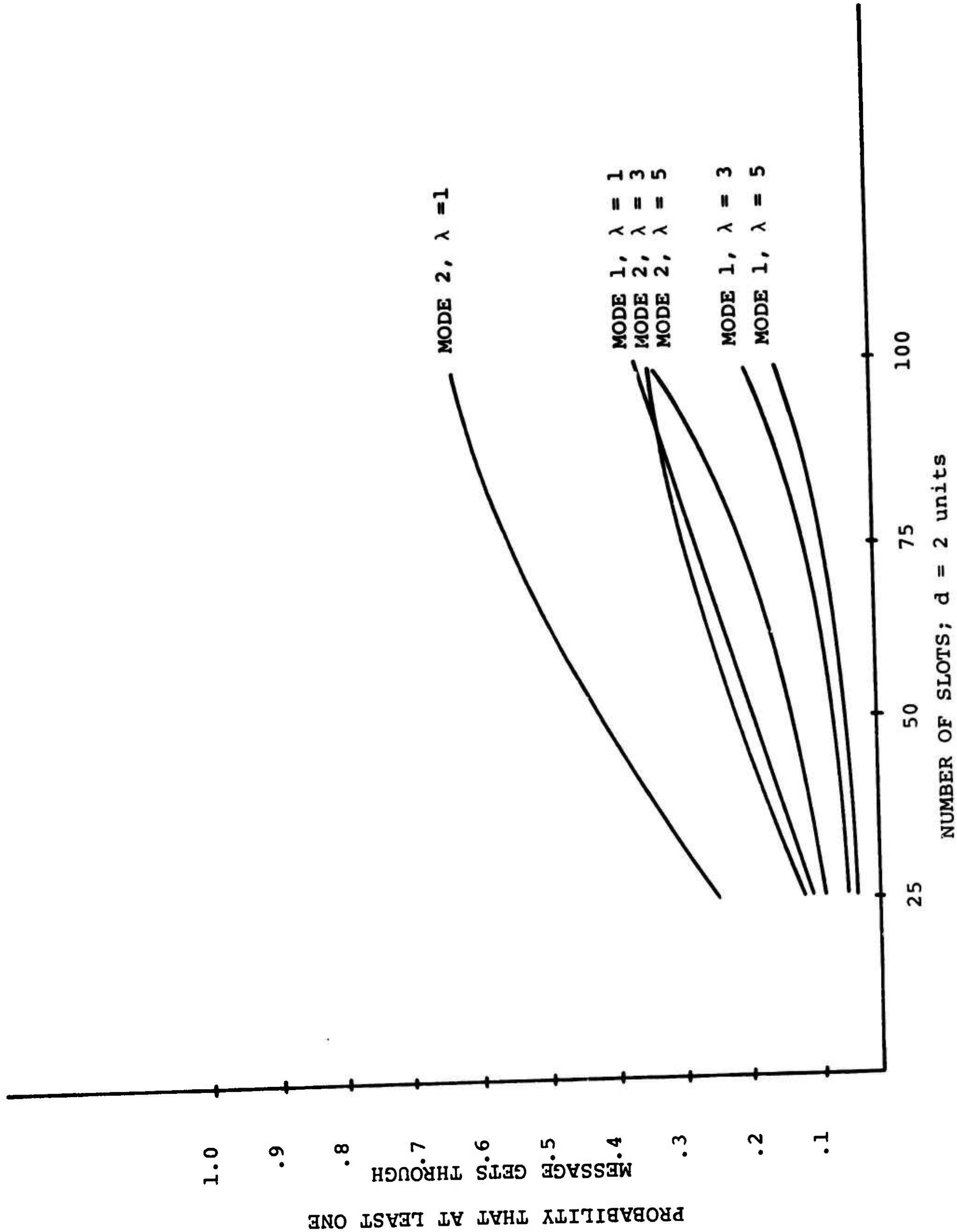

MODE 2, λ =1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
NUMBER OF SLOTS; d = 2 units
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
PROBABILITY THAT AT LEAST ONE MESSAGE GETS THROUGH

