

EXHIBIT B

GRAPH
WITH A

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1 Graphs

1.1 GRAPHS AND SIMPLE GRAPHS

Many real-world situations can be represented by a diagram consisting of a set of points, called vertices, and lines joining pairs of vertices, called edges. For example, a communication network is mainly interested in whether two vertices are connected, and the manner in which they are connected. A situation of this type is called a graph.

A graph G is an ordered pair (V, E) consisting of a nonempty set $V(G)$ of vertices and an incidence function ψ_G that maps each edge to an unordered pair of (not necessarily distinct) vertices. If u and v are vertices such that $\psi_G(e) = \{u, v\}$, then u and v are called the endpoints of e .

Two examples of graphs are given below.

Example 1

Let G be a graph with vertex set $V(G) = \{u, v, w, x, y, z\}$ and edge set $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ where

and ψ_G is defined by

$$\begin{aligned} \psi_G(e_1) &= \{u, v\}, \psi_G(e_2) = \{v, w\}, \\ \psi_G(e_3) &= \{w, x\}, \psi_G(e_4) = \{x, y\}, \\ \psi_G(e_5) &= \{y, z\}, \psi_G(e_6) = \{z, u\}. \end{aligned}$$

Example 2

where

and ψ_H is defined by

$$\begin{aligned} \psi_H(a) &= uv, \psi_H(b) = vw, \\ \psi_H(c) &= vx, \psi_H(d) = wy. \end{aligned}$$

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