

# EXHIBIT B

GRAPH  
WITH A

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# 1 Graphs

## 1.1 GRAPHS AND SIMPLE GRAPHS

Many real-world situations can be represented by a diagram consisting of a set of points, called vertices, and lines joining pairs of these points. For example, a social network is mainly interested in whether two people are friends, and the manner in which they are connected. A situation of this type is called a graph.

A graph  $G$  is an ordered pair  $(V, E)$  consisting of a nonempty set  $V(G)$  of vertices and an incidence function  $\psi_G$  that assigns to each unordered pair of (not necessarily distinct) vertices  $u$  and  $v$  a set  $\psi_G(u, v)$  of edges. Vertices  $u$  and  $v$  are called adjacent if  $\psi_G(u, v) \neq \emptyset$ .

Two examples of graphs are given below.

### Example 1

Let  $G$  be a graph with vertex set  $V(G) = \{u, v, w, x, y, z\}$  and edge set  $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$  where

and  $\psi_G$  is defined by

$$\begin{aligned} \psi_G(e_1) &= \{u, v\}, \psi_G(e_2) = \{u, w\}, \psi_G(e_3) = \{u, x\}, \\ \psi_G(e_4) &= \{u, y\}, \psi_G(e_5) = \{u, z\}, \psi_G(e_6) = \{v, w\}, \\ \psi_G(e_7) &= \{v, x\}, \psi_G(e_8) = \{v, y\}, \psi_G(e_9) = \{v, z\}, \\ \psi_G(e_{10}) &= \{w, x, y, z\}. \end{aligned}$$

### Example 2

where

and  $\psi_H$  is defined by

$$\begin{aligned} \psi_H(a) &= \{u, v\}, \psi_H(b) = \{u, w\}, \\ \psi_H(c) &= \{u, x\}, \psi_H(d) = \{u, y\}, \\ \psi_H(e) &= \{v, w\}, \psi_H(f) = \{v, x\}, \\ \psi_H(g) &= \{v, y\}, \psi_H(h) = \{v, z\}, \\ \psi_H(i) &= \{w, x, y, z\}. \end{aligned}$$

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