FIGURE 20

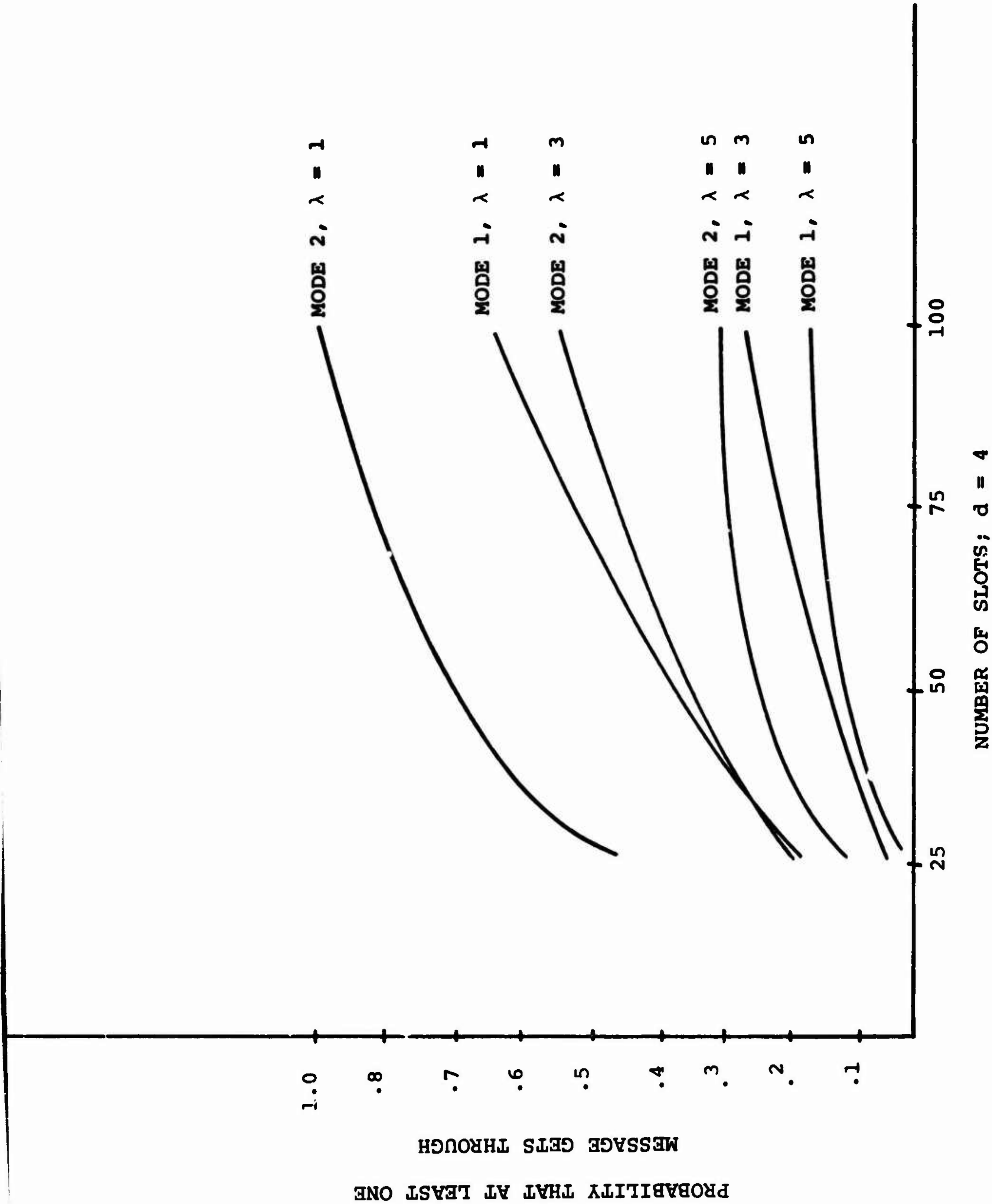

MODE 2, λ = 1
MODE 1, λ = 1
MODE 2, λ = 3
MODE 2, λ = 5
MODE 1, λ = 3
MODE 1, λ = 5
25
50
75
100
1.0
.8
.7
.6
.5
.4
.3
.2
.1
NUMBER OF SLOTS; d = 4
PROBABILITY THAT AT LEAST ONE
MESSAGE GETS THROUGH

FIGURE 21

10. SYSTEMS WITH RETRANSMISSIONS

10.1 Retransmission From Source of Origination

We now can extend the scope and generality of the basic model by including the possibility of retransmission of messages which are erased in random slotting or in technical language, "not captured". The notion of retransmission can be modelled in at least two ways. The first way, considered in this section is that when a message is wiped out, it is retransmitted from its source of origination after a fixed delay time J(d) which depends on the distance of origination from the ground station. The second type of retransmissions which we shall consider are retransmissions which occur at the point (repeater) of erasure at one time unit after wipeout. The latter type will be analyzed in Section 7.

