

EXHIBIT B

GRAPH
WITH A

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1 Graphs

1.1 GRAPHS AND SIMPLE GRAPHS

Many real-world situations can be represented by a diagram consisting of a set of points, called vertices, and lines joining pairs of these points. For example, a social network is mainly interested in whether two people are friends, and the manner in which they are connected. A situation of this type is called a graph.

A graph G is an ordered pair (V, E) consisting of a nonempty set $V(G)$ of vertices and an incidence function ψ_G that maps each edge to an unordered pair of (not necessarily distinct) vertices u and v such that $\psi_G(e) = uv$. The vertices u and v are called the endpoints of the edge e .

Two examples of graphs are given below.

Example 1

Let G be a graph with vertex set $V(G) = \{u, v, w, x, y, z\}$ and edge set $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ where

and ψ_G is defined by

$$\begin{aligned} \psi_G(e_1) &= uv, \psi_G(e_2) = vw, \\ \psi_G(e_3) &= wx, \psi_G(e_4) = xy, \\ \psi_G(e_5) &= yz, \psi_G(e_6) = zu. \end{aligned}$$

Example 2

where

and ψ_H is defined by

$$\begin{aligned} \psi_H(a) &= uv, \psi_H(b) = vw, \\ \psi_H(c) &= wx, \psi_H(d) = xy, \\ \psi_H(e) &= yz, \psi_H(f) = zu. \end{aligned}$$

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