To begin to develop programmable equations, we need some notation:

Let $X_{(i,j),(u,v)}(t)$ be the number of messages arriving at (i,j) at time t which originated at (u,v) at time t - (u-i). (recall that the first coordinate refers to distance from the origin).

Let $Y_{(i,j),(u,v)}(t)$ be the number of messages accepted at (i,j) at time t which originated at (u,v) at time t - (u-i).

Let $X_{(i,j)}(t)$ be the number of messages arriving at (i,j) at time t.

Let $Y_{(i,j)}(t)$ be the number of messages accepted at (i,j) at time t.

Let $Z_{(i,j)}(t)$ be the number of retransmissions at (i,j) at time t.

We develop equations to compute $Z_{(i,j)}(t)$. To begin, we have the following assumptions.

$$X_{(i,j),(i,j)}(t) = \text{Poisson Variate} + Z_{(i,j)}(t) \quad (30)$$

$Y_{(i,j),(i,j)}(t) = X_{(i,j)}(t)$ randomized over mode 1 or mode 2 distribution. (31)

Obviously;

$$X_{(i,j)}(t) = \sum_{(u,v)\varepsilon I(i,j)} X_{(i,j),(u,v)}(t) \quad (32)$$

Where $I_{(i,j)}$ is the set of all repeaters which arein the input set to (i,j), i.e., all repeaters for which there exists <u>directed</u> path <u>to</u> (i,j).

If we assume that each arriving message is equally likely to be accepted, it follows that:

$$Y_{(i,j),(u,v)}(t) = Y_{(i,j)}(t) \frac{X_{(i,j),(u,v)}(t)}{X_{(i,j)}(t)}, \quad (33)$$

and that

$$Z_{(i,j)}(t) = \sum_{\substack{(u,v) \text{ in} \\ O(i,j)}} \left[X_{(u,v),(i,j)}(t-J(i)) - Y_{(u,v),(i,j)}(t-J(i))\right], \quad (34)$$

where $O_{(i,j)}$ is the "outward set of (i,j)" defined as the set of all repeaters which receive messages from (i,j), including (i,j) itself. The quantity J(i) is the delay factor which depends on i the distancefrom the origin, but is independent of the source of message wipeout.

The only part of the equation(9.5)which is not accounted for is $X_{(u,v),(i,j)}(t)$. The next equation is obvious:

$$X_{(i,j),(u,v)}(t) = \sum_{\substack{(K,W) \text{ in} \\ II_{(i,j)}}} Y_{(K,W),(u,v)}(t-1); \quad (35)$$

where $II_{(i,j)}$ is the immediate input set to (i,j), that is those repeaters one unit of distance further than (i,j) which repeat to (i,j) in one time unit.

The equations (30) to (35) were successfully programmed for the square grid net of repeaters at the lattice points of the Euclidean plane, five units or less distance from the origin. This net has a total of 61 repeaters. We do not include numerical data since many time points must be computed to obtain meaningful steady state results. This can be done at any time since the program is available.

10.2. Retransmissions at Point of Loss

In this model we assume that retransmissions of wiped out messages occur at the point of wipeout one time unit later, independently of where the message originated. For this type of assumption, we need to compute, $Z_{(i,j),(u,v)}(s,t)$ the number of retransmissions at (i,j) at time t of messages which originated at (u,v) at time s, s=0, 1, 2,..., $t-|u-j|$. The quantity Z is computed for repeaters (u,v) in the input set to (i,j). The quantities $Z_{(i,j)}(t)$ and $Z_{(i,j),(u,v)}(t)$ are defined as in Section 13.

Since we are assuming that retransmissions occur at the point of wipeout, to compute delays we must keep track of time and place of origination of messages. We therefore define the quantities $X_{(i,j),(u,v)}(s,t)$ and $Y_{(i,j),(u,v)}(s,t)$ as the number of messages arriving and respectively accepted at (i,j) at time t which originated at (u,v) at time s.

According to our assumptions, the required quantity can be computed from:

$$Z_{(i,j),(u,v)}(s,t) = X_{(i,j),(u,v)}(s, t-1) - Y_{(i,j),(u,v)}(s, t-1) \quad (36)$$

According to (5), we need to compute X and Y which can be done recursively.